



SKILLS AND ERRORS ANALYSIS IN READING GRAPHS AMONG HIGH SCHOOL LEARNERS

Jhette Althea C. Jadloc,
Noreen Mae D. Tumamao,
Alma Mae A. Aguisando,
Marian Kaye F. Gallegoⁱ
University of Mindanao,
Davao City, Philippines

Abstract:

This study examined the skills and errors of high school students in reading graphs, specifically pie charts, bar graphs, line graphs, histograms, and ogives, at the University of Mindanao, Matina, Davao City. Anchored in Curcio's Perspective Theory and Bertin's Matrix Theory of Graphics, the research employs a descriptive qualitative design to assess students' graph literacy across three levels of reading: literal, interpretative, and extrapolative. Using validated researcher-made questionnaires and thematic analysis, findings revealed that a majority of students demonstrated low proficiency in graph interpretation, particularly at the extrapolative level. Common errors included misreading data entries, miscalculating angles and frequencies, constructing inaccurate graphs, and providing insufficient explanations or inferences. These findings highlight critical gaps in students' graphical comprehension and reasoning, underscoring the need for explicit instruction in data interpretation and visual communication. The results aim to inform instructional practices and curriculum development, enhancing students' graph literacy.

Keywords: graph reading skills, error analysis, literal reading, interpretative reading, extrapolative reading, mathematics education in the Philippines

1. Introduction

Reading graphs enables students to understand what the graph represents by identifying data values of interest and drawing a summary of the information in the graph (McKenna *et al.*, 2023). Understanding how to read graphs is fundamental because it improves geometry learning (Žakelj and Klančar, 2022). Reading the graphs is the first step, but proper understanding requires interpreting the data, making inferences, and grasping the underlying trends and patterns. According to the results of the Trends in

ⁱ Correspondence: mgallego@umindanao.edu.ph

International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA), students encounter difficulties in interpreting and constructing graphs (Bursal & Yetis, 2020). Furthermore, students struggle with reading graphs and tend to score low on graph reading assessments. This implies that students struggle and have difficulties comprehending graphs at more advanced levels when interpreting between and beyond the data (Ozmen *et al.*, 2020).

Researchers assessed students' graphical proficiency to identify areas of weakness and challenge in graphical comprehension. It focused on challenges related to variable identification, connection, selecting the right graph for a given set of data, pattern analysis, and data presentation through graph reasoning (Stefanel, 2019; Glazer, 2011; Taylor, 2010). However, students in high school have also reported errors (Ibnatul *et al.*, 2021; Lynch *et al.*, 2000). The study by Idehen (2020) and Saidi and Siew (2019) indicates that the errors generated by secondary school students primarily stem from challenges in selecting an appropriate representation and understanding fundamental concepts in graph analysis.

2. Literature Review

This study is grounded in the theories of Curcio's Perspective Theory (1987) and Bertin's Matrix Theory of Graphics (2001). Curcio (1987) proposed three levels of graph interpretation: reading the data (finding explicit information), reading between the data (uncovering the relationship), and reading beyond the data (making predictions or inferences). The skills of high school learners in reading graphs, pie charts, bar graphs, line graphs, histograms, and ogives are determined based on the three stages of graph reading and the three levels of graph interpretation. Moreover, Bertin proposes three stages involved in reading a graph: external identification (understanding the title, axis labels, and scale), internal identification (recognizing bars, lines, or dots), and perception of pertinent correspondences (the combination of external and internal features to comprehend the information). Once the graph has been read, these three stages in reading a graph can identify single pieces of information, notice patterns, and draw conclusions. Furthermore, Curcio's Perspective Theory states that the graph interpretation of students is influenced by students' prior knowledge, mathematical content, and graphical form.

This research examines the skills and errors that high school learners exhibit in reading graphs. Learning different types of graphs is vital to study, and all graphs are essential in today's world, especially in mathematics. However, it is crucial to have a technique that incorporates concepts and skills from various other practices, such as modeling, inquiry, experimentation, and visualization. Furthermore, visual model data may result from observation and experimentation (Gardner *et al.*, 2022; Pelaez *et al.*, 2022). Students frequently make two common errors while graphing: first, they misread the graph because they struggle to understand the problem; second, they misinterpret it due to unfamiliarity with the graph, which leads to confusion (Whitaker & Jacobbe, 2017). Additionally, it is established that this graph enhances the standard of education and

inspires students to acquire relevant skills and capabilities (Kusumaningpuri & Fauziati, 2021).

One previous study investigated middle school students' graph literacy based on cognitive and affective factors, examining the differences and the basis of these differences in students' viewpoints when reading different types of graphs. However, the study is limited to Turkish learners, and graph literacy, influenced by cognitive and affective factors, is a frequently examined topic. Future researchers must not only focus on the impact of these cognitive and affective factors on graph literacy but also address the failure associated with using different types of graphs and the aversion students may have towards certain graphs (Bursal & Polat, 2020).

Moreover, Bahtaji (2020) noted that many students struggle with reading and interpreting graphs, as they lack the necessary skills in the Philippine context. Implementing supportive graphical interventions is considered the most effective strategy for guiding and enhancing students' comprehension of graphs, ultimately contributing to an overall improvement in their learning. Additionally, according to the study by Galabo *et al.* (2018), first-year students exhibit significantly low mean scores in graph-related tasks, indicating poor performance in mathematics. This highlights the importance of incorporating graphs into intervention plans. It also suggests that graphs should be considered when solving mathematical problems, especially since high school students struggle to read, interpret, or construct graphs.

The previous research, "Determining the Graphical Literacy Levels of 8th Grade Students," primarily concentrated on the graphical literacy levels of 8th-grade students in areas such as reading, interpreting, drawing, comparing, and evaluating characteristics. This type of study has become increasingly prevalent, but it does not delve into the complexities of interpreting graphs. The present study examines the graph-reading skills and errors of high school students using advanced graph problems. The goal is to identify common skills and errors among students when reading graphs. This information helps teachers develop effective teaching strategies for discussing graphs, ultimately enhancing student engagement and improving their mastery of reading graphs. Understanding the skills and challenges also aids educational sectors in implementing necessary changes in the curriculum, particularly given that time constraints can hinder teachers from fully addressing competency goals during math lessons. Students are also better equipped to recognize areas needing improvement when reading graphs and learn to apply contextual knowledge of graphs more effectively.

This study aims to analyze the skills and errors in reading graphs among high school learners. Specifically, it seeks to answer the following questions: to ascertain the reading graph skills of high school students in terms of pie charts, bar graphs, line graphs, histograms, and ogives, and to determine the errors in reading graphs of high school learners.

3. Material and Methods

3.1 Research Respondents

The respondents in this study are high school students from the University of Mindanao, Matina Campus, Davao City, with a total population of 154. The researchers employ fishbowl sampling, a suitable method for randomly selecting 26 respondents from the population. The researchers initially compile a list of all potential participants. Each student's name or identification is written on a piece of paper and placed into a small container. The slips are well shuffled to ensure randomization. After mixing the slips, the researchers randomly drew the desired number of participants, 26 in this study, one at a time without looking into the container. Each drawn slip is recorded and removed from the container to prevent duplication, ensuring that sampling is done without replacement. Qualitative research typically requires 20 to 60 surveys to achieve data saturation (Bekele & Ago, 2022).

Moreover, the respondents must be high school students officially enrolled at the University of Mindanao, Matina Campus, Davao City, and who have undergone lessons on graphs. Students from other branches or schools, as well as those who have dropped out, are excluded. The researchers allow participants to withdraw from the study.

3.2 Research Instruments

In this study, qualitative data are collected by administering researcher-made questionnaires to measure high school learners' skills and errors in reading graphs. The first part of the questionnaire profiles participants (name, gender, age, and grade level), and the data comprises five main questions and two open-ended probing questions. The qualitative data are checked using the rubrics outlined in Curcio's Perspective Theory (1987).

Table 1: Rubric for Assessing Students' Graph Reading Skills

Level	Literal Reading (reading the data)	Interpretative Reading (reading between the data)	Extrapolative Reading (reading beyond the data)
4	Students provide fully correct information/justification directly from the graph (e.g., identifying values or trends).	Students provide fully correct comparisons, patterns, and drawing relationships between data points.	Students provide fully correct predictions or inferences based on the trends and patterns in the graph.
3	Students provide correct information/justification directly from the graph (e.g., identifying values or trends).	Students provide correct comparisons, patterns, and drawing relationships between data points.	Students provide correct predictions or inferences based on the trends and patterns in the graph.
2	Students provide partial/general information/justification directly from the graph (e.g., identifying values or trends).	Students provide partial/general correct comparisons, patterns, and drawing relationships between data points.	Students provide partial/general predictions or inferences based on the trends and patterns in the graph.
1	Students provide incorrect information/justification directly	Students provide incorrect comparisons, patterns, and	Students provide incorrect predictions or inferences

	from the graph (e.g., identifying values or trends).	drawing relationships between data points.	based on the trends and patterns in the graph.
0	Students make no attempt to provide information/justification directly from the graph (e.g., identifying values or trends).	Students make no attempt comparisons, patterns and drawing relationships between data points.	Students make no attempt predictions or inferences based on the trends and patterns in the graph.

Moreover, the questionnaires used in this study have been validated by trusted experts. The rubrics for the qualitative data are validated by trusted validators and mathematics teachers and analyzed using the kappa formula.

3.3 Design and Procedure

In this research, the proponents use a qualitative research design. This study involves gathering data through interviews, observations, focus group discussions, and the analysis of documents and artifacts. These methods enable researchers to collect detailed and descriptive information about participants' experiences, perspectives, and contexts. Qualitative research design also allows researchers to understand and interpret complex data (Jain, 2023). The researchers need to obtain a more comprehensive and in-depth understanding of the results to identify the skills and errors made by the students when reading graphs.

This study utilizes a descriptive qualitative design. The researchers needed to create a factual, general description of the facts, systematic characteristics, and accurate interpretation of the data gathered. Descriptive qualitative research aims to describe necessary information about the primary purpose of the research (Furidha, 2024). This design is suitable for this study because, in investigating skills and errors among high school learners, the researchers must perform a thorough analysis and understanding of the data within a limited time.

In gathering the data, the researchers first wrote a letter to the Dean of the University of Mindanao College of Teacher Education and the principal of Basic Education, requesting permission to conduct the study. Second, the researchers distribute the questionnaires to the high school students. Lastly, the researchers collect and analyze the data gathered from the test questionnaire. The data are analyzed and interpreted using statistical tools.

Furthermore, the researchers communicate their commitment to maintaining the confidentiality of the records and data obtained from the participants. The data gathered is strictly confidential and used solely for research purposes. The selected respondents have the right to withdraw from the study at any time, even if they initially agreed to participate. To ensure confidentiality, participants have the freedom to stay anonymous. Additionally, participants can refuse to share any information not directly related to this study.

Thematic analysis was used in this study. Thematic analysis is a process that analyzes qualitative data sets (such as the transcriptions of respondents' answers) and recognizes patterns in meaning throughout the data set to derive themes. This involves the personal experiences and perspectives of the respondents playing a pivotal role in

understanding the meaning of the data (Delve Tool, 2020). This method is employed to address the second statement of the problem and is carefully analyzed and interpreted.

4. Results and Discussion

4.1 Skills in Reading Graphs

Table 2 presents the skills of high school students in reading pie charts, bar graphs, line graphs, histograms, and ogives. Specifically, it describes their reading graph skills in terms of literal reading, interpretative reading, and extrapolative reading.

Table 2: Level in Reading Graph Skills of High School Learners

Types of Graphs	Indicators	Reading Graph Skills				
		4	3	2	1	0
Pie Chart	LR	6 (23%)	1 (4%)	2 (8%)	17 (65%)	0 (0%)
	IR	2 (8%)	2 (8%)	2 (8%)	18 (69%)	2 (7%)
	ER	1 (4%)	0 (0%)	1 (4%)	3 (12%)	21 (81%)
	Reading Pie Chart	1 (4%)	1 (4%)	4 (15%)	15 (57%)	5 (19%)
Bar Graph	LR	0 (0%)	2 (8%)	8 (31%)	15 (58%)	1 (4%)
	IR	0 (0%)	1 (4%)	3 (12%)	18 (69%)	4 (15%)
	ER	0 (0%)	0 (0%)	4 (15%)	17 (65%)	5 (20%)
	Reading Bar Graph	0 (0%)	1 (4%)	2 (8%)	17 (65%)	6 (23%)
Line Graph	LR	0 (0%)	3 (12%)	5 (19%)	18 (69%)	0 (0%)
	IR	0 (0%)	0 (0%)	2 (8%)	18 (69%)	6 (23%)
	ER	0 (0%)	0 (0%)	0 (0%)	15 (58%)	11 (42%)
	Reading Line Graph	0 (0%)	0 (0%)	2 (8%)	16 (61%)	8 (31%)
Histogram	LR	1 (4%)	1 (4%)	17 (65%)	6 (23%)	1 (4%)
	IR	1 (4%)	1 (4%)	6 (23%)	14 (54%)	4 (15%)
	ER	0 (0%)	1 (4%)	1 (4%)	7 (27%)	17 (65%)
	Reading Histogram	1 (4%)	0 (0%)	5 (19%)	17 (65%)	3 (12%)
Ogive	LR	0 (0%)	2 (8%)	1 (4%)	17 (65%)	6 (23%)
	IR	0 (0%)	0 (0%)	1 (4%)	17 (65%)	8 (31%)

	ER	0 (0%)	0 (0%)	1 (4%)	4 (15%)	21 (81%)
	Reading Ogive	0 (0%)	0 (0%)	2 (8%)	14 (54%)	10 (38%)
Overall	LR	7 (5%)	9 (10%)	33 (25%)	73 (56%)	8 (6%)
	IR	3 (2%)	4 (3%)	14 (11%)	85 (65%)	24 (19%)
	ER	1 (1%)	1 (1%)	7 (5%)	46 (35%)	75 (58%)
	Reading Graph Skills	0 (0%)	0 (0%)	2 (8%)	21 (81%)	3 (11%)

Note: LR (Literal Reading), IR (Interpretative Reading), and ER (Extrapolative Reading).

The data show the overall reading graph skills of high school students. In overall reading graph skills, the majority of students (81%) demonstrated low graph reading skills, scoring level 1. None of the students reached the highest level, which is level 4. Other students' scores were distributed between 3 (8%) and 2 (11%). This indicates that most of the students lacked graph interpretation skills, with most only able to extract surface-level or basic information.

In the literal reading (LR), most of the students comparatively showed better performance in other reading skills, with 56% scoring at level 1 and only 5% achieving level 4, indicating that students understand basic information from the graph. Another is the interpretative reading (IR) skills. The students' ability to derive meaning or trends from the graph is significantly weaker, with 65% scoring level 1 across most graph types, and only 3% achieving level 4 in IR skills. Lastly, extrapolative reading (ER) skills. The highest skill, which involves inferencing and predicting, revealed the weakest performance among other skills, with 58% of students at level 0 and only 1% reaching level 4. Reveals a critical gap in higher-order data interpretation.

Furthermore, this data shows the graph-specific observations. In the pie chart, for literal reading, 65% are at level 1, for interpretative reading, 69% are at level 1; and for extrapolative reading, 81% are at level 0. This suggests that students struggle most with extrapolating information from pie charts. The bar graph shows 58% at level 1 for literal reading, 69% at level 1 for interpretative reading, and 65% at level 1 for extrapolative reading. Few students reached level 4, posing interpretative challenges. Another is the line graph. In literal and interpretive reading, both descriptors achieved 69% at level 1, and in extrapolative reading, 58% at level 1, highlighting students' difficulty using line graphs for prediction or trend analysis. Next is the histogram, showing 65% at level 2 for literal reading, 54% at level 1 for interpretative reading, and 65% at level 0 for extrapolative reading. Numerous students misinterpret histograms as bar graphs, confusing frequency and distribution. Lastly, the ogive graph. Across all descriptors, the majority of students scored at levels 1 and 0. In literal and interpretative reading, 65% of students are at level 1 and 81% at level 0 for extrapolative reading. This points out a notable gap in understanding ogive graphs, specifically in making predictions and inferences.

Findings from a broad-based study using TIMSS- and PISA-based graph items indicate that high school students often perform better at literal reading but have substantial difficulties with interpretative and inferential levels (Bursal & Yetis, 2020). This corresponds with the findings of this research that students are able to retrieve basic information but cannot proceed to make deeper inferences or predictions.

4.2 Error Analysis of Reading Graph Skills

Table 3.1 presents the emergent theme of students committing errors in reading a pie chart, clustered under literal reading, interpretative reading, and extrapolative reading.

Table 3.1: Errors in Reading a Pie Chart

Reading Graph Skills	Emergent Theme The students committed errors in...	Specific Observations
Literal Reading	- Providing data entries - Calculating the angles - Calculating the frequencies - Providing partial or isolated correctness	- Left frequency and angle for Volkswagen blank. (6) - Left the rest of the table blank after the attempt at the frequency of Nissan. (1) - Incorrect angle calculations for Vauxhall and Volkswagen. (17) - Incorrect frequency calculations for Audi, Nissan, and Volkswagen. (19) - Correct frequency for Nissan only. (1) - Correct angle for Vauxhall, but incorrect for others. (1) - Attempted for Nissan frequency but incorrect and left the rest of the table blank. (1)
Interpretative Reading	- Constructing pie charts - Providing labels - Providing an explanation or justification from the data	- Disproportionate or inaccurately sized slices of a pie chart. (20) - Incorrectly drawn based on flawed data. (13) - Unrelated values used. (6) - Explanations showed a misunderstanding of the angle-frequency relationship. (2)
Extrapolative Reading	Inferencing the data	- Incorrect inference about what the pie chart represents. (1) - Made inferences based on miscalculated values. (1) - Repeated answer without analysis or new insights. (1)

Under the literal reading indicator, several students struggled to provide accurate data entries, calculate angles, and determine frequencies, often providing only partial or isolated correctness. In terms of interpretative reading indicators, the most prominent errors were with constructing pie charts, providing labels, and providing an explanation or justification. Finally, for the extrapolative reading indicator, a few students showed difficulty in inferring the data.

- **Providing Data Entries**

This error occurs when students fail to correctly extract factual data information from graphs or tables, such as copying or reading values accurately. The study by Pallauta *et al.* (2021) revealed multiple errors in data entry, including many students copying values incorrectly, misreading pictogram icons, or computing marginal frequencies incorrectly.

Make of Car	Frequency	Angle
Ford	15	45
Nissan	25	75
Audi	21	63
Vauxhall	20	60
Volkswagen		

Figure 1: Error Committed by Student Participant 2 in Providing Data Entries

Several students struggled with providing data entries, leaving parts of the table incomplete, and incorrectly calculated the answer. In the study by Oksela *et al.* (2019), students often left the table empty and failed to show complete calculations, resulting in incomplete data processing.

- **Calculating the Angles**

This error occurs when students incorrectly convert data into angles for pie chart sectors due to a misunderstanding of the formula, using the wrong total, or making computational errors. As a result, the angles do not accurately reflect the given data, which aligns with the findings of Ozturk and Saban (2023).

Make of Car	Frequency	Angle
Ford	15	45
Nissan	20	75
Audi	~~20~~ 10	63
Vauxhall	20	55
Volkswagen	9	45

Figure 2: Error Committed by Student Participants 23 and 1 in Calculating the Angles

Most students miscalculated the angles, leading to incorrect answers. For example, the angle for Vauxhall was written as 50 degrees instead of 60 degrees, showing a failure to correctly reflect data frequency. The research results align with those of Sari *et al.* (2021), which found that most students had difficulty with angle concepts and measurements.

- **Calculating the Frequencies**

This error occurs when students misidentify frequencies in datasets or graphs, leading to misunderstandings of graphs, improper selection of class intervals, and calculation errors. The results align with the findings of Bustamante-Valdés and Díaz-Levicoy (2024), who found that students struggle to interpret frequency scales and apply the correct methods, revealing gaps in visual data interpretation.

Make of Car	Frequency	Angle
Ford	15	45
Nissan	45? 25	75
Audi	33? 21	63
Vauxhall	20	50
Volkswagen	25? 39	55?

Figure 3: Errors Committed by Student Participants 7 and 10 in Calculating the Frequencies

Most students are having trouble calculating frequencies, resulting in incorrect answers, such as mismatched values for Nissan and Audi. The study by Lian *et al.* (2022) shows that students struggled to interpret and construct statistical representations, including pie charts.

- **Providing Partial or Isolated Correctness**

This error occurs when a student provides partially accurate answers by focusing on a single aspect of the task while ignoring others, indicating incomplete data processing and limited comprehension. The study of Hicks *et al.* (2021) found that many students often complete only one task, demonstrating incomplete statistical reasoning.

Make of Car	Frequency	Angle
Ford	15	45
Nissan	16	75
Audi	19	63
Vauxhall	20	59
Volkswagen	29	177

Figure 4: Error Committed by Student Participant 24 in Providing Partial or Isolated Correctness

The students showed partial correctness by calculating Vauxhall's angle, but made errors in Volkswagen, Nissan, and Audi. In the study by Idehen (2020), it was revealed that students' answers are frequently correct for pie chart parts but make errors in related values, indicating inconsistent data processing.

- **Constructing the Pie Charts**

This error occurs when students misuse the given data, such as incorrect angle calculations, wrong sector sizes, missing labels, or failing to complete the full circle, revealing difficulties in proportional reasoning and data visualization. These observations aligned with Yun *et al.* (2019), noting that learners frequently misapply

measurement concepts and fail to correctly construct statistical graphs, including pie charts.

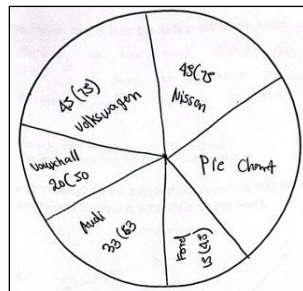


Figure 5: Error Committed by Student Participant 9 in Constructing the Pie Charts

Most of the students failed to construct the pie charts. Based on Figure 5, the divisions were imprecise, the answer was incorrect, and the segments did not add up to 360 degrees. Lian *et al.* (2022) found that many students produced imprecise segment divisions and inconsistent numeric labels, resulting in inaccurate outcomes.

- **Providing Labels**

Errors occur when students label graphs and charts incorrectly, including missing labels, vague names, or labels placed in the wrong spot, revealing a struggle to convey the data's meaning. Willis *et al.* (2020) found that students' ability to understand and reason about graphs is affected, leading to incorrect conclusions and misread scales.

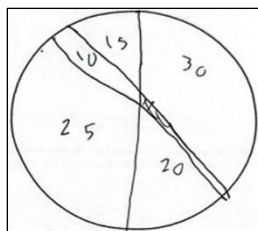


Figure 6: Error Committed by Student Participant 25 in Providing Labels

Several students were unable to provide correct labels when constructing a pie chart, giving only numerical values without specifying categories and inconsistent placement, which reduced readability. King (2019) emphasized that pie chart sectors must be consistently labeled and understandable.

- **Providing an Explanation or Justification from the Data**

This error occurs when students provide unclear or unsupported explanations. Sarumaha and Rizkianto's (2022) research found that students initially gave vague, insufficient explanations but failed to justify the reasoning.

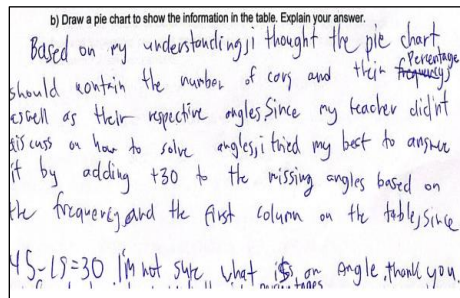


Figure 7: Error Committed by Student Participant 7
in Providing Explanation or Justification from the Data

Some students find it difficult to justify or explain responses, such as a student only adding 30 as the missing angles without explanation, rather than converting frequencies into 360° proportions. Some students had trouble describing the angle calculation procedure, according to Ozturk and Saban's (2023) study, which demonstrates a lack of proportional reasoning and a seeming inability to defend the approach

- **Inferencing the Data**

Errors arise when students answer inaccurately or draw unsupported conclusions from graphs, such as drawing conclusions not supported by the data or making unwarranted inferences. This phenomenon is observed in Xiong *et al.* (2019), who found that students frequently misinterpreted visualizations of correlational data, highlighting how visualization influences misinterpretation.

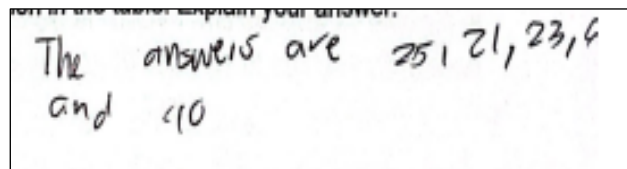


Figure 8: Error Committed by Student Participant 26 in Inferencing the Data

Based on Figure 8, some students list numbers without explaining how those values are derived or what it represents, showing a lack of logical reasoning and poor inference-making. Kurnia *et al.* (2023) found that explanations of relevance or drawing conclusions based on those values were often absent.

The error analysis of the students' reading pie chart shows that numerous errors were found in the three indicators: literal, interpretative, and extrapolative reading. Students often showed partial or isolated correctness under the literal reading indicator. In the interpretative reading indicator, common errors included constructing pie charts, missing or incorrect labeling, inadequate explanations or justifications, inadequate graph communication, and inadequate proportional reasoning. Lastly, for the extrapolative reading indicator, few students struggled to make inferences or spot trends. In conclusion, these errors highlight the difficulty of creating and understanding graphs and the importance of comprehensive work in data reasoning, computation, and visual

communication. This result is consistent with a study by Güngör (2024) that shows students struggle to understand, create, and draw trustworthy conclusions from graphs, even though the majority could read basic data

Table 3.2 presents the emergent theme of students committing errors in reading a bar graph, clustered under literal reading, interpretative reading, and extrapolative reading.

Table 3.2: Errors in Reading a Bar Graph

Reading Graph Skills	Emergent Theme The students committed errors in...	Specific Observations
Literal Reading	- Extracting specific numerical values - Calculating the difference - Describing the trend - Identifying the target population - Providing an explanation directly to the graph	- Incorrect frequency for 6-10 servings. (2) - Incorrect frequency for over 20 servings. (1) - Attempted to calculate the difference but erased or left it incomplete. (2) - Ignored adult data or misidentified the most frequent trend. (2) - Provided only a general identification of the trend. (2) - Misidentification of the target population between adults and children. (7) - Vague explanations of trends. (2) - Lacked data to support observations. (2) - Unclear frequency reasoning. (1) - Confusion between frequency and servings. (1)
Interpretative Reading	- Interpreting group comparisons - Focusing on specific categories - Recognizing patterns - Providing explanation and analytical depth based on the data	- Incorrect comparisons between adults' and children's consumption trends. (10) - Incorrect focus, such as misinterpreting between "number of servings" and "number of children". (1) - General interpretations only. (2) - Lacks specified values. (1) - Incorrect relationships. (3) - Lacks explanation or deeper analysis. (6)
Extrapolative Reading	Analyzing and Projecting data trends	- Lacks inference or prediction. (5) - General inference. (1) - Vague extrapolations. (6)

		• Basic or flawed inference. (1) • External or groundless assumptions. (5) • Unsupported claims. (5) • Descriptive response only. (1)
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Under the literal reading indicator, several students committed an error in extracting specific numerical values, calculating the difference, describing the trend, identifying the target population, and providing explanations. In terms of interpretative reading indicators, the errors were in interpreting group comparisons, focusing on specific categories, recognizing patterns, and providing explanations and analytical depth. Lastly, for the extrapolative reading indicator, most of the students committed errors in analyzing and projecting.

• **Extracting Specific Numerical Values**

This error occurs when students misread or fail to copy the correct numbers from a table, chart, or graph, which leads to incorrect answers, for example, misreading "25 children" as "10 children". Susac *et al.* (2024) found that students often misinterpret graph features and standard errors, showing misconceptions about the graphs consistently.

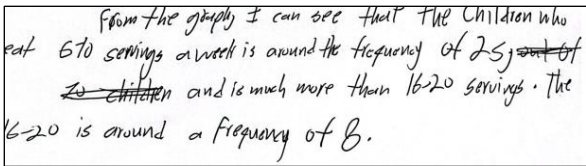


Figure 9: Error Committed by Student Participant 24 in Extracting Specific Numerical Values

Some students misread numerical values when extracting data from graphs, such as inaccurately claiming that children who consume 16–20 servings have a frequency of 4, not 8. Kerns and Wilmer (2021) similarly found that students often misread the bar graph, leading to an inaccurate frequency distribution

• **Calculating the Difference**

Errors happen when a student miscalculates differences, reverses values, or uses the wrong numbers in graphs or tables, such as computing 25-4 as 30. The study by Lin *et al.* (2025) noted that students often made step-by-step calculation errors, leading to inaccurate determination of differences in graph data.

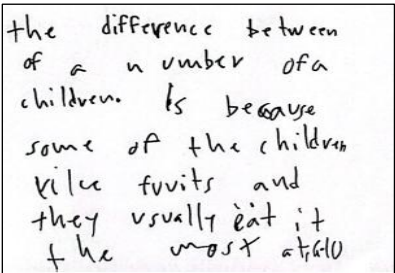


Figure 10: Error Committed by Student Participant 23 in Calculating the Difference

Several students made an error in calculating the difference from graphs; instead, the student explained why children eat more fruit, which result, no numerical comparisons. The study of Friedrich *et al.* (2024) found similar issues, showing a lack of deep understanding of fundamental data principles.

- **Describing the Trend**

This error happens when a student incorrectly depicts trends of a graph, leading to vague, incorrect, or unsupported statements, such as claiming that children eat more fruits and vegetables than adults when the graph shows the opposite. Bustamante-Valdez *et al.* (2024) found that many students exhibit significant difficulty in explaining and comprehending the data.

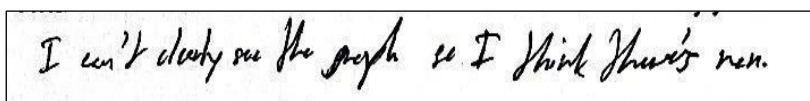
A rectangular box containing handwritten text in black ink. The text reads: "I can't clearly see the graph so I think there's more."

Figure 11: Error Committed by Student Participant 11 in Describing the Trend

When describing the trend, some students make an error. For instance, in Figure 11, a student incorrectly responded, "children eat more, more than the adults.", contrary to the graph clearly showing that adults eat more servings of fruit and vegetables each week than children. Bursal and Yetis (2020), found that students primarily struggle with interpreting and constructing graphs, making accurate analysis difficult.

- **Identifying the Target Population**

This error happens when the student is unable to understand who or what the data represents, frequently resulting in incorrect interpretations or conclusions. One specific example is when a student discusses adults when the graph shows children. According to Chang *et al.* (2024), students face significant challenges when constructing graphs, including accurately identifying the data

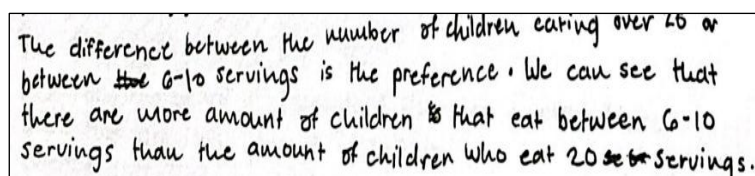
A rectangular box containing handwritten text in black ink. The text reads: "The difference between the number of children eating over 40 or between the 6-10 servings is the preference. We can see that there are more amount of children to that eat between 6-10 servings than the amount of children who eat 20 or more servings."

Figure 12: Error Committed by Student Participant 15 in Identifying the Target Population

A few students made an error in identifying the target population. While students correctly focused on "children" as the subject, they then discussed "preference" rather than the numerical count, failing to answer the requested quantitative comparison. Groth and Choi's (2023) similarly highlight that students have difficulty correctly interpreting the data and overcoming misconceptions when connecting graphs.

- **Providing an Explanation Directly to the Graph**

These errors occur when students fail to provide a clear explanation, failing to make a clear connection to the graph, logic, and supporting information; for example, when the response is "because it's higher" without elaboration. Knoll *et al.* (2023) pointed out that students frequently misinterpret charts, leading to varying and unsupported explanations.

We see that the Adults and the children are very different. As you can see the children eat more than the adults

Figure 13: Error Committed by Student Participant 21 in Providing an Explanation Directly to the Graph

Most learners' answers demonstrate inaccuracies in explaining by claiming without providing specific data points or comparisons to support the claims. According to Bruckner *et al.* (2020), students show a high level of confidence in interpreting graphs; students' understanding does not correspond with the deeper notion, reflecting shallow comprehension.

- **Interpreting Group Comparisons**

The error occurs when a student misunderstands the comparison among multiple groups on a graph, leading to incorrect judgments regarding which group has higher or lower values. For example, claiming that the boys consume more fruit than girls, which is the opposite of the actual figure provided, means it lacks interpretation of group differences. According to the study by Saske *et al.* (2024), graph interpretation is a difficult skill that students frequently struggle with when it comes to interpreting the data.

The difference between children who eat over 20 servings of fruit and vegetable and children who eat 6-10 servings of fruit and vegetables is about 4+. I think it's 4+ because in my logic $6+10=16$, $20-16=4$, and since it's 20+ I guess that it'll be 4+.

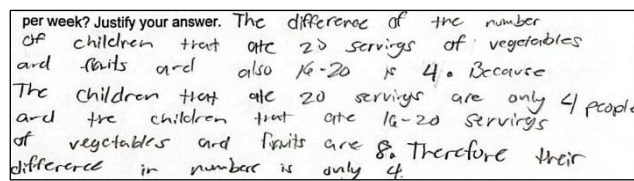
Figure 14: Error Committed by Student Participant 1 in Interpreting Group Comparisons

The student misinterprets the graph by performing incorrect mathematical operations, rather than extracting and comparing the frequencies. This illustrates a fundamental misunderstanding of interpreting and comparing data from visual representations. Abt *et al.* (2025) state that students frequently struggle to accurately compare data sets.

- **Focusing on Specific Categories**

This error occurs when a student fails to focus on the specific group or category to which the question pertains, which leads to inaccurate or irrelevant answers. The study by

Rahmatina and Mohd Zaid (2019) found that such inattention contributes to misunderstanding in graph interpretation, including misreading means in bar graphs.



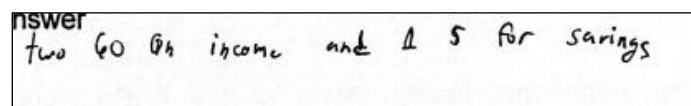
per week? Justify your answer. The difference of the number of children that ate 20 servings of vegetables and fruits and also 16-20 is 4. Because the children that ate 20 servings are only 4 people and the children that ate 16-20 servings of vegetables and fruits are 8. Therefore their difference in number is only 4.

Figure 15: Error Committed by Student Participant 10 in Focusing on Specific Categories

Several student responses illustrate a flaw in focusing on specific categories for comparison. Instead of comparing children who consume over 20 servings and those consuming between 6-10 servings, the student's response implies the 16- 20 category, leading to incorrect answers. Kang *et al.* (2021) explained that learners commonly experience difficulties due to fundamental misconceptions.

- **Recognizing Patterns**

This error occurs when students are unable to recognize recurrent patterns or consistent trends in data, such as cyclical or steady increases and decreases. As a result, incomplete or incorrect interpretations of data sequences. One study found that, including an "unsystematic jumping" strategy in which students were unable to recognize repeating units (Baumanns *et al.*, 2024), which means it was difficult to spot patterns or cyclical behaviors.



answer
two 60 on income and 1 5 for savings

Figure 16: Error Committed by Student Participant 13 in Recognizing Patterns

Some students made an error in recognizing patterns. For example, the student's response "two 60 on income and 1 5 for savings" instead of the bar graph's data on fruit and vegetable servings shows misinterpreting the entire graph's subject and failing to recognize relevant patterns. According to Holder and Xiong's (2022) study, a lack of proficiency, which can result in incorrect graph learning.

- **Providing Explanation and Analytical Depth Based on the Data**

This error arises when a student is unable to provide a comprehensive explanation that goes beyond what is instantly apparent when considering the evidence. This demonstrates a weak understanding or lack of ability to connect the evidence to larger concepts or consequences. Binali *et al.* (2022) found that students frequently perceive data in a literal sense, without providing deeper knowledge or context analysis, showing weaknesses in graph interpreting skills.

We noticed how many fruits and vegetables adults and children eat per week.

Figure 17: Error Committed by Student Participant 12 in Providing Explanation and Analytical Depth Based on the Data

Several students made a mistake when providing a clarification and critical insight. For example, the response of the student just restates the graph's premise ("We have observed how many vegetables and fruits both adults and children eat per week") without providing analysis, recognizing patterns or making inferences, demonstrating a lack of deeper meaning or connection to larger significance. Pols *et al.* (2021) similarly found that interpretive replies were usually tautological or shallow, repeating obvious information, without identifying key aspects or drawing conclusions.

- **Analyzing and Projecting Data Trends**

This error when a student struggles to use data trends to make solid forecasts about possible future outcomes. This issue may emerge as erroneous assumptions, unsupported predictions, or a failure to capitalize on discovered patterns, which can lead to a failure to make arrangements for upcoming activities. According to Becker *et al.* (2023), students often fail to engage with data and continue to struggle with interpreting graphs correctly.

The adults eat 20 servings, but the kids eat 10. This is shown on the chart.

Figure 18: Error Committed by Student Participant 14 in Analyzing and Projecting Data Trends

Many students exhibited an error in analysis and projection. For example, a student restated current data ("The adults eat 20 servings, but the kids eat 10") without identifying trends or making predictions. The response showed inaccurate interpretations and neglected the broader pattern for projecting future behavior. The study of Taylor *et al.* (2024) stated that numerous students struggle when analyzing the graphs effectively, particularly when identifying the data.

This study presents an error analysis of students' performance in interpreting and analyzing bar graphs. The common errors included literal, interpretative, and extrapolative readings. In literal reading, students fail to extract accurate, factual information, often misreading numerical values. In interpretative reading, the majority of the learners fail to comprehend the meaning of the information, make exact comparisons, and draw inferences. In extrapolative reading, learners struggle to generate inferences and recognize numerous patterns that extend beyond specific facts. These findings highlight the need for specialized educational programs that progressively

advance abilities. According to Kim *et al.* (2020), students typically find it difficult to relate graph elements to real-world implications.

Table 3.3 presents the emergent theme of students committing errors in reading a line graph, clustered under literal reading, interpretative reading, and extrapolative reading.

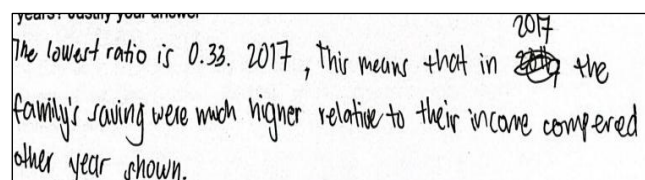
Table 3.3: Errors in Reading a Line Graph

Reading Graph Skills	Emergent Theme The students committed errors in...	Specific Observations
Literal Reading	- Examining the graph - Understanding the ratios - Reading of questions	- Struggling to interpret the graph. (1) - Conceptual misunderstanding of ratios. (2) - Misinterpretation or misreading of questions. (3)
Interpretative Reading	- Providing justification with evidence from the graph - Understanding data relationships - Comparing trends - Justifying assumptions - Interpreting percentage decrease - Comparing ratios - Comparing yearly data - Relating variables	- Lack of justification or support. (8) - Exhibited limited understanding of the relationship between income and savings. (5) - Inaccurate or unclear trend comparison across years over time. (4) - Lack of evidence-based reasoning. (2) - Inadequate interpretation of the trend of expenditure decreases over time. (5) - Incomplete or missing yearly comparisons of ratios. (8) - Incorrect comparison of expenditure from the current and previous year. (3) - Misinterpretation of variable relationships in income, savings, and expenditure. (1)
Extrapolative Reading	- Understanding patterns - Predicting future trends - Understanding historical patterns	- Inability to identify the pattern or direction in the data. (1) - Unclear or unsupported predictions about future trends. (13) - Lack of understanding of historical patterns to relate current data trends to past patterns. (3)

Under the literal reading indicator, several students committed errors in examining the graph, justifying understanding of the ratios, and reading the questions. In terms of interpretative reading indicators, the errors occurred in understanding data relationships, comparing trends, reasoning, interpreting percentage decreases, comparing ratios, year-on-year comparisons, and relating variables. Finally, for the extrapolative reading indicator, the errors are in understanding patterns, predicting future trends, and understanding historical patterns.

- **Examining the Graph**

This error occurs when students struggle to interpret the basic visual elements of a graph, such as the meaning of lines. Mathai *et al.* (2024) found that students often have trouble with line graphs, including interpreting what the lines represent, integrating spatial cues with numeric values, and extracting meaning from the visual presentation.



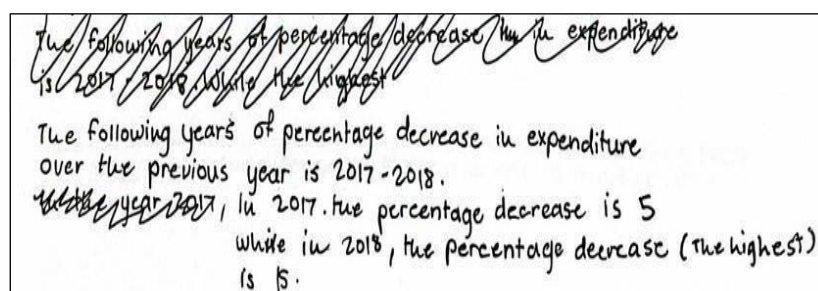
The lowest ratio is 0.33. 2017, this means that in ~~2017~~ the family's saving were much higher relative to their income compared other year shown.

Figure 19: Error Committed by Student Participant 6 in Examining the Graph

Some students made errors in analyzing graphs by providing superficial values without addressing the main task. Ruf *et al.* (2024) found that K-12 students often extract surface-level information from graphs without engaging in deeper relational reasoning.

- **Understanding Ratios**

This error occurs when students misunderstand ratios and use them to compare quantities in graphs. Arenas-Peñaloza *et al.* (2024) found that many students misinterpret ratios or rely on superficial visual cues that lead to misinterpretation of data.



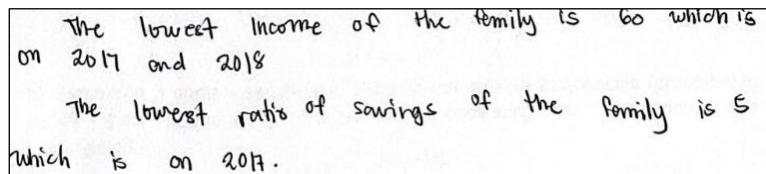
The following years of percentage decrease in expenditure is 2017-2018, while the highest is 2017-2018. The following years of percentage decrease in expenditure over the previous year is 2017-2018. ~~The year 2017~~, in 2017, the percentage decrease is 5 while in 2018, the percentage decrease (the highest) is 5.

Figure 20: Error Committed by Student Participants 15 in Understanding Ratios

Some students misunderstand ratios or percentages, confusing raw numeric differences with proportional calculations. Khasawneh *et al.* (2022) found that seventh graders often make the same error, reporting absolute differences rather than calculating percentage changes.

- **Reading the Questions**

This error occurs when students misread or overlook the question, leading to incomplete or incorrect answers despite understanding the graph. Kenney and Ntow (2024) found that such reading and comprehension errors are common when a problem is asked.



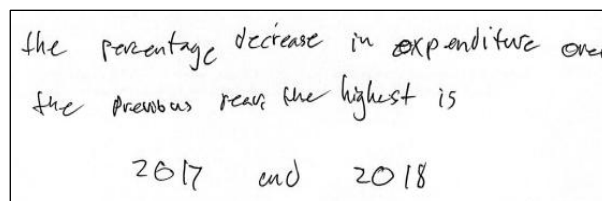
The lowest income of the family is 60 which is on 2017 and 2018
The lowest ratio of savings of the family is 5 which is on 2017.

Figure 21: Error Committed by Student Participant 9 in Understanding Ratios

Several students misunderstood the task by focusing on individual values rather than calculating the required ratios. Miao *et al.* (2022) similarly found that students often fail to establish the correct mathematical relationship between quantities.

- **Providing Justification with Evidence from the Graph**

This error occurs when students give an answer without providing a clear or logical explanation from the graph. Hughes *et al.* (2020) found that most students failed to show computations, making justification incomplete.



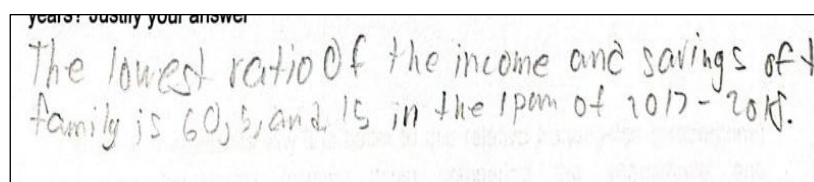
the percentage decrease in expenditure over the previous year the highest is 2017 and 2018

Figure 22: Error Committed by Student Participant 8 in Providing Justification with Evidence from the Graph

Many students drew conclusions without supporting evidence, making the response incomplete and lacking analytical depth. Ruf *et al.* (2024) found that students often provide conclusions in graph-based tasks without justification.

4.2.23 Understanding Data Relationships

This error occurs when students fail to recognize relationships, trends, or correlations in a graph, leading to being unable to explain how one data set relates to another. Jungjohann *et al.* (2022) found that students focused only on subsets of data instead of the overall pattern, leading to fragmented interpretations of relationships and trends.



years: usually your answer
The lowest ratio of the income and savings of a family is 60, 5, and 15 in the 1pm of 2017-2018.

Figure 23: Error Committed by Student Participant 23 in Understanding Data Relationships

Some students listed numerical values without explaining how they relate or how ratios are formed, and no computations or comparisons were provided. Kim *et al.* (2020) similarly found that students often cite numerical values without establishing mathematical relationships, reflecting fragmented understanding.

- **Comparing Trends**

This error occurs when students misinterpret patterns and changes over time, overlooking trends or misjudging rates of change. Chang *et al.* (2024) found that students often struggle to recognize overall patterns and interpret changes correctly.

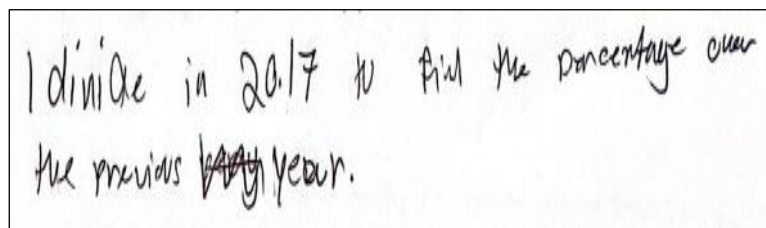


Figure 24: Error Committed by Student Participant 13 in Comparing Trends

Many students provided vague or unsupported comparisons of trends and gave no evidence, data values, or calculations. Lian *et al.* (2022) similarly found that lower secondary students often make vague or unsupported statements when describing statistical trends.

- **Justifying Assumption**

This error occurs when students provide conclusions that are not supported by data or logical reasoning. Ozmusul and Bindak (2022) found that 7th-grade students often gave confident but unsupported conclusions, lacking a valid data-based foundation.

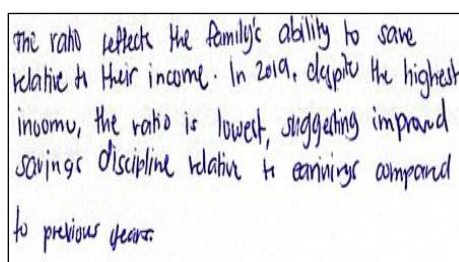
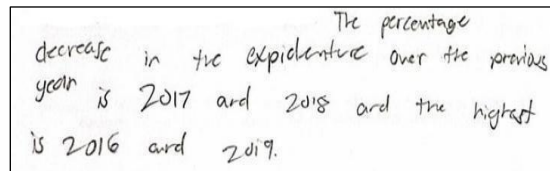


Figure 25: Error Committed by Student Participant 5 in Reasoning

Many students struggled with analyzing ratios and failed to compare actual values, and conflated absolute values with relative measures, leading to unsupported conclusions. Yanishevskaya (2023) similarly found that students often misunderstand proportional reasoning and rely on superficial approaches, leading to unsupported conclusions.

- **Interpreting Percentage Decrease**

This error happens when students misinterpret or incorrectly calculate percentage decreases, commonly using incorrect operations or confusing values. Okumuş and Güveli (2023) found that students often misapply percentage concepts, leading to inaccurate interpretations.



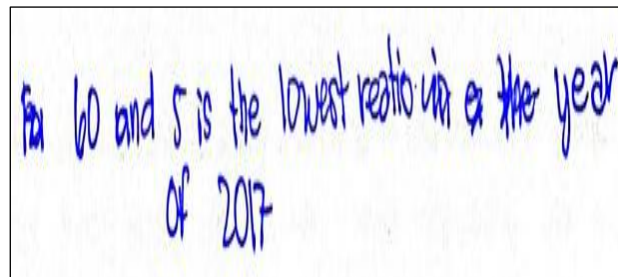
The percentage decrease in the expenditure over the previous year is 2017 and 2018 and the highest is 2016 and 2019.

Figure 26: Error Committed by Student Participant 10 in Interpreting Percentage Decrease

Many students struggled to interpret trends listed as decreases without providing numerical justification, often confusing general decreases with the largest percentage drop. Kuntz *et al.* (2023) similarly found that students often have common difficulties in distinguishing overall trends from specific quantitative changes, lacking clear numerical support.

- **Comparing Ratios**

This error occurs when students cannot accurately evaluate or compare ratios over time. Failing to simplify ratios or misunderstanding what the ratios represent, leading to false conclusions. Adak and Aliustaoğlu (2020) found that seventh graders often struggle with equivalent ratios and have difficulty making accurate ratio comparisons.



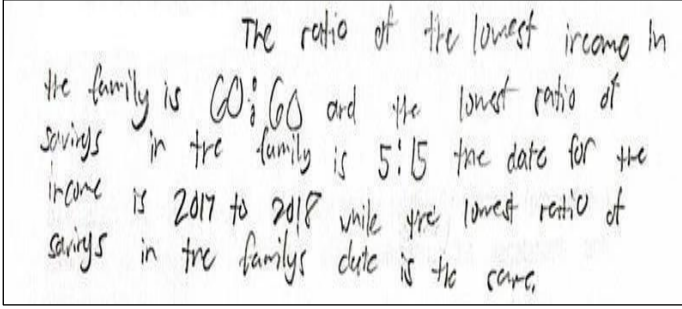
for 60 and 5 is the lowest ratio in a the year of 2017

Figure 27: Error Committed by Student Participant 16 in Comparing Ratios

Many students failed to compare and calculate ratios across all years. The students labeled it the “lowest ratio”, indicating a lack of understanding of how to evaluate ratios systematically. Khasawneh *et al.* (2022) similarly found that students often focus on isolated values rather than making comprehensive comparisons.

- **Compare Yearly Data**

This error occurs when students fail to accurately analyze changes across different years, leading to misunderstandings of changes over time. Chang *et al.* (2024) found that students often use inconsistent strategies and overlook meaningful temporal relationships when interpreting data across years.



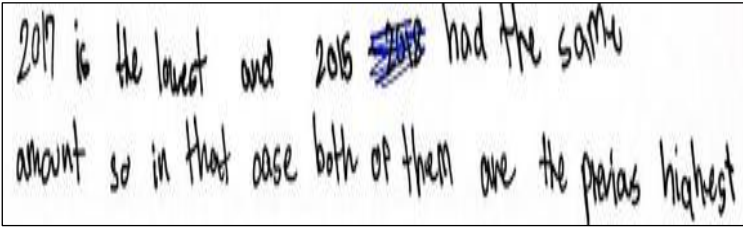
The ratio of the lowest income in the family is 60:60 and the lowest ratio of savings in the family is 5:15 the date for the income is 2017 to 2018 while the lowest ratio of savings in the family's date is the same.

Figure 28: Error Committed by Student Participant 10 in Compare Yearly Data

Some students made incorrect comparisons between current and previous year expenditures, confused raw values with ratios and did not use proper ratio computation. Kurnia *et al.* (2023) similarly found that students often misunderstand ratios, confusing raw numbers with ratios and neglecting proper calculations.

- **Relating Variables**

This error occurs when students cannot accurately interpret relationships between two or more variables, leading to incorrect conclusions or missed patterns. Johnson *et al.* (2024) found that students often fail to coordinate changing quantities, showing students struggle to draw meaningful conclusions.



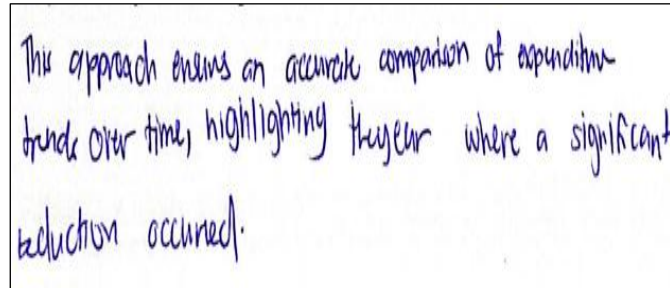
2017 is the lowest and 2016 ~~was~~ had the same amount so in that case both of them are the periods highest

Figure 29: Error Committed by Student Participant 3 in Relating Variables

Some students misunderstood how key variables - income, savings, and expenditure - interact over time, providing vague and incorrect conclusions. Donovan *et al.* (2024) similarly found that students often struggle with calculating percentage changes and articulating the relationships between variables over time.

- **Understanding Patterns**

This error occurs when students fail to recognize recurring trends or behaviors in a dataset or graph. Consequently, struggling to predict future values based on patterns. Altindis *et al.* (2024) found that students' covariational reasoning hinders deeper graphical interpretation.



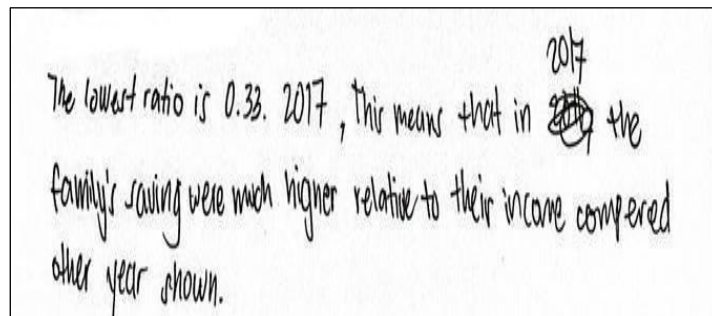
This approach ensures an accurate comparison of expenditure trends over time, highlighting the year where a significant reduction occurred.

Figure 30: Error Committed by Student Participants 3 and 5 in Understanding Patterns

Some students failed to extend them accurately and misinterpreted data trends, leaving the response unsupported and incomplete. Sterner (2024) similarly found that students often struggle to move beyond procedural descriptions to meaningful data interpretation and evidence-based justification.

- **Predicting the Future**

This error occurs when students make predictions that are incorrect or unsupported, leading to misreading the direction of the trend or predicting too far into the future. Mema and Zela (2024) found that such prediction mistakes often result from an incomplete understanding of data trends and the limitations of predictive models.



The lowest ratio is 0.33. 2017, this means that in 2017 the family's saving were much higher relative to their income compared other year shown.

Figure 31: Error Committed by Student Participant 6 in Predicting the Future

Many students made vague or unsupported predictions about future trends without basing them on accurate data. Lathifaturrahmah *et al.* (2024) similarly found that students often struggle to ground predictions in correct data interpretation, resulting in inaccurate conclusions.

- **Understanding Historical Patterns**

Many students made vague or unsupported predictions about future trends without basing them on accurate data. Lathifaturrahmah *et al.* (2024) similarly found that students often struggle to ground predictions, resulting in inaccurate conclusions.

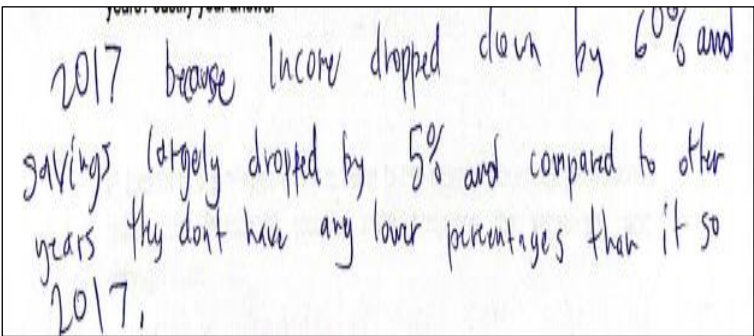


Figure 32: Error Committed by Student Participant 7 in Understanding Historical Patterns

Some students misinterpreted historical data trends, failing to accurately compare percentage changes across years, resulting in misunderstandings of historical patterns. Boels *et al.* (2024) note that students often struggle to synthesize data across a dataset, which leads to flawed conclusions about historical trends.

The error analysis of students’ performance in reading line graphs revealed difficulties across literal, interpretative, and extrapolative reading. Students struggled with basic graph skills, linking data points, recognizing trends and ratios, calculating percentage changes, making accurate year-to-year comparisons, identifying patterns, predicting future trends, and explaining changes in past patterns. Many provide unsubstantiated guesses and think in fragmented ways, hindering accurate graph interpretation. These findings align with Brückner *et al.* (2020), who reported that students often focus on surface features rather than underlying relationships, limiting deep, context-based graph interpretation.

Table 3.4 presents the emergent theme of students committing errors in reading a histogram, clustered under literal reading, interpretative reading, and extrapolative reading.

Table 3.4: Errors in Reading Histogram

Reading Graph Skills	Emergent Theme The students committed errors in...	Specific Observations
Literal Reading	• Description of distributions	• Lacked specific details about the distribution center and variability. (3) • Provides vague observations about heights instead of detailed distribution analysis. (6) • Generalized descriptions for non-vegetarians. (1) • Incomplete description of maximum observed heights. (1)
	• Evidence-based comparison	• Identified a general trend but lacks a detailed explanation of the distribution. (1)
	• Reading of graphical data.	• Misreads the graph and makes unsupported conclusions. (1)

	• Conceptualizing relative frequency • Providing explanations about relative frequencies	• Incorrectly states that vegetarians eat only vegetables, and non-vegetarians eat any food. (1) • Incorrectly identifies model height for vegetarians, stating “170 cm” instead of 160 cm. (1) • Reasoning about a vegetarian graph having higher bars is invalid despite being visually true, due to relative frequency. (1) • Provided unclear reasoning for using relative frequencies. (3) • Focused on personal difficulty reading the graph instead of explaining concepts. (1) • Explanation is difficult to follow or lacks clarity. (1)
Interpretative Reading	• Providing a specific statistical comparison. • Understanding relative frequency and proportional reasoning • Comparing or justifying height distribution	• Makes broad claims such as “nonvegetarians are taller than vegetarians” without graphical evidence. (2) • Lacked comparisons of central tendency (mean, median), spread (range, standard deviation), and shape (e.g., skewness). (8) • Provided shallow or surface-level comparison without depth. (2) • Provides an incomplete comparison due to a misunderstanding of frequency, relative frequency, and sample size. (1) • Does not fully explain why relative frequencies are better for unequal group comparisons. (7) • Provided general benefit of relative frequencies but not connected to the unequal group size context. (1) • Does not clearly articulate the concept of proportions and normalization. (3) • Lacked comprehensive comparisons or relational analysis of the data. (4)
Extrapolative Reading	• Providing observations from the data • Providing predictive reasoning in relative frequencies	• Lacks detail or accuracy in inference about height. (1) • Inferences based on incorrect trends. (2) • Providing descriptive reasoning without moving toward predictive or inferential statements. (1)

Under the literal reading indicator, students struggled with describing distributions, interpreting relative frequencies, and providing evidence-based explanations. In interpretative reading, errors included making specific statistical comparisons, applying

proportional reasoning, and justifying distributions. For extrapolative reading, students had difficulty drawing observations and making predictive inferences from the data.

- **Description of Distributions**

This error pertains to students' difficulties in explaining and accurately analyzing the key characteristics of data distribution in the graph, such as shape, center, spread, and specificity, rather than vague or generalized statements. According to the study by Boels *et al.* (2024), students displayed a tendency to focus on local peaks and individual bars instead of evaluating the distribution holistically for its shape, center, and spread.

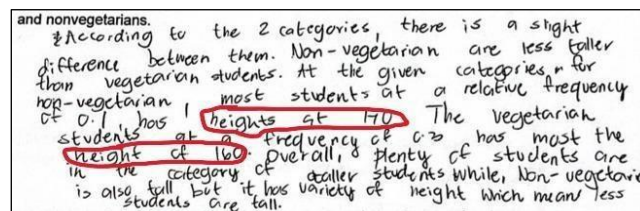


Figure 33: Error Committed by Student Participant 26 in Detailing the Shape, Center, and Spread

Several students only provide a general comparison without presenting detailed shape, center, and spread of the distributions. This relates to the findings of Schreiter and Vogel (2023): students mostly used a "local view" that only focuses on isolated data points rather than using a "global view" that underscores the distribution structure of the graph.

- **Evidence-based Comparison**

This error refers to students' unsupported claim comparisons among categories with specific data points or quantitative evidence from the graph. This resulted in unclear or incomplete analysis of data. Similarly, in the study of Ruf *et al.* (2024), both the construction and interpretation of the data in the graph. Students often have difficulties with anchoring to exact values, coordinates, or 60 specific data points.

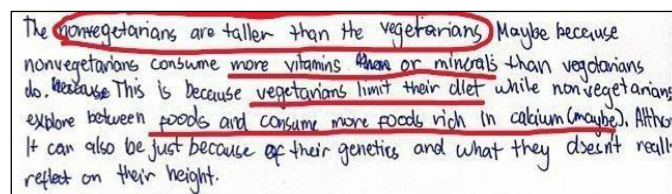


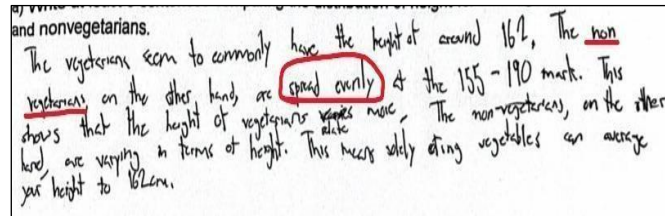
Figure 34: Error Committed by Student Participant 1 in Evidence-based Comparison

Many students failed to support any specific height values or numerical data from the graph to substantiate or defend these claims. In Sterner (2024), students' interpretations are limited without having support from the graph with evidence.

- **Reading of Graphical Data**

This error pertains to students' underlying misinterpretations or misunderstanding of the data or information, which leads to unsupported or inaccurate claims, focusing only

on what is visually obvious without understanding the relative frequency or scaling. In the study of Driessen *et al.* (2022), the students make claims from what is expected rather than what is presented in 61 the graph, ignoring the main context.



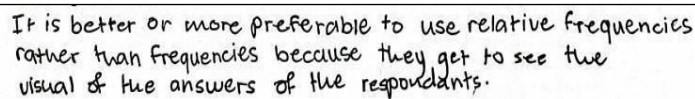
and nonvegetarians.
The vegetarians seem to commonly have the height at around 161. The non-vegetarians, on the other hand, are spread evenly at the 155-190 mark. This shows that the height of vegetarians varies more. The non-vegetarians, on the other hand, are varying in terms of height. This means solely eating vegetables can average your height to 162cm.

Figure 35: Error Committed by Student Participant 14 in Reading of Graphical Data

Some students misinterpreted the spread of non-vegetarians' heights, describing it as "evenly spread," which implies a uniform distribution. In reality, the data show a right-skewed distribution with greater variability. Incorrect strategies substantially weaken accurate graph interpretation, as documented by Boels *et al.* (2024).

- **Conceptualizing Relative Frequency**

This error reflects students' difficulty in understanding, calculating, and interpreting relative frequencies, especially when comparing groups of unequal sizes. Similarly, Kazak and Pratt (2021), found that learners often rely on raw counts instead of proportional reasoning, resulting in inaccurate comparisons between data sets.



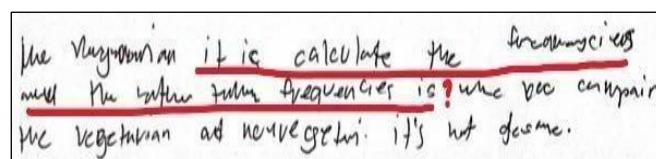
It is better or more preferable to use relative frequencies rather than frequencies because they get to see the visual of the answers of the respondents.

Figure 36: Error Committed by Student Participant 15 in Conceptualizing Relative Frequency

This error reflects students' tendency to focus on the visual appearance of a graph rather than the statistical purpose of relative frequencies. Similarly, Friedrich *et al.* (2024) found that learners often rely on superficial visual assessment, overlooking the role of relative frequency in understanding proportional relationships.

- **Providing Explanations about Relative Frequencies**

This error reflects students' difficulty in clearly communicating understanding of the graph. Explanations may be vague or incomplete. Gao *et al.* (2025) note that missing key terms or structure in reasoning prevents proper assessment of understanding.



The vegetarian it is calculate the frequencies and the other frequencies is? when you compare the vegetarian and non-vegetarian it's not same.

Figure 37: Error Committed by Student Participant 18 in Providing Explanations about Relative Frequencies

Some students provide explanations that are unclear or difficult to follow. Kazak and Pratt (2021) note that students often fail to clearly articulate why relative frequencies are useful for comparing unequal groups, reflecting the same lapse observed in this participant.

- **Providing a Specific Statistical Comparison**

This error occurs when students cannot make accurate, evidence-based comparisons between groups using statistical measures from the graph. Similarly, Abt *et al.* (2025) noted that students may rely on visual heuristics reflecting a superficial interpretation rather than a meaningful statistical comparison.

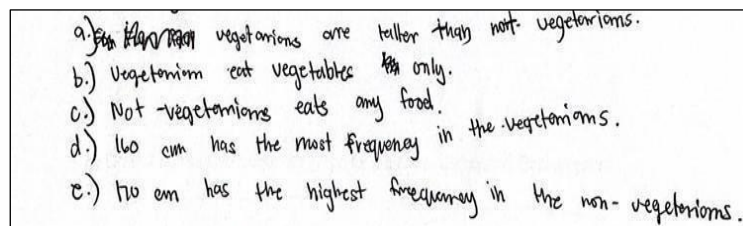


Figure 38: Error Committed by Student Participant 9
in Providing a Specific Statistical Comparison

This error occurs when students make inaccurate or unsupported comparisons between groups. Drawing conclusions without evidence from the graph can lead to incorrect interpretations, particularly when comparing groups of unequal sizes (Cummiskey *et al.*, 2020).

- **Understanding Relative Frequency and Proportional Reasoning**

This error occurs when students struggle to apply proportions, normalization, or relative frequency, especially for groups of unequal sizes. Sari *et al.* (2024) noted that this reflects a common difficulty in proportional reasoning.

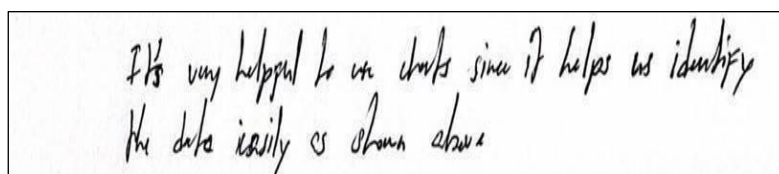


Figure 39: Error Committed by Student Participant 11
in Understanding Relative Frequency and Proportional Reasoning

The response ignored differences in sample sizes and the need for normalization when comparing unequal groups. Boynton (2025) noted that the true analytical value comes from accounting for sample-size differences, highlighting the gap in students' reasoning.

- **Comparing or Justifying Height Distribution**

This error occurs when students struggle to make data-based comparisons between groups or categories from the graph. Oz and Ciftci (2024) found that while students often recognize differences, few provide explicit, graph-based evidence.

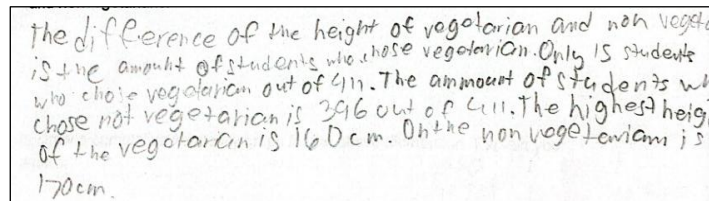


Figure 40: Error Committed by Student Participant 23 in Comparing or Justifying Height Distribution

In Figure 40, the student noted “the highest height of vegetarians is 160 cm” without exploring trends, relationships, or overall distribution patterns. Schreiter *et al.* (2024) similarly found that learners often focus on local features rather than global patterns.

- **Providing Observations from the Data**

This error occurs when students struggle to make accurate, detailed, and well-reasoned inferences from graphical data. Altindis *et al.* (2024) found that limited disciplinary knowledge hinders effective correlational analysis and interpretation.

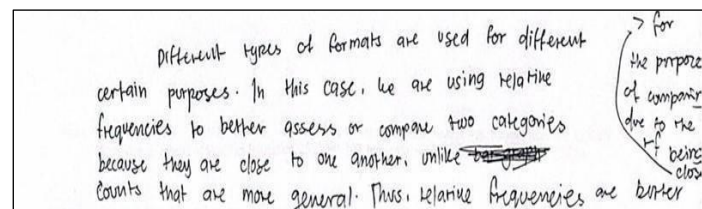


Figure 41: Error Committed by Student Participant 21 in Providing Observations from the Data

Some students attempted to make inferences but failed to fully explain them, such as mentioning fairer comparisons but not mention 37 normalization or proportions. Stump *et al.* (2023) found that students often recognize differences between datasets, yet fail to articulate why normalization or relative frequencies are necessary.

- **Providing Predictive Reasoning in Relative Frequencies**

This error pertains to students' inability to proceed to describe beyond what the data in the graph show. According to the study of Shimizu and Kang (2025), a major gap is students' inability to practice predictive inference or connect it to inferential predictions.

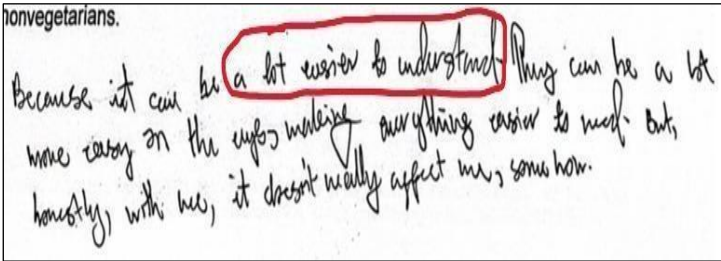


Figure 42: Error committed by Student Participant 4
in Providing Predictive Reasoning in Relative Frequencies

This error reflects students’ reliance on descriptive or intuitive reasoning rather than statistical principles. Kazak and Pratt (2021) found that novice learners often base models on surface-level context, neglecting the underlying mathematical principles.

This error analysis of students’ interpretations in the histogram revealed persistent difficulties of students across literal, interpretative, and extrapolative reading levels. Several students failed to accurately describe and often omitted key features such as center, shape, and spread, made unsupported claims, misreading of frequencies, superficial explanations that overlook crucial elements, frequently made broad comparisons and struggled to apply concepts of proportional reasoning or normalization, frequently lacked clarity, depth, and justifications that utilizes measures such as mean, median, or mode, and had difficulty in creating predictions or inferences beyond the presented data. These findings highlighted the students’ tendency to rely only on surface-level characteristics or personal impressions and the challenges posed in transitioning from visual observation to analytical interpretation or reasoning. The study of Boels *et al.* (2023) confirms that students have a shallow engagement with histogram features.

Table 3.5: Emergent theme of students committed errors in reading an ogive, clustered under literal reading, interpretative reading, and extrapolative reading

Reading Graph Skills	Emergent Theme The student committed errors in:	Specific Observation
Literal Reading	● Providing a process for calculating the median	● Correctly calculates the median but does not show steps or process. (2)
	● Identifying frequency and cumulative frequency.	● Treated frequencies as individual points rather than cumulative values. (1)
	● Creating a graphical representation	● Provides an inaccurate or unclear description of graphing cumulative frequency. (2)
	● Conceptualizing data organized into class intervals	● Uses the wrong formula, uses the formula for ungrouped data instead of grouped data. (1) ● Incorrectly calculates cumulative frequencies or fails to complete them. (4)

	• Procedural and conceptual aspects of median calculation • Literal approach in visualizing the graph	• Attempts to find the median but uses an illogical or unclear method. (5) • Incorrectly includes or calculates percentages not explicitly asked for. (1) • Adds unclear text like "1-10 1" that does not represent frequency meaningfully. (1)
Interpretive Reading	• Understanding of the ogive graph and its purpose • Determining the media • Interpreting cumulative frequency	• Comprehends cumulative frequency but struggles to connect it to ogive construction. (5) • Shows no understanding of how to determine the median from a frequency table. (11) • Did not interpret how the number of students accumulates across mark ranges or how it relates to the cumulative distribution. (3)
Extrapolative Reading	• Synthesizing data and applying understanding beyond basic definitions	• Made simple observations about the most frequent score but failed to extend reasoning or make predictive claims. (1)

Under the literal reading indicator, students' errors provide a process for calculating the median, identifying frequency and cumulative frequency, creating graphical representations, conceptualizing grouped data, and applying both procedural and conceptual approaches. In terms of interpretative reading indicators, the most prominent errors were related to understanding the ogive graph and its purpose, determining the median, and interpreting cumulative frequency. Finally, for the extrapolative reading indicator, a few students showed errors in synthesizing data and applying understanding beyond basic definitions.

- **Providing a Process for Median Calculation**

This error referred to students correctly identifying or calculating the median but failing to demonstrate or explain the steps students took to reach that answer, such as locating the middle value. The study by Oslington *et al.* (2023) showed that while many students could identify the trends and central values, only a smaller proportion of students could provide or illustrate reasoning behind their answers.

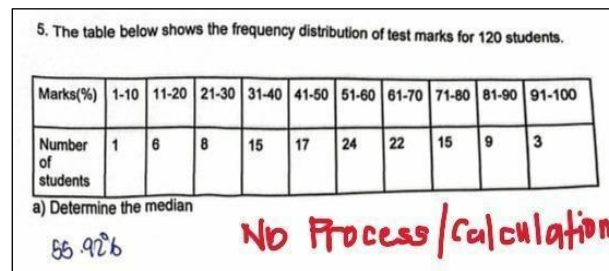


Figure 43: Error committed by Student Participant 1 in Providing a Process for Median Calculation

Some students were able to correctly calculate the median; however, they failed to show the steps or processes involved in the calculation, which is important for clarity and understanding of students' reasoning. Amiruzzaman and Kosko (2020) found that students frequently could state the median but failed to explain the steps, calculations, reasoning, or process.

- **Identifying Frequency and Cumulative Frequency**

This error occurs when students misinterpret frequency or cumulative frequency when finding the median position and class interval. According to Nabilah and Triyana (2023), many students failed to use cumulative frequency to identify the median class and instead employed midpoint-based or incorrect aggregation heuristics.

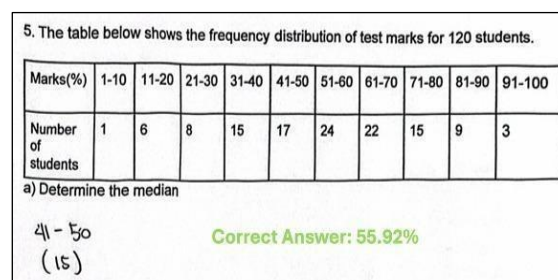


Figure 44: Error committed by Student Participant 9 in Identifying Frequency and Cumulative Frequency

Several students misidentified the median as a frequency value instead of within a class interval, such as picking a class near the middle without using cumulative frequencies. The study of Palluata *et al.* (2024) revealed that students frequently failed to apply cumulative frequency tables correctly, which leads to incorrect interpretations of grouped data medians.

- **Graphical Representation**

This error pertains to students' improper use of graphing methods, including choosing the appropriate graph, accurately plotting, and clearly labeling when displaying cumulative frequency data. According to Pallauta *et al.* (2024), a large number of students made computational mistakes, which indicates that the graphing task has misplaced points or even incorrect graph types or plotted frequencies.

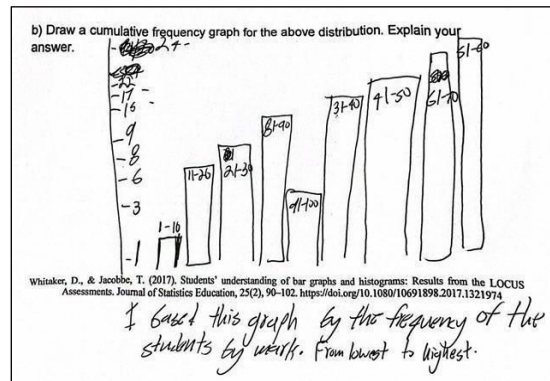


Figure 45: Error Committed by Student Participant 24 in Graphical Representation

Many students failed to draw an ogive graph (also known as a cumulative frequency graph) or substituted it with an incorrect graph type, such as a bar chart, line graph, or frequency polygon. According to a study by Ruf *et al.* (2024), many students lacked a conceptual understanding of how a data set's structure should inform the kind of graph that is selected and how it is constructed.

- **Conceptualizing Data Organized into Class Intervals**

This error refers to students' difficulty in understanding the data that is organized into class intervals, leading to incorrect application of statistical methods for grouped data. The study of Oslington *et al.* (2023) emphasizes that without correctly conceptualizing class intervals as aggregates, students cannot develop higher-level statistical reasoning and apply statistical methods effectively.

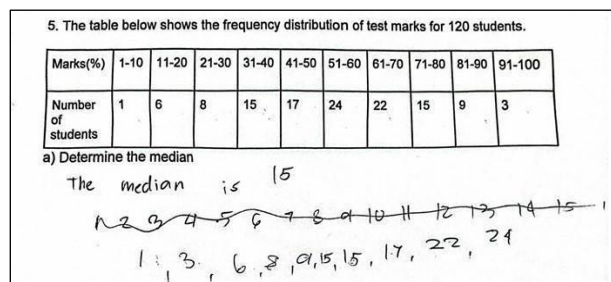


Figure 46: Error committed by Student Participant 26 in Conceptualizing Data Organized into Class Intervals

Many students misunderstood what the median represents in grouped data, often selecting the middle frequency without proper calculations. This is aligned with the APOS (Action-Process-Object-Schema) framework used in the study by Ng and Chew (2023). This is a memorized procedure, without moving on to processes that call for conceptual reasoning.

- **Procedural and Conceptual Aspects of Median Calculation**

This error refers to breaking down in both the method (procedure) and the understanding (conceptual basis) required to determine a statistical measure accurately. The study by

Delastri and Lolang (2023) highlighted skipping steps or misapplying formulas, suggesting that procedural fluency often fails without a conceptual foundation.

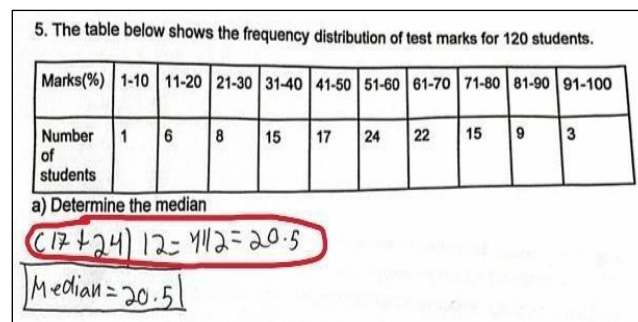


Figure 47: Error committed by Student Participant 18 in Procedural and Conceptual Aspects of Median Calculation

Many students attempted to find the median but used a different or unclear method by averaging the two student numbers rather than using cumulative frequencies to find the middle score, confusing frequency with data value. Bugay *et al.* (2021) states when calculating measures of central tendency and figuring out the median, students may apply ungrouped data reasoning incorrectly to grouped data.

- **Literal Approach in Visualizing the Graph**

This error implies that students often interact with the graph superficially, concentrating on copying, rewriting, or restating information instead of deriving an analysis from the graph itself. According to a study by Becker-Genschow *et al.* (2023), students' frequent fixation on the graph's literal elements without a deeper contextual trend analysis.

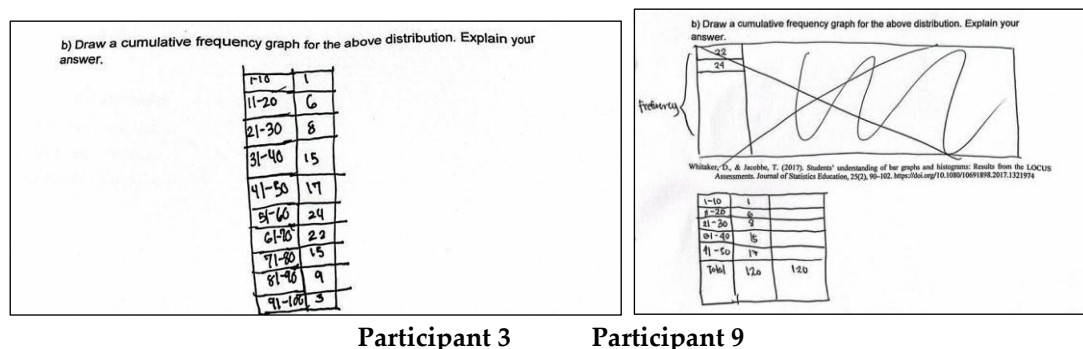


Figure 48: Error committed by Student Participants 3 and 9 in Literal Approach in Visualizing the Graph

Some students simply rewrote the frequency table without any additional analysis or calculations, demonstrating a lack of understanding of cumulative frequency. Aleifat and Tabieh (2024) showed that students frequently misunderstand cumulative frequency and provide superficial treatments of data.

- **Understanding of the Ogive Graph and Its Purpose**

This error refers to students' partial grasp of the concept of cumulative frequency, but a weak connection between the construction and interpretation of an ogive. The study of Bruckner *et al.* (2020) found that even university students who understood the concept verbally struggled to apply it graphically.

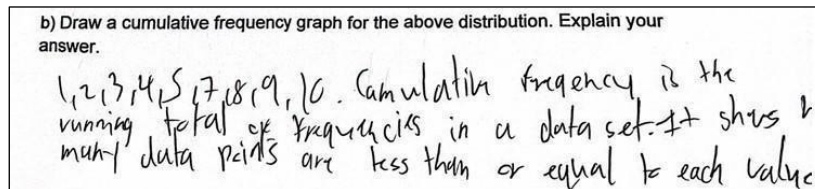


Figure 49: Error Committed by Student Participant 20 in Understanding the Ogive Graph and Its Purpose

Many students understand cumulative frequency but find it difficult to relate it to a graphical representation, such as including unrelated numbers that do not contribute to the solution. According to the study by Maries and Singh (2020), it highlights how students frequently omit or mis-plot graphs despite having correct conceptual explanations, which indicates a rift between knowing and doing.

- **Determining the Median**

This error indicates a fundamental misunderstanding of how to locate the median in a frequency table, lacking procedural steps nor the conceptual foundation. Pallauta *et al.* (2022) found students frequently failed to accumulate frequencies and consequently chose incorrect median groups.

5. The table below shows the frequency distribution of test marks for 120 students.

Marks(%)	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Number of students	1	6	8	15	17	24	22	15	9	3

a) Determine the median

15

Correct Answer: 55.92%

Figure 50: Error Committed by Student Participant 23 in Determining the Median

Numerous students demonstrated a lack of understanding of how to determine the median from a frequency table, assuming or confusing it with a random midpoint value. Cazorla *et al.* (2023) found that students prefer intuitive to cumulative reasoning, resulting in misconceptions.

- **Interpreting Cumulative Frequency**

This error reflects a student's difficulty in understanding how data values build up across intervals and are used to describe distribution. According to the study by Pallauta *et al.*

(2021), even when students correctly identify the class intervals, learners struggle to translate or read cumulative totals.

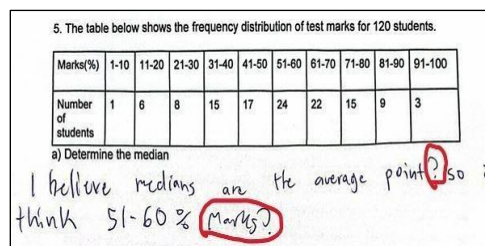


Figure 51: Error Committed by Student Participant 7 in Interpreting Cumulative Frequency

Some students failed to interpret the frequency (i.e., the number of students) accumulated across mark ranges, how it relates to the cumulative distribution, or mistakenly equated the median with an average. This is consistent with the research by Nabilah and Triyana (2023), in which students determine the median class by incorrectly using "compound averages" and neglecting cumulative frequency.

- **Synthesizing Data and Applying Understanding Beyond Basic Definitions**

This error is the result of students' limited capacity to go beyond quick observations to more in-depth analysis or deduction from the data. This aligns with Celik's (2022) research, in which students rarely make progress in interpreting or extrapolating conclusions from the data.

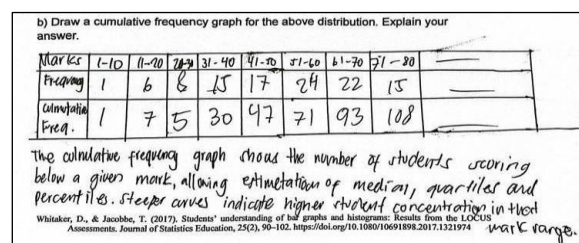


Figure 52: Error Committed by Student Participant 6 in Synthesizing Data and Applying Understanding Beyond Basic Definitions

Some students provided only a general inference about the curves or graph's shape, and their concentration was on the mark range or class interval, implying a lack of depth in analysis. Friedrich *et al.* (2024) show a common trend: students usually interpret graphs superficially, describing only broad trends without delving deeply into the underlying dataset.

The error analysis of students' interpretations of ogive graphs found significant challenges across literal, interpretative, and extrapolative levels of reading. At the literal level, many students struggled to create an ogive graph, failing to demonstrate the procedures involved in calculating the median, misinterpreted frequencies as individual values, and had conceptual misunderstandings of grouped data and procedural errors. At the interpretative level, students show difficulties in connecting cumulative frequency to ogive construction and determining the median from frequency tables, indicating a

lack of both procedural fluency and conceptual foundation. At the extrapolative level, students hardly go beyond the surface-level description to conclude, make predictions, or apply what they have learned to a larger context. According to the study by Lian *et al.* (2022), students relied on rote procedural steps without the necessary conceptual clarity, which leads to mechanical mistakes in graphing and interpretation.

5. Recommendations

A number of suggestions are made to improve students' graph literacy in light of the study's limitations and conclusions. In order to improve and strengthen students' abilities in interpretative reasoning (IR) and extrapolative reasoning (ER), teachers should first incorporate interpretative and extrapolative tasks into class activities. These tasks should consist of answering various graph types with an emphasis on trend analysis, reasoning, and prediction. Furthermore, it is important to diversify graph exposure in the classroom by making sure that students regularly interact with and are exposed to a variety of graph types, especially the more complex ones like ogives and histograms, which are often given little attention. Incorporating real-life data and problem-solving scenarios into lessons can help make graph reading relevant and meaningful, which will increase students' motivation and retention. Visual-analytical scaffolds, like step-by-step instructions, graphic organizers, and models, should also be incorporated into lessons to assist students in understanding and visualizing the relationship between variables. Future researchers or studies should investigate the long-term impacts of instructional interventions on graph literacy in order to advance this research, potentially employing an experimental design to gauge learning outcomes. Lastly, because this study relies on small, localized samples, it is critical to recognize its limitations. To validate and generalize the results across different educational contexts, future researchers should aim to include a larger sample size and a more diverse population.

6. Conclusion

The research reveals that high school students' abilities to read graphs of all five types are lacking. In terms of overall graph reading, the majority of students received a level 1 score, 81% demonstrated only rudimentary comprehension, and none attained the highest level. The majority of students struggle with interpretative and extrapolative reading, particularly when it comes to drawing conclusions or spotting trends and patterns, while a small percentage demonstrate partial proficiency in literal reading. For example, 81% of students receive a level 0 score in extrapolative reading, indicating that they struggle greatly with ogive and histogram interpretation. Given that most students can only extract surface-level information without further analysis or prediction, these results highlight the need for higher-order graph comprehension instruction to be strengthened.

Furthermore, the study finds several common mistakes that students make when reading graphs, which are divided into three categories: literal, interpretative, and

extrapolative reading. Miscalculating frequencies, reading values incorrectly, and leaving data entries blank are examples of literal reading errors that students commonly make. Errors in interpretative reading include poorly constructed graphs, improper labeling, and explanations that are ambiguous or lack evidence. Across all graph types, students' responses demonstrate that they are unable to infer patterns or make logical predictions, which is the most notable aspect of extrapolative reading errors. These results suggest that in addition to having trouble comprehending graph components, students also have trouble applying graphs as tools for reasoning and decision-making. Students' skills in interpreting data can be greatly enhanced by filling in these gaps through explicit instruction and scaffolded practice.

The study also highlights the importance of putting in place instructional interventions that help students develop a deeper comprehension and critical analysis skills beyond simply identifying data. By recognizing these gaps, educators and curriculum designers can design lessons that raise or strengthen students' general graph literacy.

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About the Author(s)

Jhette Althea C. Jadloc is a 4th-year Bachelor of Secondary Education major in Mathematics student at the University of Mindanao, Main Campus, who has a strong interest in mathematics education and learner development. She is an active member of the University of Mindanao Teacher College Mathematics Society (UMTCMS), engaging in an organization and academic activities promoting excellence in mathematics teaching. She is also a lead researcher and statistician in her senior years in the Science, Technology, Engineering, and Mathematics Strand, and she aspires to become a competent mathematics educator dedicated to enhancing students' critical thinking, problem-solving, and data interpretation skills.

Noreen Mae D. Tumamao is a 4th-year student pursuing a Bachelor of Secondary Education major in Mathematics at the University of Mindanao, Matina Campus. She is also an active member of the University of Mindanao Teacher College Mathematics Society (UMTCMS), where she represents her organization in some contests and events. She is passionate about making mathematics accessible and meaningful for learners, with a strong focus on using visual aids and practical examples to deepen understanding. Her academic work reflects a commitment to evidence-based teaching and to helping students build confidence in problem-solving and critical thinking.

Alma Mae A. Aguisando is a passionate and goal-oriented student currently pursuing a Bachelor of Secondary Education major in Mathematics at the University of Mindanao, Davao City. She is deeply committed to becoming an effective and inspiring educator, with a strong interest in helping students understand and appreciate the value of mathematics in everyday life. During her second year in college, she stepped into a leadership role as a volunteer officer, where she actively participated in organizing activities and supporting community initiatives. This experience helped her develop essential skills such as communication, teamwork, leadership, and adaptability. She continues to strive for personal and academic growth, guided by her dedication to service and her vision of making a meaningful impact in the field of education.

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