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THE ECONOMICS OF STUDENT TIME IN THE AI ERA: MODELING STUDY OPTIMIZATION FOR STUDENT ARCHETYPES

Porto Massimiliano[i]
Associate Professor,
College of Asian Pacific Studies,
Ritsumeikan Asia Pacific University,
1: 1-1 Jumonjibaru, Beppu, Oita 874-8577
Japan
orcid.org/0000-0002-5851-3429

Abstract:
This paper develops a microeconomic framework for analyzing how students optimally allocate study time across academic disciplines under heterogeneous cognitive constraints and evolving technological environments. By embedding physiological frictions—within-session study fatigue (α_i) and non-discretionary background stress (S^{σ})—into a two-good production setting, the model generates a strictly concave Study Possibilities Frontier (SPF) consistent with empirical patterns of diminishing cognitive returns. Preferences and difficulty parameters jointly determine the student's optimal study mix, yielding closed-form interior solutions for most technological specifications. The model is calibrated to four archetypal student profiles defined along dimensions of cognitive efficiency and academic motivation. Simulation results show that general technology uniformly expands the SPF but interacts asymmetrically with student heterogeneity. An autonomous output floor (TECH2) provides the strongest welfare gains for high-fatigue or low-engagement students, while multiplicative productivity tools (TECH3) benefit high-aptitude students once baseline efficiency is sufficiently high. Domain-specific artificial intelligence further mitigates the opportunity cost of difficult subjects, though misuse that substitutes for active problem-solving can induce cognitive depreciation. Overall, the framework demonstrates that educational interventions are not one-size-fits-all. As artificial intelligence and automation reshape academic environments, institutions must transition from passive mass instruction to targeted, structurally informed pedagogical strategies that align with the heterogeneous friction profiles of the student population.

JEL: A22, I21, O33, C61

[i] Correspondence: email m-porto@apu.ac.jp

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1. Introduction

The rise of digital technology has democratized access to information that was previously held exclusively at the university level, while the introduction of Artificial Intelligence (AI) has fundamentally transformed study methodologies. Whereas past academic debates focused primarily on the necessity of increasing total instructional time, this new era shifts the analytical focus toward the efficiency and quality of knowledge acquisition. Empirically, Bratti & Staffolani (2013) demonstrate that self–study is a far stronger predictor of performance than course attendance, stating that omitting self–study from empirical models leads to omitted variable bias. While numerous studies document a positive correlation between attendance and achievement (e.g., Romer (1993), attendance defined merely as physical classroom presence yields an imprecise proxy for effective learning.

Furthermore, contemporary students face a significantly higher opportunity cost of attending class than previous generations. The widespread availability of high–quality, freely accessible learning materials—such as hundreds of instructional videos on the same topic—renders traditional lectures inefficient for students who find in-class explanations opaque and can master the material more effectively via autonomous alternatives. Crucially, as Romer (1993, p. 170) observes, attendance is endogenous; it reflects a deliberate choice heavily moderated by students' perceptions of instructional quality.

Beyond instructional quality and primary study effort, academic outcomes are intrinsically linked to the allocation of time across non-academic domains. In his seminal treatise, Becker (1965, p. 494) highlights the relevance of "*other non-working uses of time*" on study performance. Time-diary and survey data compiled by Stinebrickner & Stinebrickner (2008) confirm that college students navigate complex trade-offs between study effort, classroom attendance, sleep, and paid employment. Consequently, non-study time allocations influence academic performance through channels far more nuanced than a simple reduction in available hours.

To formalize these aspects, our framework distinguishes between non-study activities which consume cognitive energy (W), such as part-time work, and those that facilitate its recovery (R), such as leisure. Cui et al. (2019) discuss the different stances of the literature on the impact of leisure on productivity. While classical economic views typically treat leisure as a pure substitute for labor, often overlooking its potential positive externalities, psychological and sociological perspectives argue that leisure plays a functional role in sustaining performance. Aligning with the latter view, we model recovery activities as a mechanism to compensate for cognitive fatigue—defined as "*a ubiquitous human condition, the result of sustained cognitive engagement that taxes our mental* *resources*" (Mullette-Gillman et al. 2015, p. 2)—thereby enabling the student to refocus more effectively on their studies.

Similarly, the literature on academic stress identifies various stressors and distinguishes between positive stress ("eustress"), which maintains student motivation, and negative stress, which leads to burnout and hampers academic performance (Iqra 2024). Contrary to the common assumption that academic stress inevitably leads to poorer outcomes, empirical findings on the relationship between stress levels and grades remain inconclusive. For instance, recent studies report non-significant effects (Pérez-Jorge et al. 2025), significant negative impacts (Qin et al. 2025, Al-Roomy 2025), or even increases in academic performance (Benítez-Agudelo et al. 2025). In our framework, we model stress—which also includes academic stress—as the residual of cognitive fatigue that leisure and resting cannot compensate for, adjusted by an individual sensitivity parameter.

By following Levin & Tsang (1987), who conceptualize the student as both a producer of activities subject to time and effort constraints and a consumer with a strictly quasi-concave utility function, we develop a microeconomic framework for analyzing how students optimally allocate study time across academic disciplines under heterogeneous cognitive constraints and evolving technological environments. Within this framework, a student's broader academic achievement is determined directly by the multidimensional knowledge outputs $(Q_1, \dots, Q_n)$ they successfully absorb during their active study time.

To evaluate the impact of technological change, we construct a baseline specification alongside three alternative scenarios characterized by distinct structural channels through which general technology ($TECH$) enters the production function. The integration of Artificial Intelligence (AI) is then applied to each of these scenarios, modeling its effects to capture both documented improvements in learning and the potential negative consequences reported in recent studies—such as declines in cognitive skills, problem-solving skills, critical thinking (Labadze et al. 2023, Kasneci et al. 2023, Sova et al. 2024, Farrokhnia et al. 2024, Wang et al. 2024).

The remainder of this paper is organized as follows. Section 2 derives the analytical solution to the student's constrained optimization problem for the n-subject case. Without loss of generality, Section 3 restricts this framework to a two-subject environment to show that the slope of the study possibilities frontier (SPF) remains strictly negative across all specifications, preserving standard production-theoretic properties. Section 4 characterizes the interior solutions for the two-good model, establishing the theoretical baseline for the numerical simulations in Section 5. These simulations are calibrated around four archetypal student profiles structured across a 2×2 parameter matrix defined by cognitive efficiency (the subject difficulty parameter, $\alpha_i > 1$) and academic preferences (the utility weight, $\gamma_i \in (0,1)$). Each archetype represents a distinct combination of aptitude and motivation. By holding these internal traits (α, γ) constant within each profile, we systematically simulate how external shocks —specifically the introduction of general technology ($TECH$) and asymmetric, AI-assisted study technologies (θ) — reconfigure the optimal production frontiers, knowledge quantities, and time allocation vectors for each respective student type. Section 6 presents and discusses the simulation results across the different student archetypes. Finally, Section 7 offers concluding remarks, pedagogical reflections, and policy considerations.

2. Generalized Model to n Subjects

Knowledge output for each specific field of study is governed by the following structural production function:

$$Q_i = f_0(k_i, T_i, \alpha_i, S^{\sigma}) = \frac{k_i T_i^{\frac{1}{\alpha_i}}}{S^{\sigma}} \tag{1}$$

where:

- Q_i is the knowledge output of subject i;
- $k_i > 0$ is the baseline learning rate of subject i $\left(\frac{\partial f_0}{\partial k_i} > 0\right)$;
- T_i is the active study time devoted to subject i $\left(\frac{\partial f_0}{\partial T_i} > 0 \text{ and } \frac{\partial^2 f_0}{\partial T_i^2} < 0\right)$;
- The parameter $\alpha_i > 1$ represents the inherent difficulty or **study–fatigue rate** of subject i, with higher α_i implying faster fatigue accumulation per unit of study time. This structural restriction ensures that the exponent $1/\alpha_i$ remains strictly between 0 and 1, mathematically enforcing diminishing marginal returns to active study time T_i; and
- S^{σ} represents the global environmental stress penalty, driven by the background residual stress $\epsilon = \epsilon(T_w, T_R) > 0$. We introduce three parameters to capture the physiological reality that non-discretionary environmental drains induce cognitive fatigue, whereas rest recharges a student's baseline capacity. These parameters are treated as exogenous parameters determined prior to the study session:
    - **Non-Study Fatigue Sensitivity Parameter ($\boldsymbol{\omega}$)**: This measures the marginal degradation of cognitive ability per hour of energy-consuming environmental activities (T_W). We define T_w broadly to include non-discretionary drains such as commuting or household responsibilities. Because these stressors are inherent to the student's environment, T_w is assumed to be strictly positive, ensuring that some baseline fatigue is always present.
    - **Recharging Sensitivity Parameter ($\boldsymbol{\rho}$)**: This represents the brain's efficiency in neutralizing fatigue through rest or leisure (T_R). Without further expanding, $T_R = T_L + T_S$, where T_L is time for leisure and $T_S \geq T_S^{MIN}$ is time for sleep, which must meet a minimum biological requirement.

Furthermore, we impose the environmental constraint that rest serves specifically to mitigate existing background tiredness:

$$\omega T_W > \rho T_R \tag{2}$$

The residual

$$\epsilon = \omega T_W - \rho T_R > 0 \tag{3}$$

measures the background stress of the student. An underlying stress level persists despite resting activities.

- **Overall Stress Sensitivity Parameter ($\boldsymbol{\sigma}$)**: This captures the nonlinear impact of environmental stress on cognitive baseline capacity, where the structural base S is defined as:

$$S = 1 + \epsilon = 1 + \omega T_W - \rho T_R \tag{4}$$

where "1" represents the baseline cognitive state in the absence of environmental stress. Higher values of σ represent an accelerated decline in overall knowledge absorption capacity due to environmental pressure. High levels of S^σ model cognitive burnout, dampening the marginal productivity of study hours. Conversely, optimal environmental conditions allow $S^\sigma \to 1$, minimizing the baseline penalty and allowing the student to approach peak structural efficiency.

Equation 1 represents the baseline framework, which we extend to incorporate general technological study assistance (*TECH*) and field-specific AI productivity boosts. These two distinct technological processes impact the study possibilities frontier (SPF) through different structural mechanisms:

- **General Technology** (*TECH*): We model three different specifications of general technological assistance: an extension of the time endowment (TECH1), a fixed bonus of knowledge output (TECH2), and a multiplicative productivity bonus (TECH3). Because these variables impact all fields symmetrically, they do not alter the relative ratio of baseline learning rates. Consequently, any positive shock to *TECH* induces an outward shift of the study possibilities frontier across all specifications.
- **Artificial Intelligence** (*AI*): We model this as an integration of advanced domain-specific tools that directly augment learning efficiency. We assume that AI exhibits a directional bias, providing a more pronounced relative efficiency advantage to quantitative subjects where it systematically solves and explains complex algorithmic or mathematical tasks. Formally, we define the AI-adjusted learning rate for any subject i via the mapping:

$$k_i^{AI} = \kappa(k_i, \theta) \tag{5}$$

where $\theta \geq 1$ captures the intensity of a positive AI shock. To establish a baseline of asymmetric technological change, we assume that the marginal productivity boost is heavily skewed toward quantitative disciplines, such that:

$$\frac{\partial \kappa_q}{\partial \theta} > \frac{\partial \kappa_{nq}}{\partial \theta} \geq 0 \tag{6}$$

where q and nq denote quantitative and non-quantitative subjects, respectively. This parameterization introduces an asymmetric efficiency bias. Unlike global technology, the AI parameter alters the relative slopes of the production functions, shifting the marginal rate of transformation (MRT) along the frontier. This changes the structural opportunity cost of allocating study time between quantitative and nonquantitative disciplines.

Depending on the channels through which technology alters the learning process, we consider three distinct variations of the knowledge production function:

- **Specification 1 (TECH1):** General technology functions purely as an exogenous expansion of the available time budget, leaving the underlying structural engineering of learning unaffected:

$$Q_i = f_1(k_i, \theta, T_i, \alpha_i, S^{\sigma}) = \frac{k_i^{AI} T_i^{\frac{1}{\alpha_i}}}{S^{\sigma}} \tag{7}$$

where the total time allocated to studying is bounded by the baseline effective time plus a technology-enabled time-saving equivalent ($TECH$):

$$\sum_{i=1}^{n} T_i \leq T_{EFF} + TECH \tag{8}$$

Here, $T_{EFF} = T - T_W - T_R$ represents the baseline effective study time remaining after non-discretionary commitments. We assume a fixed daily time endowment ($T = 24$) as the representative baseline unit of analysis.

- **Specification 2 (TECH2):** Technology injects an autonomous, fixed baseline quantity of knowledge output directly into each subject domain. This bonus is entirely decoupled from active study time and is immune to cognitive fatigue:

$$Q_i = f_2(k_i, \theta, T_i, \alpha_i, S^{\sigma}) + TECH = \frac{k_i^{AI} T_i^{\frac{1}{\alpha_i}}}{S^{\sigma}} + TECH \tag{9}$$

- **Specification 3 (TECH3):** Technology behaves as an interactive, multiplicative shift factor that directly complements the student's structural learning efficiency across all domains:

$$Q_i = (1 + TECH) f_3(k_i, \theta, T_i, \alpha_i, S^\sigma) = \frac{k_i^{AI} T_i^{\frac{1}{\alpha_i}} (1+TECH)}{S^\sigma} \tag{10}$$

where $TECH > -1$. Parameter values in the range $-1 < TECH < 0$ capture negative technological disruptions, administrative friction, or digital distractions that reduce baseline efficiency.

Assuming that f is strictly increasing in study time, we invert the functions to isolate the active time requirements:

$$\text{(Baseline): } T_i = g_0(k_i, Q_i, \alpha_i, S^\sigma) = \left[\frac{Q_i S^\sigma}{k_i}\right]^{\alpha_i} \tag{11}$$

$$\text{(TECH1): } T_i = g_1(k_i, \theta, Q_i, \alpha_i, S^\sigma) = \left[\frac{Q_i S^\sigma}{k_i^{AI}}\right]^{\alpha_i} \tag{12}$$

$$\text{(TECH2): } T_i = g_2(k_i, \theta, Q_i - TECH, \alpha_i, S^\sigma) = \left[\frac{(Q_i - TECH) S^\sigma}{k_i^{AI}}\right]^{\alpha_i} \tag{13}$$

where we assume the target knowledge output always exceeds the autonomous technological bonus ($Q_i > TECH, \forall i$) to ensure positive active study time requirements.

$$\text{(TECH3): } T_i = g_3\left(k_i, \theta, \frac{Q_i}{1+TECH}, \alpha_i, S^\sigma\right) = \left[\frac{Q_i S^\sigma}{k_i^{AI}(1+TECH)}\right]^{\alpha_i} \tag{14}$$

The student maximizes a strictly quasi-concave utility function $U(Q_1, Q_2, \dots, Q_n)$ that is strictly increasing in all arguments $\left(\frac{\partial U}{\partial Q_i} > 0\right)$, ensuring a unique global maximum. To simplify the notation and streamline the analytical framework, we assume that the student has already allocated work time (T_W) and resting time (T_R) in a prior stage. Consequently, the student treats the global stress level as a fixed parameter and optimizes only the residual effective time available for studying.

To further simplify the budget constraint, we assume that any time inefficiency is fully subsumed into T_R, allowing us to express the constraint as a strict equality. Thus, the resulting constrained optimization problem under any given model specification $m \in \{0,1,2,3,4\}$ is expressed as:

$$\max_{Q_1, Q_2, \dots, Q_n} U(Q_1, Q_2, \dots, Q_n) \quad s.t. \sum_{i=1}^{n} T_i = \bar{T} \tag{15}$$

where the precise active study time requirements T_i are defined by equations (11)–(14).

Here, $\bar{T}$ represents the total effective time budget allocated to study regardless of the model specification. This implies that under Specification 1 (TECH1), the time budget implicitly includes the additive technology term (i.e., $\bar{T} = T_{EFF} + TECH$). Because $TECH$

is strictly exogenous from the perspective of the academic optimization stage, it acts as an invariant parameter and does not alter the structural optimization conditions. Forming the Lagrangian for a generalized technological state:

$$\mathcal{L}(Q_1, Q_2, \dots, Q_n, \lambda) = U(Q_1, Q_2, \dots, Q_n) + \lambda\left[\bar{T} - \sum_{i=1}^{n} g_m\left(k_i, \theta, \tilde{Q}_i, \alpha_i, S^\sigma\right)\right] \tag{16}$$

where the specific functional arguments of g_m correspond to the mappings defined in equations (12)–(14) and

$$\tilde{Q}_i = \begin{cases} Q_i & m = 0,1 \\ Q_i - TECH & m = 2 \\ \frac{Q_1}{1+TECH} & m = 3 \end{cases} \tag{17}$$

The first-order necessary conditions yield the following equilibrium relationship for any pair of subjects *i* and *j*:

$$\frac{\frac{\partial U}{\partial Q_i}}{\frac{\partial U}{\partial Q_j}} = \frac{\frac{\partial g_m}{\partial Q_i}}{\frac{\partial g_m}{\partial Q_j}} \tag{18}$$

where we assume interior solutions ($T_i > 0\ \forall i$) across all domains.

Equation (18) expresses the student's allocation rule as a standard equi–marginal condition. The left–hand side is the Marginal Rate of Substitution (MRS), capturing the student's willingness to trade off knowledge gains across subjects according to their preferences. The right–hand side is the ratio of marginal time requirements—the Marginal Rate of Transformation (MRT)—which reflects the technological and cognitive cost of producing additional knowledge in each domain.

At the optimum, the student adjusts target knowledge levels until the marginal utility obtained from an additional hour of study is equalized across all subjects. This alignment of MRS and MRT ensures that no reallocation of study time can increase overall utility.

With the generalized structure in place, we now turn to explicit analytical solutions by restricting attention to a two–subject setting, which provides a transparent and tractable benchmark for understanding the model's core mechanisms.

3. The Two-Good Parametric Model and Frontier Solutions

To obtain explicit analytical solutions, characterize the geometry of the study possibilities frontier, and calibrate the model for numerical simulations, we restrict the general *n-subject* framework to a two-subject environment. Let $i \in \{E, IR\}$, where Economics (E) represents the quantitative field of study and International Relations (IR) represents the non-quantitative technological baseline field:

$$k_{IR}^{AI} = k_{IR} \text{ and } k_{E}^{AI} = k_E \cdot \theta \tag{19}$$

Let α_i denote the subject-specific study-fatigue parameter for $i \in \{E, IR\}$, defined via the mapping:

$$\alpha_i = \begin{cases} \alpha \text{ if } i = E \\ \beta \text{ if } i = IR \end{cases} \tag{20}$$

where $\alpha > 1$ and $\beta > 1$ represent the unique cognitive friction profiles of Economics and International Relations, respectively.

Following the baseline functional parameterization, we assume the student possesses Cobb-Douglas preferences over the two subjects, where $\gamma \in (0,1)$ represents the relative utility weight allocated to quantitative knowledge:

$$U(Q_E, Q_{IR}) = Q_E^{\gamma} Q_{IR}^{1-\gamma} \tag{21}$$

The closed-form boundary equations of the Study Possibilities Frontier (SPF) for the baseline and each of the technological specifications are, respectively:

$$\text{(Baseline): } Q_{IR} = \left[T_{EFF}\left(\frac{k_{IR}}{S^{\sigma}}\right)^{\beta} - \frac{k_{IR}^{\beta}}{k_E^{\alpha}} Q_E^{\alpha} S^{\sigma(\alpha-\beta)}\right]^{\frac{1}{\beta}} \tag{22}$$

$$\text{(TECH1): } Q_{IR} = \left[(T_{EFF} + TECH)\left(\frac{k_{IR}}{S^{\sigma}}\right)^{\beta} - \frac{k_{IR}^{\beta}}{k_E^{AI\,\alpha}} Q_E^{\alpha} S^{\sigma(\alpha-\beta)}\right]^{\frac{1}{\beta}} \tag{23}$$

$$\text{(TECH2): } Q_{IR} = TECH + \left[T_{EFF}\left(\frac{k_{IR}}{S^{\sigma}}\right)^{\beta} - \frac{k_{IR}^{\beta}}{k_E^{AI\,\alpha}} (Q_E - TECH)^{\alpha} S^{\sigma(\alpha-\beta)}\right]^{\frac{1}{\beta}} \tag{24}$$

$$\text{(TECH3): } Q_{IR} = \left[T_{EFF}\left(\frac{k_{IR}(1+TECH)}{S^{\sigma}}\right)^{\beta} - \frac{k_{IR}^{\beta}}{k_E^{AI\,\alpha}} Q_E^{\alpha} \frac{S^{\sigma(\alpha-\beta)}}{(1+TECH)^{(\alpha-\beta)}}\right]^{\frac{1}{\beta}} \tag{25}$$

The formulation for TECH2 (24) reveals that technology acts as a "safety net", shifting the entire frontier outward regardless of the stress level S^{σ}. Furthermore, the structural term $(Q_E - TECH)^{\alpha}$ signifies that the student only incurs a cognitive fatigue cost for the specific target output they personally study for, after subtracting the fixed knowledge endowment provided automatically by technology.

Conversely, the formulation for TECH3 (25) reveals that, unless the subject-specific difficulties are perfectly balanced ($\alpha = \beta$), a multiplicative technological shock warps the frontier asymmetrically.

We verify that across all specifications, the slope of the frontier remains strictly negative in conformity with production theory.

We simplify equation (23) for TECH1 by grouping the terms that are constant relative to study allocation:

$$\mu_0 = (T_{EFF} + TECH)\left(\frac{k_{IR}}{S^{\sigma}}\right)^{\beta} \text{ and } \mu_1 = \frac{k_{IR}^{\beta}}{k_E^{AI\,\alpha}} S^{\sigma(\alpha-\beta)} \tag{26}$$

This allows us to express the frontier in the condensed form:

$$Q_{IR} = (\mu_0 - \mu_1 Q_E^{\alpha})^{\frac{1}{\beta}} \tag{27}$$

The derivative of Q_{IR} with respect to Q_E yields:

$$\frac{dQ_{IR}}{dQ_E} = -\frac{\alpha}{\beta}\mu_1 Q_E^{\alpha-1}(\mu_0 - \mu_1 Q_E^{\alpha})^{\frac{1}{\beta}-1} \tag{28}$$

Similarly, for the TECH2 specification, we define:

$$\phi_0 = T_{EFF}\left(\frac{k_{IR}}{S^{\sigma}}\right)^{\beta} \text{ and } \phi_1 = \frac{k_{IR}^{\beta}}{k_E^{AI\,\alpha}} S^{\sigma(\alpha-\beta)} \tag{29}$$

Rewriting the frontier as

$$Q_{IR} = TECH + [\phi_0 - \phi_1(Q_E - TECH)^{\alpha}]^{\beta} \tag{30}$$

The derivative yields:

$$\frac{dQ_{IR}}{dQ_E} = -\frac{\alpha}{\beta}\phi_1(Q_E - TECH)^{\alpha-1}[\phi_0 - \phi_1(Q_E - TECH)^{\alpha}]^{\frac{1}{\beta}-1} \tag{31}$$

For the TECH3 specification, we define:

$$\delta_0 = T_{EFF}\left(\frac{k_{IR}(1+TECH)}{S^{\sigma}}\right)^{\beta} \text{ and } \delta_1 = \frac{k_{IR}^{\beta}}{k_E^{AI\,\alpha}} \frac{S^{\sigma(\alpha-\beta)}}{(1+TECH)^{(\alpha-\beta)}} \tag{32}$$

Allowing us to express the frontier and its derivative as:

$$Q_{IR} = (\delta_0 - \delta_1 Q_E^{\alpha})^{\frac{1}{\beta}} \tag{33}$$

$$\frac{dQ_{IR}}{dQ_E} = -\frac{\alpha}{\beta}\delta_1 Q_E^{\alpha-1}(\delta_0 - \delta_1 Q_E^{\alpha})^{\frac{1}{\beta}-1} \tag{34}$$

To guarantee that the study possibilities frontier is strictly concave to the origin $\left(\frac{d^2Q_{IR}}{dQ_E^2} < 0\right)$, ensuring a well-behaved optimization problem with a unique global interior maximum, we evaluate the second-order structural conditions. Differentiating (28) yields:

$$\frac{d^2Q_{IR}}{dQ_E^2} = -\frac{\alpha}{\beta}\mu_1 Q_E^{\alpha-2}(\mu_0 - \mu_1 Q_E^{\alpha})^{\frac{1}{\beta}-2}\left[(\alpha-1)\mu_0 + \mu_1 Q_E^{\alpha}\left(1-\frac{\alpha}{\beta}\right)\right] \quad (35)$$

Given that $\alpha > 1, \beta > 1, \mu_0 > 0, \mu_1 > 0$ by our structural parameter definitions, the outside multipliers are strictly negative, and the interior bracketed terms remain strictly positive for any active allocation ($Q_E > 0$). In fact, from the frontier definition (27) we obtain
$Q_{IR}^{\beta} = \mu_0 - \mu_1 Q_E^{\alpha}$

Hence,

$$\begin{cases} Q_{IR}^{\beta} = 0 \implies \mu_0 - \mu_1 Q_E^{\alpha} = 0 \implies Q_E^{\alpha} = \frac{\mu_0}{\mu_1} \\ Q_{IR}^{\beta} > 0 \implies \mu_0 - \mu_1 Q_E^{\alpha} > 0 \implies Q_E^{\alpha} < \frac{\mu_0}{\mu_1} \end{cases} \quad (36)$$

The term in brackets is strictly positive for all active allocations over the feasible set:

$$\left[(\alpha-1)\mu_0 + \mu_1\frac{\mu_0}{\mu_1}\left(1-\frac{\alpha}{\beta}\right)\right] = \mu_0\alpha\left(1-\frac{\alpha}{\beta}\right) > 0 \quad (37)$$

This establishes $\frac{d^2Q_{IR}}{dQ_E^2} < 0$, confirming that within-session study fatigue naturally generates a strictly concave transformation frontier across all technology specifications. Because TECH1 and TECH3 share the same functional form, and TECH2 is analogous with Q_E replaced by $Q_E - TECH$, concavity holds across all technology specifications.

Figure 1 and Figure 2 simulate, respectively, the effect on studying of the study–fatigue parameter and stress sensitivity on the shape of the study possibilities frontier.

Figure 1 illustrates how increasing the fatigue parameter α —representing a student who struggles more with quantitative study—raises the marginal time cost of producing Economics knowledge. Holding all other variables constant, we observe that under model specification TECH1, increasing α from 1.2 to 1.8 approximately halves the maximum attainable "knowledge output" in Economics when the student allocates all available time to that subject.

The top panels in Figure 2 compare a low level of stress sensitivity ($\sigma = 1$) with a higher level ($\sigma = 2$). Even with identical daily commitments and identical preferences over subjects, a student with higher stress sensitivity produces strictly less knowledge in both domains. This reflects the amplifying effect of stress on the effective time cost of studying.

The bottom-left panel of Figure 2 shows that increasing the background stress level (ϵ) while holding sensitivity constant, reduces output in both subjects. Finally, the bottom-right panel illustrates the effects of "eustress" framed as optimal stress-resilience. Here, excellent rest and recovery shrink residual cognitive fatigue toward zero ($\epsilon \longrightarrow 0$), minimizing the denominator penalty and shifting the study possibilities frontier to its maximum outward structural capacity.

A notable feature of the TECH2 specification is that the frontier does not intersect the axes. Instead, it has endpoints at the coordinates $(TECH, Q_{IR}^{MAX})$ and $(Q_E^{MAX}, TECH)$. This "offset" illustrates the exogenous nature of the technological bonus which effectively establishes a "technological floor" for academic output independently of the student's active study time allocation.

4. The Student's Constrained Optimization Problem

The student maximizes their utility subject to the constraint represented by their production possibilities, the SPF. We will consider in turn each SPF, from the baseline (22) to the TECH3 specification (25).

Figure 1: Sensitivity Analysis: The Impact of the Study–Fatigue Parameter (α) on the SPF

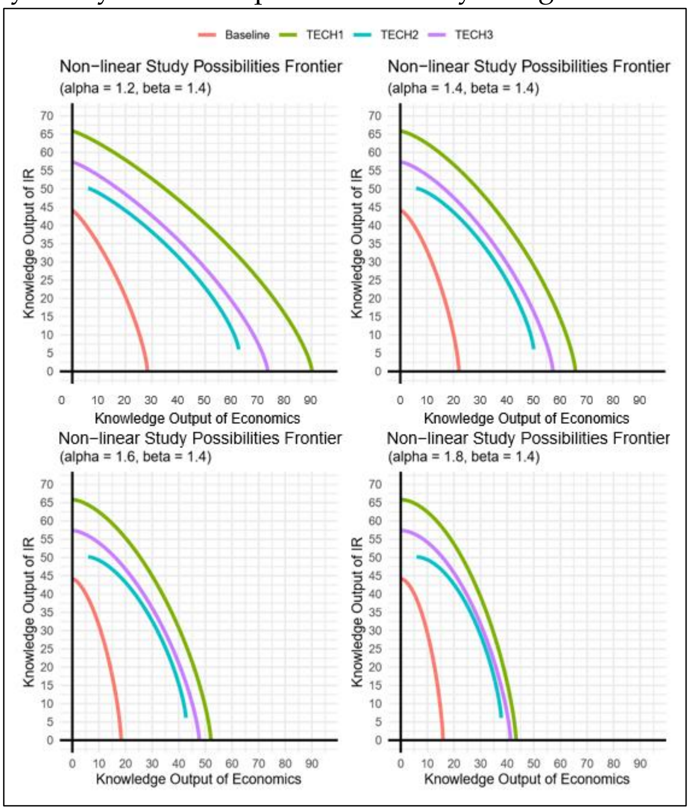
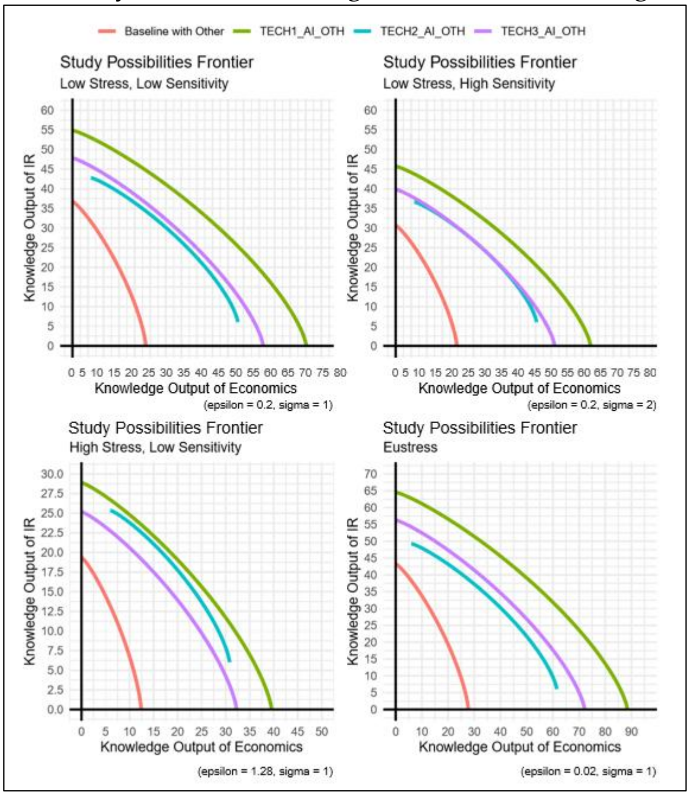


Figure 2: Sensitivity of the SPF to Cognitive Stress and Fatigue Resilience



To simplify the constrained optimization problem, we assume that the student has already allocated time T_W and T_R, so that the student only allocates the residual effective time available to study. Consequently, we group the parameters that are exogenous to the student's study allocation in the time constraint. Specifically, we define the scaling parameters Π_0 and Ψ_0 as the effective unit time requirements for each subject:

$$\Psi_0 = \frac{(1+\omega T_W-\rho T_R)^\sigma}{k_E} \quad \text{and } \Pi_0 = \frac{(1+\omega T_W-\rho T_R)^\sigma}{k_{IR}} \tag{38}$$

This allows us to express the study possibilities frontier (22) in the following condensed implicit form:

$$(Q_E\Psi_0)^\alpha + (Q_{IR}\Pi_0)^\beta = \bar{T} \tag{39}$$

where $\bar{T}$ denotes the total available time for studying regardless of the model specification.

The following constrained maximization problem is solved using the Lagrangian technique:

$$\max_{Q_E,Q_{IR}} Q_E^\gamma Q_{IR}^{1-\gamma}$$
$$s.t. (Q_E\Psi_0)^\alpha + (Q_{IR}\Pi_0)^\beta = \bar{T} \tag{40}$$

The corresponding Lagrangian is:

$$\mathcal{L} = Q_E^{\gamma} Q_{IR}^{1-\gamma} + \lambda\left[\bar{T} - (Q_E \Psi_0)^{\alpha} - (Q_{IR}\Pi_0)^{\beta}\right] \tag{41}$$

The first-order conditions (FOC) are:

$$\frac{\partial \mathcal{L}}{\partial Q_E} = \gamma Q_E^{\gamma-1} Q_{IR}^{1-\gamma} - \lambda\alpha\Psi_0 (Q_E\Psi_0)^{\alpha-1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial Q_{IR}} = (1-\gamma) Q_E^{\gamma} Q_{IR}^{-\gamma} - \lambda\beta\mathit{\Pi}_0 (Q_{IR}\mathit{\Pi}_0)^{\beta-1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{T} - (Q_E\Psi_0)^{\alpha} - (Q_{IR}\Pi_0)^{\beta} = 0 \tag{42}$$

The solution to the maximization problem yields:

$$Q_{IR}^{*} = \frac{1}{\Pi_0}\left[\frac{(1-\gamma)\alpha}{\gamma\beta+(1-\gamma)\alpha}\bar{T}\right]^{\frac{1}{\beta}} \tag{43}$$

$$Q_E^{*} = \frac{1}{\Psi_0}\left[\frac{\gamma\beta}{\gamma\beta+(1-\gamma)\alpha}\bar{T}\right]^{\frac{1}{\alpha}} \tag{44}$$

By rearranging these expressions, we identify the optimal time allocation (T^{*}) for both subjects:

$$(Q_{IR}^{*}\Pi_0)^{\beta} = \frac{(1-\gamma)\alpha}{\gamma\beta+(1-\gamma)\alpha}\bar{T} = T_{IR}^{*} \tag{45}$$

$$(Q_E^{*}\Psi_0)^{\alpha} = \frac{\gamma\beta}{\gamma\beta+(1-\gamma)\alpha}\bar{T} = T_E^{*} \tag{46}$$

As a confirmation check, the sum of these allocations exhausts the total available time ($T_E^{*} + T_{IR}^{*} = \bar{T}$).

These results provide several key insights into the student's decision-making process:

- **Interaction of Preference and Difficulty:** The optimal time allocation is determined by the interaction of preferences (γ) and the difficulty parameters (α, β). Specifically, time spent on Economics (T_E^{*}) is driven by the weighted term $\gamma\beta$, implying that the relative difficulty of the *other* subject also influences time allocation.
- **Asymmetric Output:** While time allocation shares are symmetric in structure, the actual knowledge output (Q^{*}) is governed by the exponents $1/\alpha$ and $1/\beta$. This

implies that for a "struggling" student (high α), even a significant time investment results in lower total and marginal output compared to a *Natural* student.

- **Scale of Production:** The shadow prices Π_0 and Ψ_0 act as productivity scalers. While external factors like work and rest (T_W, T_R) affect the overall scale of what can be produced, they do not alter the *proportion* of time the student chooses to allocate between subjects. Preferences and difficulty alone determine the mix.

The optimal solutions for TECH1 and TECH3 specifications are straightforward to derive. In fact, the baseline solution logic applies directly to TECH1; the difference lies in the definition of $\bar{T}$, which increases to account for the technological factor $TECH$, and k_E^{AI}. On the other hand, for TECH3, once we define the updated time-costs Ψ_1 and Π_1 based on (14) as:

$$\Psi_1 = \frac{(1+\omega T_W - \rho T_R)^\sigma}{k_E^{AI}(1+TECH)} \text{ and } \Pi_1 = \frac{(1+\omega T_W - \rho T_R)^\sigma}{k_{IR}(1+TECH)} \tag{47}$$

the optimal solutions for Q_{IR} and Q_E become, respectively:

$$Q_{IR}^* = \frac{1}{\Pi_1}\left[\frac{(1-\gamma)\alpha}{\gamma\beta+(1-\gamma)\alpha}\bar{T}\right]^{\frac{1}{\beta}} \tag{48}$$

And,

$$Q_E^* = \frac{1}{\Psi_1}\left[\frac{\gamma\beta}{\gamma\beta+(1-\gamma)\alpha}\bar{T}\right]^{\frac{1}{\alpha}} \tag{49}$$

Conversely, the TECH2 specification does not yield a closed-form solution. The TECH2 specification differs fundamentally because the technological bonus enters outside the concave transformation.

For TECH2, the unit time-costs are still Ψ_0 and Π_0, accounting for AI, but the student solves the following constrained optimization problem:

$$\begin{gathered}\max_{Q_E, Q_{IR}} Q_E^\gamma Q_{IR}^{1-\gamma} \\ s.t. [(Q_E - TECH)\Psi_0]^\alpha + [(Q_{IR} - TECH)\Pi_0]^\beta = \bar{T}\end{gathered} \tag{50}$$

The corresponding Lagrangian is

$$\mathcal{L} = Q_E^\gamma Q_{IR}^{1-\gamma} + \lambda\{\bar{T} - [(Q_E - TECH)\Psi_0]^\alpha - [(Q_{IR} - TECH)\Pi_0]^\beta\} \tag{51}$$

where we assume the student chooses production levels that strictly exceed the technological floor $(Q_i > TECH)$.

The first-order conditions (FOC) are:

$$\frac{\partial \mathcal{L}}{\partial Q_E} = \gamma Q_E^{\gamma-1} Q_{IR}^{1-\gamma} - \lambda \alpha \Psi_0 [(Q_E - TECH)\Psi_0]^{\alpha-1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial Q_{IR}} = (1-\gamma) Q_E^{\gamma} Q_{IR}^{-\gamma} - \lambda \beta \Pi_0 [(Q_{IR} - TECH)\Pi_0]^{\beta-1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{T} - [(Q_E - TECH)\Psi_0]^{\alpha} - [(Q_{IR} - TECH)\Pi_0]^{\beta} = 0 \quad (52)$$

The first two first-order conditions lead to:

$$\frac{\gamma}{1-\gamma}\frac{Q_{IR}}{Q_E} = \frac{\alpha}{\beta}\frac{\Psi_0}{\Pi_0}\frac{[(Q_E - TECH)\Psi_0]^{\alpha-1}}{[(Q_{IR} - TECH)\Pi_0]^{\beta-1}} \quad (53)$$

As shown in (53), a closed-form solution is unattainable unless the restriction $\alpha = \beta = 1$ is imposed.

5. Simulation Calibration and Student Archetypes

To operationalize the parametric two-good model and evaluate the welfare and distributive effects of technological interventions, we introduce four distinct student archetypes. These archetypes are structurally parameterized across a 2×2 matrix defined by cognitive processing efficiency (the subject difficulty parameter, $\alpha_i > 1$) and academic preferences (the utility weight, $\gamma_i \in (0,1)$). Each archetype represents a distinct combination of aptitude and motivation:

- **The "Natural":** Students who demonstrate a high innate aptitude for Economics ($\alpha = 1.2$) and place a high subjective value on the field ($\gamma = 0.8$). Benefiting from efficient knowledge absorption and strong personal motivation, this archetype achieves elevated output levels.
- **The "Calculative Struggler":** Students who find the quantitative discipline inherently challenging ($\alpha = 1.8$) but consider it professional or strategically essential ($\gamma = 0.8$). This archetype represents individuals who face steep marginal time costs but persist because they view economic literacy as a critical tool for their future careers.
- **The "Efficient Minimalist":** Students who possess a high aptitude for Economics ($\alpha = 1.2$) but lack personal interest or strategic motivation ($\gamma = 0.2$). Characterized by high cognitive efficiency paired with low academic engagement, they produce only a baseline output before reallocating resources to alternative pursuits.
- **The "Uninterested Struggler":** Students who find Economics structurally difficult ($\alpha = 1.8$) and lack interest in the discipline ($\gamma = 0.2$). Typically, these students enroll only because the course is a mandatory curricular requirement, exerting the minimum effort necessary to pass.

By holding these internal traits (α, γ) constant within each profile, we can systematically simulate how external shocks—specifically the introduction of general technology $(TECH)$ and asymmetric, AI-assisted study technologies (θ)—reconfigure the optimal production frontiers, knowledge quantities, and time allocation vectors for each respective student type. Table 1 reports the explicit numerical calibration matrix used to initialize the simulations.

Table 1: Parametric Calibration Matrix for Student Archetypes

Archetype	**Difficulty** (α)	**Preference** (γ)	**Expected Simulation Behavior**
1. The Natural	$\alpha = 1.2$ (Low)	$\gamma = 0.8$ (High)	High Investment / High Volume Output
2. The Calculative Struggler	$\alpha = 1.8$ (High)	$\gamma = 0.8$ (High)	High Time Drain / Target-Driven Resilience
3. The Efficient Minimalist	$\alpha = 1.2$ (Low)	$\gamma = 0.2$ (Low)	Low Time Investment / Shift to Preferred Subject
4. The Uninterested Struggler	$\alpha = 1.8$ (High)	$\gamma = 0.2$ (Low)	High Failure Risk / Bound to Minimum Floor

Figures 3 to 10 show the results of simulations based on the student archetypes. Figures 3–6 illustrate the effect of general technology only, while Figures 7–10 include the effect of AI tools on the learning rate. Table 2, Table 3, and Table 4 report the numerical results, which are discussed in detail in Section 6.

6. Discussion of Simulation Results

6.1 Analysis of Baseline Efficiency and Archetype Disparities

The baseline results compiled in Table 2 demonstrate that academic achievement is driven by the simultaneous interaction of psychological preferences (γ) and baseline cognitive friction α. For archetypes with identical preferences, structural variations in subject difficulty generate stark disparities in total knowledge production and realized utility.

Consider the contrast between the *Natural* student and the *Calculative Struggler*, both of whom allocate a high subjective weight to quantitative knowledge $(\gamma = 0.8)$. The *Natural* student, benefiting from a low fatigue rate $(\alpha = 1.2)$, achieves an elevated and well-balanced output vector, leading to a high utility baseline $(U^* = 17.67)$. Conversely, the *Calculative Struggler* faces steep diminishing marginal returns to active study time $(\alpha = 1.2)$. Despite identical strategic motivations, this cognitive friction severely suppresses their quantitative output, dragging down overall utility $(U^* = 11.72)$ and illustrating how innate difficulty limits the translation of raw student ambition into realized academic welfare.

Figure 3: Optimal Study Mix for the "Natural" under General Technology

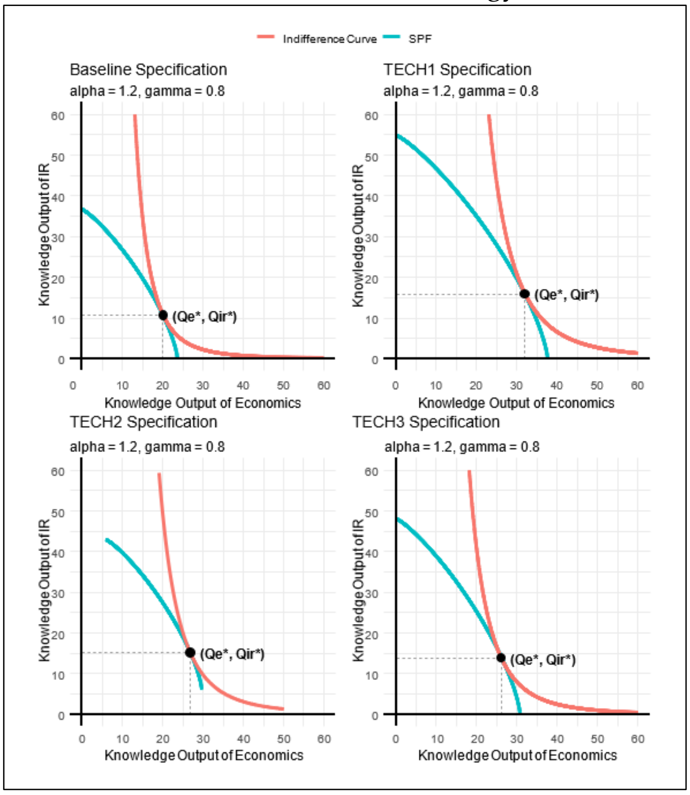


Figure 4: Optimal Study Mix for the "Calculative Struggler" under General Technology

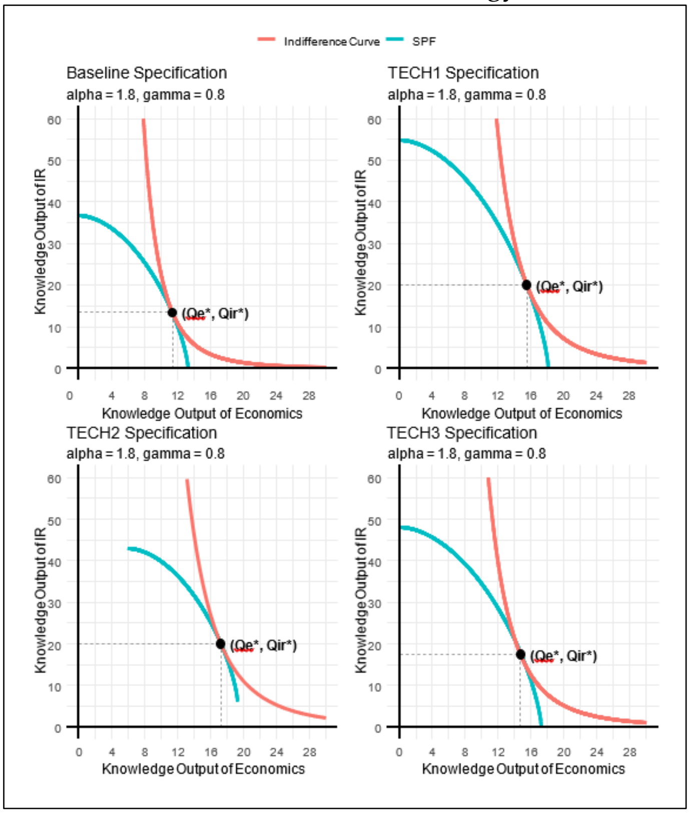


Figure 5: Optimal Study Mix for the "Efficient Minimalist" under General Technology

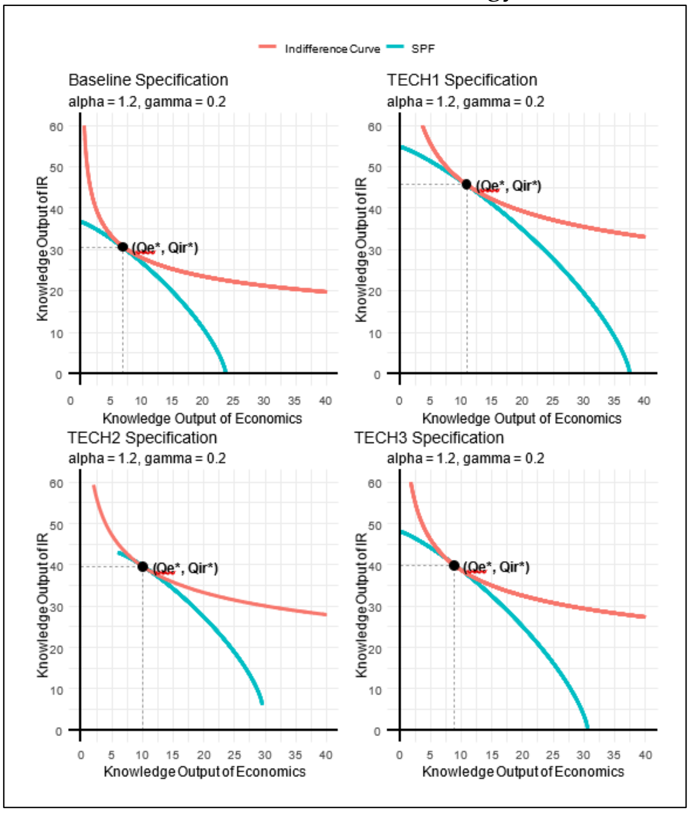


Figure 6: Optimal Study Mix for the "Uninterested Struggler" under General Technology

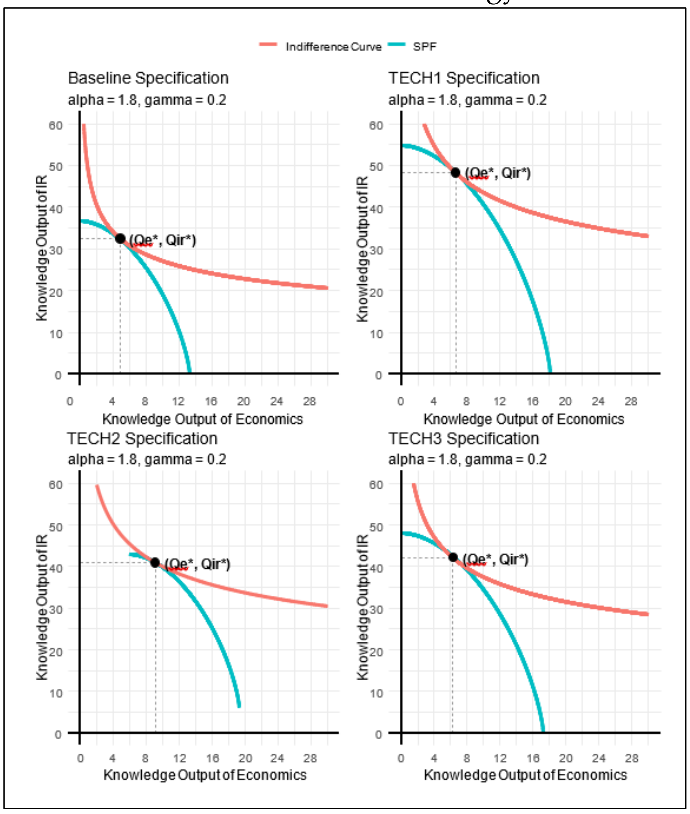


Figure 7: Optimal Study Mix for the "Natural"
under General Technology and AI Tools

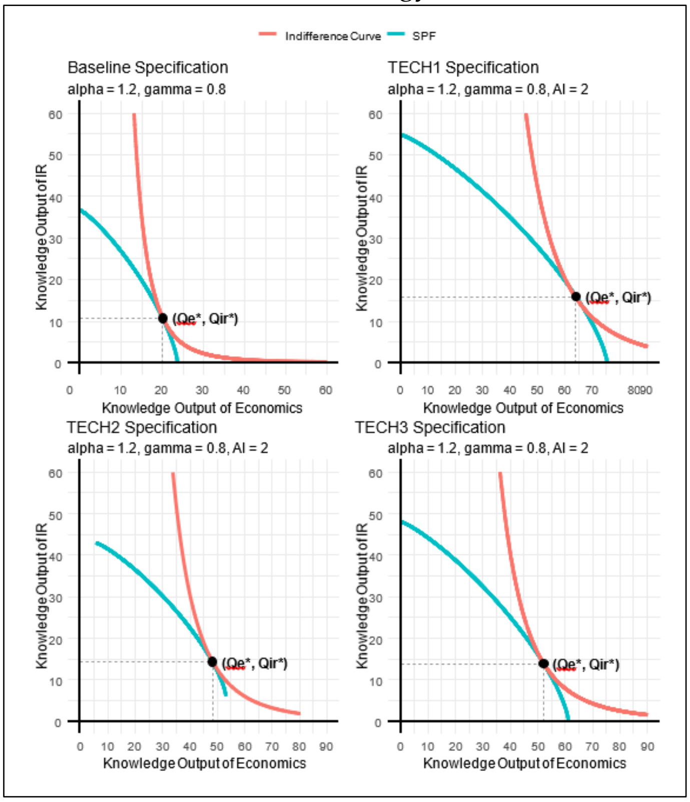


Figure 8: Optimal Study Mix for the "Calculative Struggler"
under General Technology and AI Tools

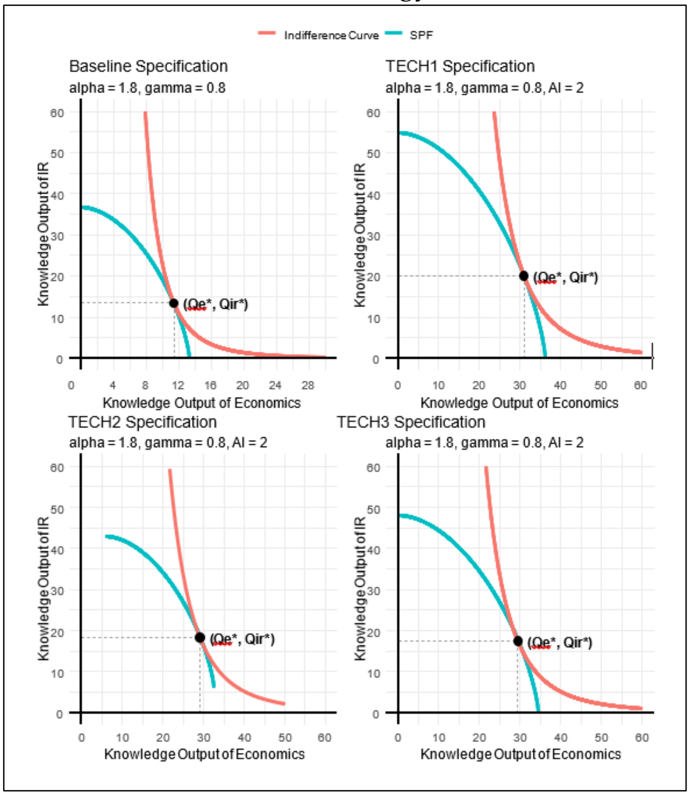


Figure 9: Optimal Study Mix for the "Efficient Minimalist" under General Technology and AI Tools

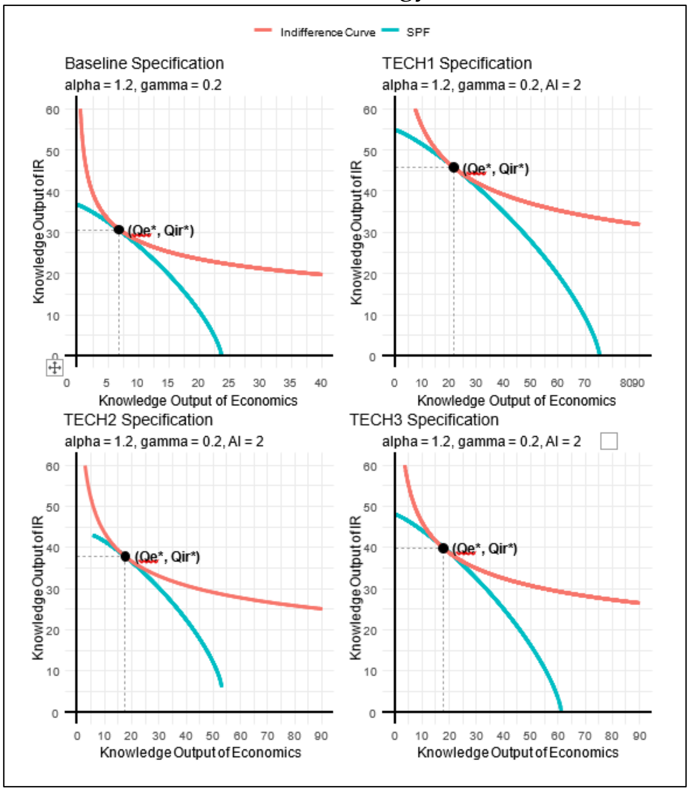


Figure 10: Optimal Study Mix for the "Uninterested Struggler" under General Technology and AI Tools

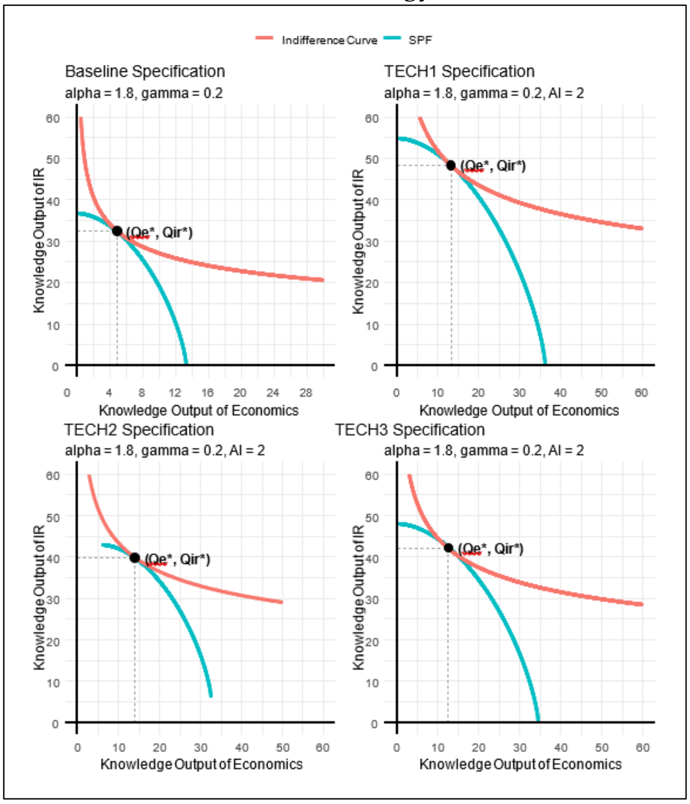


Table 2: Comparison of Optimal Study Mix for Student Archetypes (Baseline)

Student Archetype	**Model**	Q_E^*	Q_{IR}^*	U^*
Natural	Baseline	20.05	10.66	17.67
Calculative Struggler	Baseline	11.33	13.41	11.72
Efficient Minimalist	Baseline	6.82	30.65	22.70
Uninterested Struggler	Baseline	4.82	32.42	22.15

Simulation Parametrization. The four student archetypes are parameterized across the 2×2 matrix as follows: *Natural* $(\alpha = 1.2, \gamma = 0.8)$, *Calculative Struggler* $(\alpha = 1.8, \gamma = 0.8)$, *Efficient Minimalist* $(\alpha = 1.2, \gamma = 0.2)$, and *Uninterested Struggler* $(\alpha = 1.8, \gamma = 0.2)$. Exogenous baseline parameters held constant across all specifications are set to $\beta = 1.4, k_E = 5, k_{IR} = 10, T_R = 12, T_W = 4, \rho = 0.1, \omega = 0.35, \sigma = 1, \theta = 1$, and $TECH = 0$.

Table 3: Comparison of Optimal Study Mix for Student Archetypes (Only General Technology)

Student Archetype	**Model**	Q_E^*	Q_{IR}^*	U^*
Natural	TECH1	31.96	15.99	27.96
Calculative Struggler	TECH1	15.46	20.00	16.29
Efficient Minimalist	TECH1	10.87	45.72	34.30
Uninterested Struggler	TECH1	6.58	48.34	32.45
Natural	TECH2	26.73	15.18	23.86
Calculative Struggler	TECH2	17.22	19.92	17.73
Efficient Minimalist	TECH2	10.09	39.53	30.08
Uninterested Struggler	TECH2	9.05	40.91	30.25
Natural	TECH3	26.06	13.86	22.97
Calculative Struggler	TECH3	14.73	17.43	15.23
Efficient Minimalist	TECH3	8.87	39.85	29.50
Uninterested Struggler	TECH3	6.27	42.14	28.79

Simulation Parametrization. The four student archetypes are parameterized across the 2×2 matrix as follows: Natural $(\alpha = 1.2, \gamma = 0.8)$, Calculative Struggler $(\alpha = 1.8, \gamma = 0.8)$, Efficient Minimalist $(\alpha = 1.2, \gamma = 0.2)$, and Uninterested Struggler$(\alpha = 1.8, \gamma = 0.2)$. Exogenous baseline parameters held constant across all specifications are set to $\beta = 1.4, k_E = 5, k_{IR} = 10, T_R = 12, T_W = 4, \rho = 0.1, \omega = 0.35, \sigma = 1,$and $\theta = 1$. The technology shock parameters are specified as $TECH = 6$ under the TECH1 and TECH2 specifications, and scaled to $TECH = 0.3$ under the multiplicative TECH3 specification.

Table 4: Comparison of Optimal Study Mix for Student Archetypes using AI Tools (General Technology and AI Tools)

Student Archetype	**Model**	Q_E^*	Q_{IR}^*	U^*
Natural	TECH1	63.92	15.99	48.39
Calculative Struggler	TECH1	30.92	20.00	28.34
Efficient Minimalist	TECH1	21.75	45.72	39.40
Uninterested Struggler	TECH1	13.17	48.34	37.27
Natural	TECH2	48.20	14.31	37.80
Calculative Struggler	TECH2	29.15	18.21	26.53
Efficient Minimalist	TECH2	17.61	37.77	32.42
Uninterested Struggler	TECH2	13.98	39.71	32.23
Natural	TECH3	52.13	13.86	40.00
Calculative Struggler	TECH3	29.46	17.43	26.52
Efficient Minimalist	TECH3	17.73	39.85	33.89
Uninterested Struggler	TECH3	12.55	42.14	33.07

Simulation Parametrization. The four student archetypes are parameterized across the 2×2 matrix as follows: *Natural* ($\alpha = 1.2, \gamma = 0.8$), *Calculative Struggler* ($\alpha = 1.8, \gamma = 0.8$), *Efficient Minimalist* ($\alpha = 1.2, \gamma = 0.2$), and *Uninterested Struggler*($\alpha = 1.8, \gamma = 0.2$). Exogenous baseline parameters held constant across all specifications are set to $\beta = 1.4, k_E = 5, k_{IR} = 10, T_R = 12, T_W = 4, \rho = 0.1, \omega = 0.35, \sigma = 1$,and $\theta = 2$. The technology shock parameters are specified as $TECH = 6$ under the $TECH1$ and $TECH2$ specifications, and scaled to $TECH = 0.3$ under the multiplicative $TECH3$ specification.

A more subtle and compelling economic phenomenon emerges on the opposite side of the preference matrix ($\gamma = 0.2$). Both the *Efficient Minimalist* and the *Uninterested Struggler* share a low appreciation for quantitative study, heavily prioritizing the nonquantitative numeraire discipline (IR). However, because the *Uninterested Struggler* finds Economics structurally exhausting ($\alpha = 1.8$), they face an acute optimization trap. To satisfy their preference structure, they flee the high-fatigue quantitative discipline, which paradoxically drives their output in International Relations ($Q_{IR}^* = 32.42$) *higher* than that of the *Efficient Minimalist* ($Q_{IR}^* = 30.65$). Despite producing more of their preferred subject, the *Uninterested Struggler* suffers a definitive welfare penalty, achieving a lower overall utility level ($U^* = 22.15$) than the *Efficient Minimalist* ($U^* = 22.70$). This shows that severe cognitive friction in a single mandatory field creates an unavoidable allocative drag, distorting the student's entire study mix and capping their global academic welfare.

6.2 Analysis of Baseline Technology Efficiency ($\boldsymbol{\theta = 1}$)

The results compiled in Table 3 illustrate the divergent impact of general technological specifications when students operate at their baseline learning rate without domain-specific artificial intelligence ($\theta = 1$).

- **The Efficiency-Enhancing Effect**: Across all four student archetypes, the introduction of any technological infrastructure ($TECH$) induces an outward shift of the study possibilities frontier, yielding strictly higher optimized utility levels (U^*) relative to the baseline state. This confirms that symmetric, domain-neutral technological interventions effectively mitigate the global time and cognitive constraints bounding student welfare.

- **The Structural Safety Floor Effect**: For the *Calculative Struggler* ($\alpha = 1.8$), the TECH2 architecture yields a significantly higher utility level ($U^* = 17.73$) than both TECH1 ($U^* = 16.29$) and TECH3 ($U^* = 15.23$). This numerical property delivers a vital insights into academic intervention design: for students facing steep conceptual barriers, a fixed, autonomous knowledge endowment is vastly superior to either an extension of raw study time or a multiplicative productivity shift. Because the autonomous bonus is entirely decoupled from active study hours, it allows the struggling student to bypass the most inefficient, high-fatigue portion of their production function where cognitive friction makes early progress disproportionately costly.
- **The Minimalist Reallocation Strategy**: Both the *Efficient Minimalist* and the *Uninterested Struggler* ($\gamma = 0.2$) exploit technological interventions primarily to maximize knowledge accumulation in their preferred subject (Q_{IR}^*). Under the TECH1 time-expansion model, these low-engagement archetypes direct their expanded resource pool almost entirely toward International Relations, driving Q_{IR}^* to its highest respective levels (45.72 and 48.34). This behavior demonstrates that when a student possesses low subjective valuation for a discipline, they treat that subject as a minor curricular constraint to be satisfied with minimum effective effort rather than an academic asset to be mastered.

6.3 Analysis of Enhanced Efficiency ($\boldsymbol{\theta = 2}$)

Table 4 shows the results when AI-assisted study tools effectively double the learning rate ($\theta = 2$), equalizing the learning rate ratio across subjects.

- **Productivity vs. Content Bonus**: With higher base efficiency, TECH3 (the productivity multiplier) becomes more attractive for high-interest students. For the *Natural*, utility under TECH3 (40.00) now surpasses TECH2 (37.80). This suggests that once a student is efficient enough, they prefer tools that scale with their effort rather than fixed content bonuses.
- **Interest as the Primary Driver**: When AI makes Economics "easier" for everyone, the variability in Q_E^* is driven almost entirely by the preference parameter γ. The *Natural* produces nearly four times the output in Economics as the *Uninterested Struggler*, despite both having the same *AI* boost. This underscores the central role of intrinsic motivation in shaping study choices.

6.4 Cross-Table Comparison: The Structural Role of AI Integration

A cross-table comparative analysis between the baseline technology simulation ($\theta = 1$) and the artificial intelligence integration environment ($\theta = 2$) reveals several profound implications of domain-specific automation for educational production and allocative efficiency:

- **The Diminishing Relative Advantage of the Output Floor**: As the AI-driven quantitative efficiency parameter increases from $\theta = 1$ to $\theta = 2$, the relative superiority of the TECH2 "safety net" diminishes for high-aptitude profiles. By raising the baseline learning rate (k_E^{AI}), AI tools effectively smooth out the early, high-fatigue segments of the study possibilities frontier. Consequently, the guaranteed, autonomous knowledge units provided by TECH2 become less impactful compared to the compounding, multiplicative gains unlocked under the TECH3 specification.
- **The Strategic Efficiency Release for Low-Engagement Archetypes**: For the *Uninterested Struggler*, transitioning to an AI-enabled environment ($\theta = 2$) nearly doubles their quantitative output (Q_E^*). However, this archetype chooses to allocate the entirety of their "AI-saved time" to the non-quantitative discipline (Q_{IR}^*). This confirms that domain-specific AI functions primarily as an efficiency enabler; it permits low-engagement students to satisfy mandatory, high-friction curricular requirements with minimal disruption to their primary academic interests.
- **Mitigation of Cognitive Opportunity Costs**: The structural transition to $\theta = 2$ significantly flattens the utility penalty traditionally borne by the *Calculative Struggler*, yielding a non-marginal expansion in welfare that demonstrates how field-specific AI tools effectively neutralize the steep opportunity costs of pursuing a strategically vital but naturally difficult discipline.
- **Cognitive Depreciation and the Adverse AI Shock ($\mathbf{0 < \theta < 1}$)**: Thus far, the parameter $\theta \geq 1$ has been treated strictly as a positive shock to comprehension. If a student relies on AI to aggressively bypass active cognitive exercises rather than to scaffold their comprehension, modeled via the parameter range $0 < \theta < 1$, they sacrifice the critical "learning-by-doing" mechanism essential to quantitative mastery. Mathematically, this induces a negative structural shift ($k_E^{AI} < k_E$), increasing the opportunity cost of active study time in the quantitative domain.

7. Conclusion

This paper has developed a microeconomic framework to model a student's optimal time allocation across distinct academic disciplines under varying technological states. By structurally incorporating physiological constraints—specifically, within-session study fatigue (α_i) and non-discretionary environmental background stress (S^σ)—the model captures the realistic boundaries that govern cognitive absorption. The resulting structure bridges psychological realism with classical production theory, demonstrating that cognitive fatigue endogenously generates a strictly concave Study Possibilities Frontier (SPF).

The parameterization of four student archetypes shows that academic performance is not determined by effort alone, but by the equilibrium interaction between intrinsic motivation (γ) and baseline learning capacity. The simulation results yield several insights with direct relevance for educational design and policy:

- **Technology as a Structural Equalizer.** Symmetric general technology ($TECH$) shifts the SPF outward for all students, but its welfare effects differ sharply across archetypes. The autonomous output floor provided by TECH2 functions as an academic safety net, disproportionately benefiting low-engagement or high-fatigue students by allowing them to bypass the steep, high-friction region of their learning production functions.
- **AI as an Efficiency-Releasing Mechanism.** Domain-specific artificial intelligence (θ) substantially reduces the opportunity cost of mastering difficult subjects. Under stable preference weights, AI enables low-engagement students to satisfy mandatory quantitative requirements quickly, freeing effective time for their preferred fields without sacrificing welfare. For high-aptitude students, AI amplifies the returns to effort, making multiplicative productivity technologies (TECH3) increasingly attractive.
- **The Risk of Cognitive Depreciation.** The framework also highlights the potential downside of automation. If AI is used to bypass rather than scaffold cognitive engagement ($0 < \theta < 1$), the erosion of learning-by-doing reduces effective mastery and increases the opportunity cost of future study. This adverse AI shock contracts the SPF and diminishes long-run academic capacity.

Beyond digital tools, the simulations provide a theoretical critique of traditional, uniform lecture formats. Because students differ markedly in fatigue parameters (α_i) and preference profiles (γ_i), long standardized lectures push struggling students into the steep, diminishing-returns region of their learning functions, where absorption collapses. The model therefore supports reallocating instructional resources away from extended lectures and toward high-engagement, small-group environments, targeted seminars, and expanded office hours. Such interventions operationalize the logic of the TECH2 architecture: they provide personalized conceptual endowments that help students bypass initial high-friction barriers, reducing cognitive opportunity costs while enabling motivated students to accelerate along their frontiers.

Overall, the framework demonstrates that educational interventions are not one-size-fits-all. As artificial intelligence and automation reshape academic environments, institutions must transition from passive mass instruction to targeted, structurally informed pedagogical strategies that align with the heterogeneous friction profiles of the student population. Future research may extend this model by endogenizing work and rest decisions or by empirically validating the parametric predictions using time-use and academic performance data.

Declaration of Generative AI and AI-assisted Technologies in the Manuscript Preparation Process

Statement: During the preparation of this work the author used Google Gemini in order to improve text readability. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

Conflict of Interest Statement

The author declares no conflicts of interest.

About the Author(s)

Massimiliano Porto is an Associate Professor in the College of Asia Pacific Studies at Ritsumeikan Asia Pacific University (APU), Japan. He holds a PhD in Economics from Kobe University, Japan.
ORCID: https://orcid.org/0000-0002-5851-3429

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