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STUDENTS' ERRORS IN SOLVING MATHEMATICAL WORD PROBLEMS AND THEIR ABILITY IN IDENTIFYING ERRORS IN WRONG SOLUTIONS

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Abstract:
Solving mathematical word problems is a difficult, complex and intellectual activity. The formation of problem solving skills is much more difficult than calculation skills because the problems are related to many concepts and a lot of mathematical relationships. Therefore, students are inevitable to commit errors to deal with them. This study shows the errors of 160 3rd grade students regarding some kinds of word problems in Mathematics 3 in Vietnam. The results indicate that children commit many errors due to many different reasons such as: subjectivity, carelessness, wrong application of the calculation rules, incorrect identification of problem kinds and wrong calculation.

Keywords: errors, reasons, ability, mathematical word problems, problem solving

1. Introduction

1.1 Works related to problem solving and error

Problem solving is a major activity in the teaching of mathematics, so it attracts many researchers. Also, the problem solving process is rather sophisticated, so students must go through several stages before giving a solution. Polya (1973) supposed that problem solving has four stages: understanding the problem, planning the strategy, implementing the plan, and reviewing the answers. However, according to Newman (1977) problem solving consists of five steps: reading, comprehension, transformation,

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processing skills, and encoding. Here, the author gave an error analysis model related to these five elements. In addition to these four errors, other authors added language errors and careless errors (Junaedi et al., 2015). It is said that the steps above are not easy for many students, leading to difficulties for them. Tambychika and Meerahb (2010) studied the difficulties of the students in solving mathematical problems. The results showed that the students lacked many mathematical skills such as real numbers, visual and spatial information skills. Information skills were the most important. The lack of mathematical skills and cognitive abilities in learning inhibited mathematical problem solving. Besides, some difficulties students can meet in problem solving are reading and interpreting the data, determining and delegating data and making conclusions and arguments (Susanti et al., 2014). Meanwhile, Ndalichako (2013) said that the causes for difficulties in problem solving include the lack of understanding of the appropriate procedures to apply to problem solving, the complexity of the task, and process generalization of procedures even in inappropriate situations.

From the difficulties encountered, students often make errors for various reasons. Abdullah (2015) studied the errors of students regarding fractional topics. The results showed that students faced the problem of establishing relationships and implementing solution strategies. The Nigerian authors, Ekwueme and Ali (2012) divided mathematical errors into 4 kinds, such as: arbitrary, structural, executive and clerical errors. They surveyed 400 students on these errors. The results showed that the students committed the dominant structural error and there were also girls who made more mistakes in the process than boys. Another study of errors related to the topic fraction was conducted by Raduana (2010). He used Newman Error Analysis Model to interpret the errors of 374 students. The results showed that students made most types of errors, so he suggested that the teacher should ask the students about the problems they encountered during problem solving. In their study of errors in addition and subtraction of fractions among students in secondary schools, Idris and Narayanan (2011) divided errors into 3 types: a) careless errors, b) negligence errors, and c) systematic random errors. However, Munasinghe (2013) only considered systematic errors, random errors and careless errors in the study of error patterns related to "addition" in primary schools in Sri Lanka.

Similarly, Sarwadi and Shahrill (2014) investigated students' errors to learn fractions and decimals. The results showed that students' misconceptions seemed to have a significant impact on their progress and achievement in the exam. The findings also showed that students' errors and misconceptions were not only different but also had different causes. The causes of students' errors may be due to psychological or

incorrect guidance from parents or family, teacher's unawareness of the students in the classroom or the social and economic background of parents (Munasinghe, 2013).

A case study of 9th grade students' errors regarding mathematical word problems was done by Sepeng and Sigola (2013). Their data obtained proved that the errors made by the learners in the solution of the word problems appeared as a result of the lack of understanding of the mathematical vocabulary used in a problem statement. Additionally, the authors, Veloo, Krishnasamy & Abdullah (2015) studied the errors of the 10th grade students involved with symbols, graphs and problem-solving in mathematics. Some popular errors were committed by 315 students such as: conceptual, careless problem-solving and value errors. In addition, the researchers pointed out various causes for the above errors. Also, the authors implicitly introduced students to the role of graphs in real life.

Loc investigated errors of 12th grade students in calculating integrals in 2014 and solving the problems of coordination method in space in 2015. The findings revealed that students made many mistakes on related topics. He also stated that student's error analysis would help teachers recognize students' errors, adjust teaching methods, and then conduct in-depth research on relevant topics.

Students' errors in problem solving are inevitable, what ways do teachers use to help students identify and correct them? In their study, Ekwueme and Ali (2012) suggested that students should do the correct answer search and highlight the importance of the steps involved in getting to the right answers. Also, it is said that bringing Newman's Error Analysis model into the classroom assessment program is a teaching tool to assess, analyse, and interpret students' difficulties with mathematical word problems (White, 2010). Moreover, Chamundeswari (2014) studied the errors students in secondary schools made about algebra operations. Some pedagogical measures he recommended are more home assignments, periodical checking, rigorous practice and drilling at the initial stage, innovative methods and technique.

1.2 Mathematical word problems in the textbook *Mathematics 3* in Vietnam

Mathematical word problems in the textbook *Mathematics 3* in Vietnam are more diversified and updated than ever before, referring to the real life. For example, in addition to "traditional" problems, there are "multiple choice" problems in Mathematics 3, the problem of tabulation related to statistics. The problems of time, length, volume, money, etc., mostly reflect the daily activities (morning, afternoon, evening) with normal skills (weigh, measure, count...). It can be said that each problem in the textbook is often a real situation students need to know and can solve.

1.3 Some kinds of mathematical word problems and expected solutions in the textbook *Mathematics 3* in Vietnam

- Find equal portions of a number.
- Multiples of a number.
- Portions of a number.
- Solving problems with two operations.
- By how many times is the bigger number greater than the smaller number?
- What portion of the bigger number is the smaller number?
- Problems of reducing to units.
- Quantitative and metric problems.
- Problems with geometric content (calculating perimeter, area of rectangle, square).

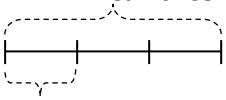
A. Kind 1: Find equal portions of a number

Problem in *Mathematics 3*: A girl has 12 candies. She gave her younger sister $\frac{1}{3}$ of the number of candies. How many candies did she give her sister?

Summary:

12 candies

is:



? candies

Solution

The number of candies she gave her younger sister

12 : 4 = 3 (candies)

Answer: 4 candies.

B. Kind 2: Multiples of a number

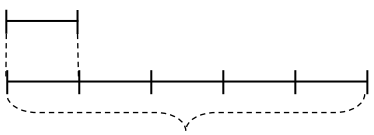
Problem in *Mathematics 3*: Segment AB is 2cm long; segment CD is 5 times longer than AB. How long is segment CD in cm?

Summary**:**

2cm

Segment AB:

Segment CD:



?cm

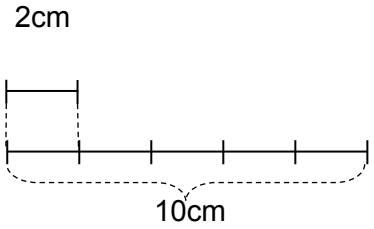
Solution

The length of segment CD is: 2 x 5 = 10 (cm)

Answer: 10cm

The textbook raises a rule: *We multiply a number by the times when we want to increase it*.

C. Kind 3: Portions of a number

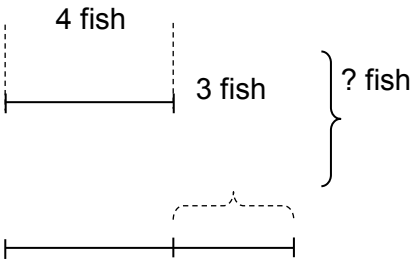


The length of segment CD is: 10cm
The length of segment AB is: 2cm
We have the length of segment AB when we decrease 5 times the length of segment CD
The textbook raises a rule: *We divide a number by the times when we want to decrease it*.

D. Kind 4: Solving problems with two operations

Problem in *Mathematics 3*: There are 4 fish in aquarium 1; there are 3 more fish in aquarium 2. How many fish are there in two aquariums?

Summary:

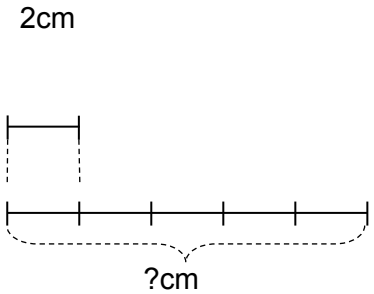


The number of fish in Aquarium 2 is: 4 + 3 = 7 (fish)
The number of fish in two aquariums is: 4 + 7 = 11 (fish)
Answer: 11 fish.

E. Kind 5: By how many times is the bigger number greater than the smaller number?

Problem in *Mathematics 3*: Segment AB is 2cm long, segment CD is 10cm long. How many times is segment CD longer than segment AB?

Summary**:**



Solution

How many times segment CD is longer than segment AB by: 10 : 2 = 5 (times).

Answer: 5 times

F. Kind 6: What portion of the bigger number is the smaller number?

Problem in *Mathematics 3*: Mother is 30 years old; her child is 6 years old. What portion of the mother's age is the child's?

Solution

The mother's age is greater than her child's by: 30 : 6 = 5 (times)

Therefore, the child's age is $\frac{1}{5}$ his mother's age.

Answer: $\frac{1}{5}$.

G. Kind 7: Problems of reducing to units

Problem in *Mathematics 3*: There are 35*l* of honey, divided equally into 7 cans. How many litters of honey are there in two of those cans?

Summary:

7 cans: 35*l*

2 cans: …*l*?

Solution

The number of liters of honey contained in each can is:

35 : 7 = 5 (*l*)

The number of liters of honey contained in 2 cans is:

5 x 2 = 10 (*l*)

Answer: 10*l* of honey.

Based on the problem solving process of the above authors and the solutions in Mathematics 3, we develop it to suit the context of Vietnam. This process usually includes 4 steps as follows:

- Reading the statement of the problem carefully: read the problem carefully, think about the problem, pay particular attention to the question of the problem, do not rush to calculate without reading the problem.
- Summarizing of the problem for establishing the relationship between given numbers by language, symbol, summarizing the conditions of the problem, or illustrating these conditions by diagrams, drawings.

- Analyzing the problem to plan a solution: Think about, to answer the question of the problem, what to know, what to do? Think about what the given numbers and conditions of the problem, what calculations can be performed, which calculations can help answer the question of the problem? Based on that, think to set the problem solving sequence.
- Doing calculations in sequences that have been set up and write the solution: Whenever a calculation is performed, it is necessary to check that the calculation is correct. Solving the problem must try to find out whether the answer has correctly answered the question of the problem. Is it consistent with the conditions of the problem?

In general, the above process requires a great deal of analysis, synthesis and problem-solving skills that not all students have, so they can make errors in solving word problems. From grade 1 to grade 2, students are already familiar with solving mathematical problems, but they are just simple ones. In the third grade, students begin to encounter the problems with more complex relationships, which must be done in two steps, so they are often embarrassed to recognize and analyze problems as well as apply the methods of solving, which leads to unfortunate errors.

2. Research questions

Q1: Do students make errors when solving mathematical word problems in *Mathematics 3*? What are the causes of the errors?
Q2: Do students identify errors in wrong solutions?

3. Research methodology

3.1 Participants

The survey included 160 3rd grade students in SocTrang province and they had learned the kinds of word problems in the textbook *Mathematics 3.*

3.2 Instrument and procedure

We designed six questions related to the above kinds of problems, including 2 questions for students to solve the problems, four questions with the given solutions, and students have to judge whether each question is right or wrong and indicate the wrong place if it's wrong.

In order to analyse and interpret data, we calculated the frequency and analysed students' works, thus pointing out the causes of their errors.

3.3 6 questions

Item 1: 72kg of rice are loaded evenly with 8 bags. How many bags are 54kg of rice loaded evenly in?

Item 2: There are 16m of fabric; 3m are required to sew a costume. How many costumes can be sewn at most and how many meters are left?

Item 3: Nam has 16 postcards. NAM has 7 fewer postcards than THANG. How many postcards do both of them have?

LAN solved the above problem as follows:

Solution

The number of postcards THANG has: 16 – 7 = 9 (postcards)

The number of postcards NAM and THANG has: 16 + 9 = 25 (postcards)

Answer: 25 postcards

According to you, is LAN's solution so right or wrong? If wrong, please point out her wrong place.

Item 4: There are 6 books on the upper shelf and 24 in the lower one. What portion of the number of books in the lower shelf is the number of books in the upper one?

NAM solved the above problem as follows:

Solution

The number of books on the lower shelf is greater than the number of books in the upper one by: 24 : 6 = 4 (books)

Answer: $\frac{1}{4}$

According to you, is NAM's solution so right or wrong? If wrong, please point out her wrong place.

Item 5: NAM has 10 calculator sticks. NAM has double the number of calculator sticks HUNG has. How many calculator sticks does HUNG have?

HA solved the above problem as follows:

Solution

The number of calculator sticks HUNG has is: 10 x 2 = 20 (calculator sticks)

Answer: 20 calculator sticks

According to you, is HA's solution so right or wrong? If wrong, please point out her wrong place.

Item 6: A rectangular parcel of land has the length of 7m. Its width is shorter 2m than its length. Find the perimeter of this land parcel.

AN solved the above problem as follows:

Solution

The width of the rectangular is: 7 – 2 = 5 (m)

The perimeter of the rectangular is: 7 x 5 = 35 (m)

Answer: 35m

According to you, is AN's solution so right or wrong? If wrong, please point out her wrong place.

3.4 The correct answers expected for 6 questions in the context of Vietnam

Table 1: The correct answers expected for 6 questions

Questions	The correct answers expected for 6 questions
1	The number of kilogram of rice in a bag is: 72 : 8 = 9 (kg) The number of bags which 54kg of rice loaded evenly in is: 54 : 9 = 6 (bags) Answer: 6 bags
2	We have: 16 : 3 = 5 (remainder 1) The number of costumes can be sewn is: 5 (costumes) The number of fabric meters left is: 1 (m) Answer: 5 costumes ; 1m
3	LAN's solution: wrong Reason: to find the number of THANG's postcards, she have to plus instead minus
4	NAM's solution: wrong Reason: 24 : 6 = 4 (times) instead of 4 (books)
5	NAM's solution: wrong Reason: 10 : 2 = 5 (sticks) instead of 10 x 2 = 20 (sticks)
6	AN's solution: wrong Reason: 7 + 5 = 12 (m) instead of 7 x 5 = 35 (m); to find the perimeter of the rectangular instead of its area

Table 2: The objectives of 6 items

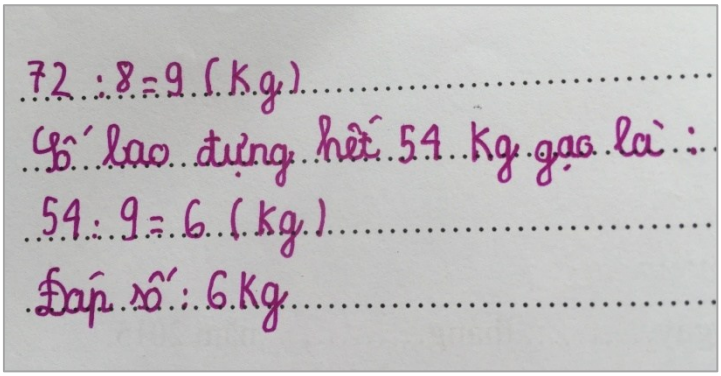
Items	Aim to answer the questions
Items 1 and 2	Q1: Do students make errors when solving mathematical word problems in *Mathematics 3*? What are the causes of the errors?
Items 3, 4, 5 and 6	Q2: Do students identify errors in wrong solutions?

4. Results and discussion

Table 3: The percentages of students with correct / incorrect answers

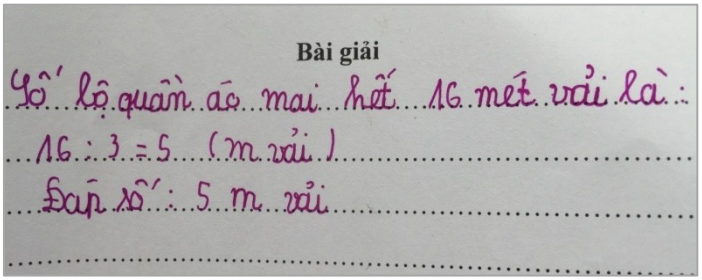
Items	The number of students with a correct answer	The number of students with an incorrect answer
1	105 (65.63%)	55 (34.37%)
2	130 (81.25%)	30 (18.75%)
3	128 (80.00%)	32 (20.00%)
4	155 (96.88%)	5 (3.12%)
5	121 (75.63%)	39 (24.37%)
6	136 (85.00%)	24 (15.00%)

Item 1: To test students about their ability to identify, analyse, and solve "problems of reducing to units". According to Table 3, the results of the survey included 105 students with the right answer, accounting for 65.63%. There were 55 students doing wrong, accounting for 34.37%; the wrong reason was that 10 students did not remember the steps to solve the problem (step 1 to find the value of a part). 31 students were confused in the second step (in the second step, we had to do the $54:9$ but students did 54×9) while 14 students wrote wrong unit (in step 1, the unit had to be "kg" but the students wrote "bag", in step 2, the unit was "bag", however the students wrote "kg"). The students' performance in item 3 showed that the students had not clearly defined the difference between the two types of problems. The main reason was that they were not sure about the solution method and their ability to deduce and analyse the problems was still weak.



Item 6: The question was to test the student's ability to solve the problem in the form of "Division with remainders". It was clear to say that 130 students did the right operation, accounting for 81.25%; while 30 students with wrong answers, accounting for 18.75%. That the reason was they were still subjective and reckless. 21 students made the wrong division $16:3$; children calculated out of 8 or 3. 5 students calculated correctly but their

conclusion was wrong. Besides, 4 students did not know how to solve the problem. Specifically, they did two solution steps, in the step 1 they did 16×3 and in the step 2 they did $48 : 3 = 16$). This showed that this kind of problem was not difficult for students, but still students did wrong because mainly they calculated carelessly and rushed to record the conclusion without reviewing the problem.

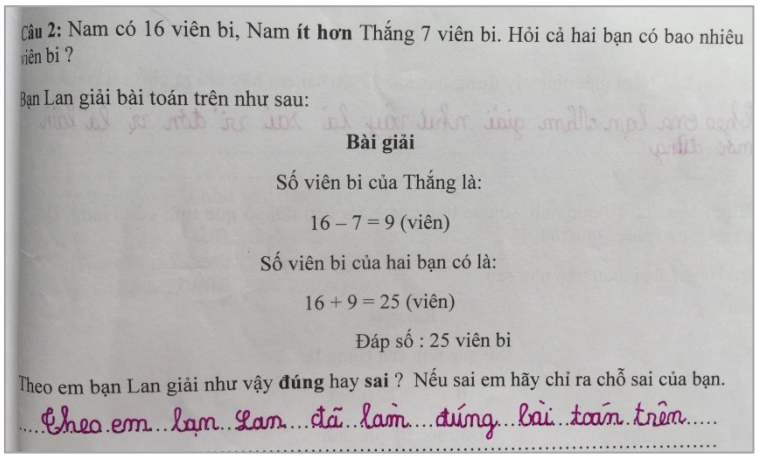

Bài giải

Số lộ quần áo mai hết 16 mét vải là:

16 : 3 = 5 (m vải)

Đáp số: 5 m vải

Item 3: To test students on reading and problem solving skills, when solving the "More, less" problems. There were 128 students with exact explanation in this question, accounting for 80.00% of the student's total. The results showed that students had good knowledge and skills in problem analysis. However, there were still 32 students with incorrect explanation, accounting for 20.00%, mainly due to their subjectivity and carelessness when reading the requirements of the problem. Furthermore, their analytical ability was limited.


Câu 2: Nam có 16 viên bi, Nam **ít hơn** Thắng 7 viên bi. Hỏi cả hai bạn có bao nhiêu viên bi ?

Bạn Lan giải bài toán trên như sau:

Bài giải

Số viên bi của Thắng là:

16 – 7 = 9 (viên)

Số viên bi của hai bạn có là:

16 + 9 = 25 (viên)

Đáp số : 25 viên bi

Theo em bạn Lan giải như vậy **đúng** hay **sai** ? Nếu sai em hãy chỉ ra chỗ sai của bạn.

Theo em bạn Lan đã làm đúng bài toán trên

Item 4: To test students' knowledge of problem solving methods in the form "What portion of the bigger number is the smaller number? ". The results of the survey included 155 students who did the right question, accounting for 96.88. This showed that students mastered the solution rule that teachers had provided in the previous lesson. Nevertheless, there were still 5 students doing wrong, accounting for 3.12% of

the total of the students' works. The reason was that they did not read the requirements of the problem and assumed that it had 2 solution steps.

Item 5: The item was to test students in reading, analysing, and solving problems in the form of "Multiples of a number". The survey consisted of 121 students who had the right answer, accounting for 75.63% and 39 students using the wrong rule, accounting for 24.37%. Their ignorance led to reading the requirements of the problem carelessly (10 students answered that this problem must be two solutions, to find the number of Nam's sticks first). Additionally, they read the problem without thinking, they misunderstood the issue of the problem (29 students think that the given solution is right after finishing readingit). This showed that the students were still subjective, reckless in doing assignments and did not master the knowledge.

Item 6: The item's aim was to test the student's applicability of the rectangular area rule to a specific problem. The results of the survey revealed that 136 students answered this question rightly, accounting for 85.00%. This proved that they mastered the learnt theoretical knowledge. However, there were also 24 students making a mistake, accounting for 15.00%. They committed errors because of their confusion between calculating perimeter and area of the rectangle.

4. The answers to Q1: Do students make errors when solving mathematical word problems in *Mathematics 3*? What are the causes of the errors?

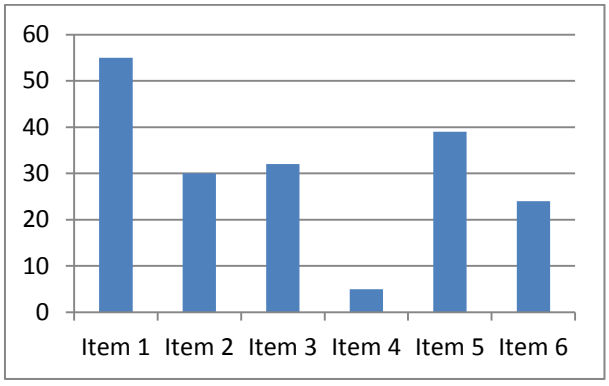


Figure 1: The number of students with wrong answers in 6 items

As shown in Figure 1, item 1 had the most number of students with wrong answers among the six items. This item had the answer with two operations, which made it difficult for students. Next, item 5 also had many students making mistakes because they did not recognize the "multiplication" relationship. Conversely, item 4 had the least number of students because most students discovered the wrong thing in NAM's solution was to write the wrong unit.

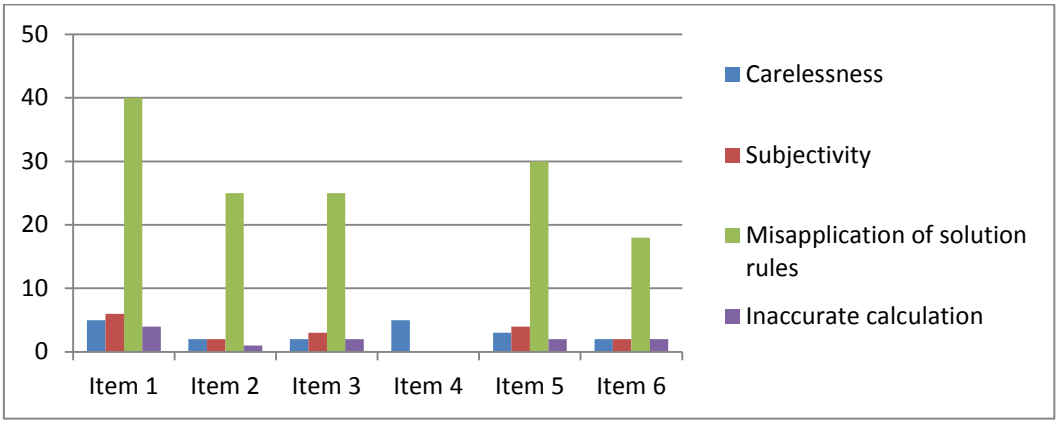


Figure 2: Reasons for students' errors in 6 items

According to Figure 2, the main cause of the errors was a misapplication of solution rules. In most of the 6 items, students committed errors because they did not master the solution rules. This could be explained as they were governed by a lot of rules for word problems in Grade 3, leading to confusion inevitably. Meanwhile, other causes of errors were not many.

4.2 The answers to Q2: Do students identify errors in wrong solutions?

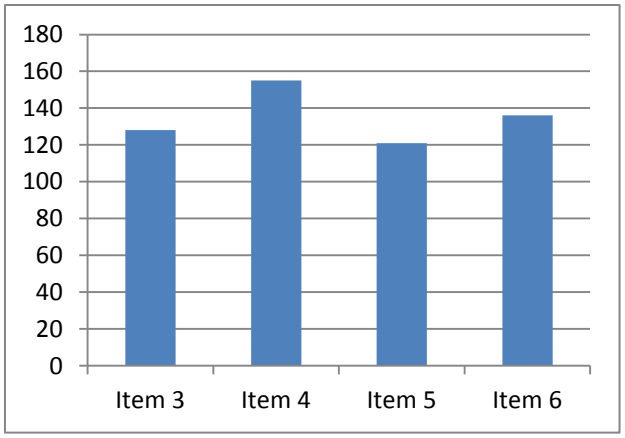


Figure 3: The number of students identifying errors in 4 items

As shown in Figure 3, the majority of the students identified the errors contained in the given answers, in particular, 155/160 students found the "unit" error in item 4. Moreover, they also corrected those errors.

5. Conclusion

In general, the findings of the survey show that a large number of students have mastered the knowledge, so they use it to analyse the hypothetical solution effectively. Besides, they also successfully solve the problems without the solutions as other

questions. However, some children make critical errors. These errors are caused by many different reasons such as: carelessness, subjectivity, misapplication of solution rules and inaccurate calculation. From here, teachers are also interested in some measures to help them prevent and correct errors. Some measures are suggested as follows:

- Paying attention to the basic knowledge when teaching new lessons.
- Studying carefully the lectures suitable for each student.
- Performing clear and specific steps, do not turn off the steps when practicing the solution.
- Carefully analysing the cause of the error to promptly correct and let students do the same exercises.
- Being interested in regularly reviewing, consolidating and systematizing the knowledge and skills in solving mathematical word problems.

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