European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available online at: www.oapub.org/edu

DOI: 10.46827/ejes.v13i6.6731 Volume 13 | Issue 6 | 2026

THE ROLE OF MATHEMATICS EDUCATION ADVISORS IN THE DESIGN AND IMPLEMENTATION OF SEMINARS AND WORKSHOPS: CONSULTANT, MANAGER, RESEARCHER, OR LEADER?

Stavroula Patsiomitou[i]
Education Advisor for Mathematics,
C Area, Athens,
Ph.D. University of Ioannina,
M.Ed. National and Kapodistrian University of Athens,
Greece

Abstract:
This paper describes the role of Mathematics Education Advisors in designing, organizing, and implementing professional development seminars and workshops aimed at supporting mathematics teachers' professional development, pedagogical innovation, and instructional practices in digital learning environments. The study conceptualizes this role as extending beyond administrative responsibilities to encompass pedagogical leadership, instructional design, and the development of professional learning communities. The study is grounded in didactical and theoretical perspectives on Mathematics Education. In relation to my own contribution, my research in dynamic geometry, together with the conceptual constructs I introduced within this framework, constitutes a central theoretical foundation not only of the present study but also of the seminars and professional development activities that I organize and implement. Within this context, I present and analyse a series of professional development initiatives that integrate dynamic geometry software, digital technologies, and emerging tools such as artificial intelligence. A key dimension of my work involves the design and structure of digital learning repositories that extend classroom practice and promote the dissemination /distribution of interactive learning materials. As an illustrative example, I present my work on the systematic transformation of assessment tasks drawn from the Item Bank of Graded Difficulty (IBGD) of the Hellenic Institute of Educational Policy (IEP), in which symbolic, verbal, and mathematical representations are converted into graphical representations within the GeoGebra dynamic geometry environment, conceptualized through a "Dynamic Calculus" approach. Overall, the study concludes that seminars and workshops, when grounded in robust didactical frameworks and supported by digital technologies, can function as powerful

[i] Correspondence: email spatsiomitou@gmail.com

mechanisms for pedagogical transformation, bridging educational theory, policy, and classroom practice.

Keywords: mathematics teacher education; seminars and workshops; digital environments; mathematics education advisor

1. Introduction: a comprehensive framework for the role of Education Advisors

The current study explores the multifaceted role of a Mathematics Education Advisor as a designer and facilitator of seminars and workshops aimed at enhancing mathematics teaching and learning through digital innovation and pedagogical leadership. The study positions this role as extending beyond administrative responsibilities to include pedagogical leadership and the cultivation of professional learning communities. This study presents a few workshops, two-day seminars or symposia for mathematics teachers which I designed, organized and coordinated, emphasizing the integration of modern pedagogical and digital tools, such as the use of dynamic geometry, incorporating also emerging technologies and artificial intelligence. The paper discusses the practical application of consultancy and presents a unique perspective on a relatively unexplored field that significantly contributes to critical policy analysis. The paper also demonstrates the contribution of these activities to teachers' professional development and the enhancement of students' learning experimental experiences.

Drawing on a series of professional development initiatives, the study highlights how this role extends beyond coordination to include instructional design, educational research and pedagogical leadership. Law 4823/2021, titled "Upgrading of the School, Empowerment of Teachers, and Other Provisions" (FEK A' 136/03.08.2021) [website 1], fundamentally restructured the support and evaluation system of Greek primary and secondary education by introducing *Education Advisors.* Law 4823/2021 introduces *Education Advisors* as key agents and components in the enhancement of education, assigned with the pedagogical and scientific support of teachers and schools. According to Article 10 of Law 4823: "*the role of the Education Advisor is to provide pedagogical and scientific guidance to teachers, to support professional development, and to foster the development of innovative initiatives in the field of education […]. More specifically, the responsibilities of the Education Advisor include supporting the fulfillment of daily teaching and educational needs, observing [and evaluating] classroom instruction and delivering model lessons, monitoring and supporting the operation of school laboratories and libraries, facilitating the effective use of their resources and equipment, and undertaking initiatives aimed at improving teaching in each subject area and ensuring the quality of educational provision* […]."

This legislative framework marks a significant shift in educational governance, positioning pedagogical guidance and quality enhancement at the centre of the educational system, while emphasizing the role of support through the work of Education Advisors. Within this context, the role of a Mathematics Education Advisor is redefined as that of an instructional designer in the field of Mathematics (e.g., Strømskag,

2017) and a facilitator of professional learning communities (PLCs) (e.g., Dimino *et al.*, 2015; Garet *et al.*, 2001). This expanded role also invites consideration of additional dimensions, such as those of manager, leader, and researcher. According to the Cambridge Dictionary [website 2], a *consultant* is defined as someone who provides expert advice on a particular subject; a *manager* as an individual responsible for organizing and, in some cases, training a team; a *researcher* as someone who systematically investigates a subject in order to generate new knowledge or understanding; and a *leader* as a person who guides or influences a group, situation or process. In this light, the role of the Education Advisor extends beyond administrative supervision to encompass active participation in the design and implementation of learning experiences that support both teachers and students across diverse educational contexts.

Within this framework, professional development activities are not approached as formal obligations; rather, they are conceptualized as dynamic spaces for interaction, exchange, critical reflection, and the co-construction of knowledge. The professional development processes I implement are not fragmented; rather, they are grounded in a clear theoretical framework based on communities of practice (Lave & Wenger, 1991; Wenger, 1998), wherein teachers learn through active participation in groups with shared interests, exchanging experiences and practices. According to Wenger-Trayner E., & B. (2015) *Communities of practice are groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly*. Within a community of practice, individuals "share" their professional knowledge, engage in common activities, and interact within the context of a shared enterprise (Lave & Wenger, 1991). A key characteristic of such communities is their alignment with the collective goals and shared practices of the group. Furthermore, teachers' professional development is conceptualized as an ongoing process of interaction, reflection, and collaborative practice, rather than as a series of isolated training events. Buchbinder (2013) presents a dual case study examining how novice teachers *recontextualize* knowledge acquired during their teacher education programs within their new classroom settings. According to Buchbinder (2013, p. 89), this perspective on learning is grounded in the situative framework of Lave and Wenger (1991), which conceptualizes learning as participation in the social practices of communities situated within specific social and physical contexts.

This study situates these practices and professional knowledge within contemporary frameworks (i.e., digital education, the development of the curriculum in class by means of a constructivist process, the use of Dynamic geometry systems (DGS) and STEAM approaches, and the utilization of multiple representations in mathematics teaching and learning process). It also emphasizes collaborative professional development, inclusive education, and the integration of artificial intelligence and digital tools in classroom practice. Mishra & Koehler's (2006) Technology, Pedagogy, and Content Knowledge (TPACK) framework offers an insightful viewpoint on the knowledge necessary for prospective teachers to effectively integrate technology into their instructional practices [website 3]. *The TPACK framework describes how effective* *teaching with technology is possible by pointing out the free and open interplay between technology, pedagogy, and content. Applying TPACK to the task of teaching with technology requires a context-bound understanding of technology, where technologies may be chosen and repurposed to fit the very specific pedagogical and content-related needs of diverse educational contexts* (Kereluik, Mishra, & Koehler, 2010; Mishra & Koehler, 2009 cited in Koehler *et al.*, 2013). Digital media and artificial intelligence have made a strong entry into the educational field (e.g., Anastasiades, 2023), opening up new prospects for improving the quality of educational work and the evaluation of school units. The World Economic Forum's report states that the primary goal for us as educators is to help students achieve their educational objectives [websites 4, 5]. Regarding knowledge, our main aim is to promote digital literacy and literacy in matters related to AI. In a broader context, adhering to the framework of the *seven principles for artificial intelligence in education* as outlined by the World Economic Forum, it is essential to assess the influence of AI on educational practices, foster academic integrity, ensure adherence to current regulations, comprehend the advantages of AI while mitigating associated risks, and preserve human oversight in decision-making processes. Specifically, ChatGPT (*Chat Generative Pre-Trained Transformer*) [website 6], widely known since 2022, is considered a natural language processing tool and an AI program that has been the subject of numerous studies (e.g., Ipek *et al.*, 2023). Some studies have expressed concerns about its introduction into education, while others highlight its potential benefits as well as possible risks (Punie, & Redecker, 2017). These rapid advancements in AI interfaces will generate new educational opportunities for personalised learning, instructional innovation, and adaptive pedagogical models (Tuomi, 2018). As a result, we need well-defined visions and policies that incorporate emerging technological possibilities into the broader context of educational transformation and the future of learning. Therefore, it may be crucial to establish appropriate visions and policies while simultaneously creating innovative models for education and teaching (Tuomi, 2018):

> *"It may, therefore, be necessary to develop appropriate visions and policies by simultaneously creating future-oriented models for education and teaching. Creating concrete experimentations in an authentic context with teachers and experts in education is important. As AI is now very high on the policy agenda, it is too easy to generate high-level visions of the future that claim that AI is the next technical revolution."* (p. 32)

Within this framework, a central role is assigned to the organization of educational events such as workshops, training days, symposia, and conferences. These collective learning spaces function as catalysts for professional dialogue, reflection, and collaboration, enabling educators to share practices, exchange ideas, and co-construct pedagogical knowledge. Such actions support the development of a reflective professional culture and promote continuous improvement in teaching and learning.

Within this evolving context, as an Education Advisor for Mathematics, I have fostered a culture of collaboration among mathematics teachers, cultivating a learning community and a cooperative network focused on the sharing of best practices, ideas, and experiences. Moreover, I assumed an active role in promoting the integration of innovative practices within the schools under my academic responsibility, placing particular emphasis on the implementation of interactive systems and artificial intelligence. I organized professional development seminars for educators focused on the effective use of these tools, alongside initiatives designed to foster students' active engagement. Through sustained communication—both face-to-face and digital, including dedicated online platforms such as my website *Mathematics with Dynamic and Static Tools* [website 7]—I have promoted a continuous dialogue aimed at improving mathematics teaching and learning. Through this platform, I have consistently conveyed the following message: "*Having served for many years as a classroom teacher, I am fully aware of the challenges we face on a daily basis, and I sincerely hope to establish meaningful communication and collaboration with you. In brief, my scientific and pedagogical positions focus on enhancing the quality of our educational work in general, and of mathematics teaching in particular. Our primary goal is to provide high-quality education to our students by integrating digital technology into contemporary teaching approaches, while also ensuring equal learning opportunities for all children. In my new role, I remain fully available to support you in your multifaceted work, to jointly address challenges, and to collaborate constructively*." The ambition of this work is twofold. *First*, it aims to enhance professional development in the teaching of Geometry and Mathematics generally, through the use of contemporary digital tools and innovative instructional practices. *Second*, it seeks to strengthen the human-centered dimension of the educational experience, emphasizing relationships among students, teachers, and knowledge as a fundamental pillar of quality education. It is therefore important to refer to the theoretical framework that I discussed during the lectures I delivered, with the primary aim of strengthening mathematics teachers' theoretical grounding in the didactics and psychology of mathematics, and more specifically in how these theories relate to the use of digital mathematical tools.

2. A Theoretical Framework for Mathematics Teacher Instruction: Instrumentation, Representations, and Learning Trajectories

The present study aims to highlight contemporary theoretical and didactical approaches to the teaching of Mathematics, as these are shaped within a complex and multifaceted educational context. Within this framework, the study of learning processes is fundamentally connected to the evolution of cognitive psychology, which has primarily concentrated on child development. The development of the curriculum in class by means of a constructivist process focuses on an *active learning process* (Piaget, 1937/1971), fuelled by the interaction between their experience, the mental processing of their knowledge (Vygotsky, 1978) and the students' sequential construction of this knowledge (Terwel, 1999). Such processes of knowledge construction are facilitated by the teachers and instructors, who scaffold students' mathematical thinking, facilitate mathematical discussions in class, use mathematical representations, and reinforce alternative learning methods (Hiebert & Carpenter, 1992; Fuson, Carrol & Drueck, 2000, p.277). In this context, Remillard (1999) argues that curriculum materials are "*the primary vehicles used […] to stimulate curricular change [and] to change the nature of students' mathematics learning opportunities*" (p.315). Nevertheless, teachers play a crucial role in developing the curriculum in the classroom, as they are responsible for identifying and addressing students' difficulties and learning needs. Alongside curriculum development, technological tools have also contributed significantly to contemporary approaches to mathematics education. Dynamic geometry systems (DGS) are microworlds (e.g., Edwards, 1998, p. 74) designed to facilitate the teaching and learning of Euclidean geometry, Algebra and Calculus. Dynamic geometry software has been used broadly in research regarding the teaching and learning process of geometry over the past several decades (e.g., Leung & Or, 2007, p. 177). There are 2-dimensional DGS packages, such as the Geometer's Sketchpad (Jackiw, 1991), Cabri II (Laborde, Baulac, & Bellemain, 1988), Geogebra (Hohenwarter, 2001), Cinderella (Richter-Gebert & Kortenkamp, 1999), etc., as well as 3-dimensional DGS packages, such as Cabri 3D (Laborde, 2004), and so on. I conceived, coined, and introduced in 2008 the notions of *Linking Visual Active Representations*, *Reflective Visual Reaction*, and in 2011, the notion of *instrumental decoding* within the context of my PhD research.

> *"Investigating, managing, reflecting on, reading, reorganizing, conceiving, activating and implementing new ideas, notions and terms in the context of an extended research study lie at the core of creativity and innovation. The most important step after you have coined a new idea is to investigate the idea's potential to be transformed into a successful implementation in class and in education more generally."* (Patsiomitou, 2023b, p. 18)

These notions emerged from the research study analysis involving a proving process, which was directly linked to the design process within the software (Patsiomitou, 2008a; Patsiomitou, 2011), as reported in Patsiomitou (2023b, p. 9):

I was led to the theoretical constructs of the notions "Linking Visual Active Representations" and "Reflective Visual Reaction" as a result of my experience designing DGS tasks and problems for my Master's dissertation (January-May 2005)- and Ph.D. thesis (January-May 2007) and the research I conducted using these tools, as reported in the following excerpt from the study "*The development of students' geometrical thinking through Linking Visual Active Representations*" (Patsiomitou & Koleza, 2009): "*For this reason, the researcher (S.P.) designed the multiple pages of the software using interaction techniques such as 'hide/show action buttons' or 'link-buttons'* (p. 159)[…] "*the meanings of Linking Visual Active Representations, and Reflective Visual Reaction during a dynamic geometry problem solving, are introduced / defined in the present study directly connected with the design process […]*"

More recently, I introduced the concept of *instrumental schemas* (Patsiomitou, 2025a), although it had already been identified in my earlier work (Patsiomitou, 2008d). As previously noted, the introduction of new theoretical constructs inevitably raises concerns regarding their academic validity. For this reason, I deliberately postponed their formal consolidation until sufficient empirical evidence had been established through subsequent studies. Furthermore, I introduced a set of strategies that I consider necessary for achieving meaningful outcomes, not only in learning processes but also in fostering students' voluntary engagement and personal participation (Patsiomitou, 2016c in Greek; Patsiomitou, 2026).

Table 1 below presents all the concepts that I have conceived, coined, formulated or reformulated over the years during my work with the software. Given that I have served as a reviewer for the PME conferences (i.e., conferences organized by the International Group for the Psychology of Mathematics Education) for 17 years, I have often revisited and critically revised my own formulations, and in subsequent research using digital tools, I have frequently identified perspectives that differed from my earlier work.

I have maintained a strong research coherence over time, repeatedly revisiting, refining, and reformulating instrumental didactical concepts in response to new insights and the use of DGS technological tools. My work is guided by a clear orientation towards pedagogical innovation, particularly through the integration of dynamic geometry environments and visualization-based approaches. I place central emphasis on the transition from representation to understanding, systematically connecting symbolic, visual, and real-world representations of mathematical ideas. This work is underpinned by a consistent commitment to scientific precision and methodological rigor, ensuring that all proposed ideas and instructional designs are carefully grounded, critically examined, and empirically informed.

Table 1: Conceptual evolution of my research program (Patsiomitou, 2005–2026)

	Dynamic Notion	**Citations**
1	Linking "alive" Representations	**Patsiomitou, 2005a**, Patsiomitou, 2005b, Patsiomitou, 2006a, b, c, d, e, f, g, Patsiomitou, 2008f, g
		Patsiomitou, 2007a, Patsiomitou, 2018a, b, Patsiomitou, 2019c
2	A "dynamic" method of exhaustion for number pi (π)	**Patsiomitou, 2006f**, Patsiomitou, 2007c
		Patsiomitou, 2018a, Patsiomitou, 2019c
3	The "dynamic" building of number fi (φ) as an approximation process	**Patsiomitou, 2006g**, Patsiomitou, 2007e
		Patsiomitou, 2019b, c
4	Structural Algebraic Units	**Patsiomitou, 2006** (oral presentation in Greek), Patsiomitou, 2007e, Patsiomitou, 2008e, Patsiomitou, 2009b; Patsiomitou, 2008c, Patsiomitou, 2009a, Patsiomitou, 2019c
5	Linking Visual Active Representations (LVAR)	**Patsiomitou, 2008a**, b, 2010; Patsiomitou, 2008h; Patsiomitou & Koleza, 2008, 2009
6	Reflective Visual Reaction	**Patsiomitou, 2008a**, b; Patsiomitou & Koleza, 2008, 2009
7	Design and definition of LVAR modes (five phases)	**Patsiomitou, 2008b**, 2010, 2019c ; Patsiomitou & Emvalotis 2009b

		Patsiomitou, 2012a, d, Patsiomitou, 2020a, Patsiomitou, 2022a
8	A proposal for a DG research -based curriculum	**Patsiomitou, 2010** ; Patsiomitou & Emvalotis 2009d, 2010a, b
		Patsiomitou, 2012a, Patsiomitou, 2020a, Patsiomitou, 2022a
9	Custom tools (scripts) as alive/active objects	**Patsiomitou, 2005a**, Patsiomitou, 2006c, d, e, g, Patsiomitou, 2007b, d, e, Patsiomitou, 2009b, c, Patsiomitou, 2022a
		Patsiomitou, 2008d, Patsiomitou, 2018a, Patsiomitou, 2019c; Patsiomitou & Emvalotis 2009c
10	Theoretical and Experimental dragging	**Patsiomitou, 2011a**, Patsiomitou, 2012b, Patsiomitou, 2014, Patsiomitou, 2018b, Patsiomitou, 2019c
		Patsiomitou, 2011b, Patsiomitou, 2012a, Patsiomitou, 2015a, c, Patsiomitou, 2016a, b, Patsiomitou, 2020a, Patsiomitou, 2021c, Patsiomitou, 2022a
11	Dynamic point, Dynamic segment, Parametrical segment, Dynamic propositions, dynamic meaning	**Patsiomitou, 2011a**, Patsiomitou, 2012b, Patsiomitou, 2014, Patsiomitou, 2018b, Patsiomitou, 2019c
		Patsiomitou, 2011b, Patsiomitou, 2012α, Patsiomitou, 2015a, c, Patsiomitou, 2016a, b, 2020a, c, 2021a, b, c, 2022a
12	Instrumental decoding, instrumental obstacle	**Patsiomitou, 2011a**, Patsiomitou, 2012b, Patsiomitou, 2014, Patsiomitou, 2018b, Patsiomitou, 2019c
		Patsiomitou, 2011b, 2012α, 2013b, 2015a, c, 2016a, b, 2020a, b, c, 2021c, 2022a
13	Discursive, visual and operational apprehension of tool's use (adaptation of Duval's (1995) cognitive notions)	**Patsiomitou, 2011a**, Patsiomitou, 2012b, Patsiomitou, 2014, Patsiomitou, 2018b, Patsiomitou, 2019c
		Patsiomitou, 2011b, Patsiomitou, 2012α, Patsiomitou, 2015a, c, Patsiomitou, 2016a, b, Patsiomitou, 2020a
14	"House of parallelograms" (adaptation of Graumann's (2005) "*house of quadrilaterals*")	**Patsiomitou, 2012b**, Patsiomitou, 2019c
		Patsiomitou, 2012α, Patsiomitou, 2015α, c, Patsiomitou, 2020a
15	A classification of the internally constructed quadrilaterals	**Patsiomitou, 2012b**, Patsiomitou, 2019c
		Patsiomitou, 2012α, Patsiomitou, 2015α, c, Patsiomitou, 2020a
16	A proposal for a qualitative upgrading of math curricula (including fractals, spirals etc.)	**Patsiomitou, 2005a**, b, Patsiomitou, 2007a, b, c, d, Patsiomitou,2009b, c, d, c, e, f, g, h, 2012a, Patsiomitou, 2015b, d, 2016c
		Patsiomitou 2007a, Patsiomitou, 2012b, c
17	A Dynamic Hypothetical Learning Path (Adaptation to Simon's (1995) *Hypothetical Learning Trajectory*)	**Patsiomitou, 2012b**, Patsiomitou, 2019c
		Patsiomitou, 2012α, Patsiomitou, 2015α, c, Patsiomitou, 2020a, b, c, Patsiomitou, 2022a

18	Introduction of a Pseudo-Toulmin model	**Patsiomitou, 2011a**, Patsiomitou, 2012b, Patsiomitou, 2014, Patsiomitou, 2018b, Patsiomitou, 2019c, Patsiomitou, 2021a, b
		Patsiomitou, 2011b, Patsiomitou, 2012α, Patsiomitou, 2015a, c, Patsiomitou, 2016a, b, Patsiomitou, 2020a, Patsiomitou, 2022a
19	Dynamic (/ perceptual) definition and arbitrary economic definition. (An adaptation of the Govender & De Villiers' (2004) clarification)	**Patsiomitou, 2013a**
		Patsiomitou, 2012α, Patsiomitou, 2015a, c, Patsiomitou, 2016a, b, 2020a
20	An adaptation of Battista (2007)'s categorization regarding the development of students' abstract processes	**Patsiomitou, 2013a**, Patsiomitou, 2019c
		Patsiomitou, 2012α, Patsiomitou,2015a, c, Patsiomitou, 2016a, b, Patsiomitou, 2020a
21	Linking Visual Active Representations (LVAR)-reformulation of the definition	**Patsiomitou, 2012b**, Patsiomitou, 2014, Patsiomitou,2019c
		Patsiomitou, 2012a, Patsiomitou, 2020a
22	Dynamic Didactic cycle (Adaptation to Simon's (1995) *The Mathematics Teaching Cycle*)	**Patsiomitou, 2014**, Patsiomitou, 2019c, Patsiomitou, 2021a, b
		Patsiomitou, 2015α, β, Patsiomitou, 2022a
23	Implementation of LVARs for the teaching of mathematics in class	**Patsiomitou, 2014**, Patsiomitou, 2019c
		Patsiomitou, 2012c, d, Patsiomitou, 2015d, Patsiomitou, 2016a, b, Patsiomitou, 2020a, b, c, d
24	Kinds of Transformations	**Patsiomitou, 2014**, Patsiomitou, 2019c, Patsiomitou, 2021a, b
		Patsiomitou, 2009a, b, Patsiomitou, 2021c, Patsiomitou, 2020a, b, c, Patsiomitou, 2022a
25	The meaning of "alive" tool	**Patsiomitou, 2005a**, b, Patsiomitou, 2006b, Patsiomitou, 2020a, Patsiomitou, 2022a
		Patsiomitou, 2018a, Patsiomitou, 2019c, Patsiomitou, 2022c
26	Dynamic Active Learning Trajectory	**Patsiomitou, 2018a**, Patsiomitou, 2019c
27	Dynamic object, Dynamic Diagram, Dynamic section,	**Patsiomitou, 2019 a**, b, c, Patsiomitou, 2020a, 2022a
28	Hybrid-dynamic objects	**Patsiomitou, 2006a**, 2022a
		Patsiomitou, 2019 a, b, c, Patsiomitou, 2021a, b
29	Procept-in-action	**Patsiomitou, 2019 b**, c
30	Empirical Classification Model for Sequential Instructional Problems in Geometry	**Patsiomitou, 2019a**, c
		Patsiomitou, 2022a
31	Representation and dynamic representations	**Patsiomitou, 2019c**
		Patsiomitou, 2020a, Patsiomitou, 2022a
32	Didactics of Mathematics	**Patsiomitou, 2019c**
		Patsiomitou, 2020a, b, d, Patsiomitou, 2022a
33	Instrumental Learning Trajectories interdependence/intra-dependence between dynamic tools and objects	**Patsiomitou 2021a**, b
		Patsiomitou, 2021c, Patsiomitou, 2022a
34	Virtual Cuisenaire Rods	**Patsiomitou, 2022 b**, c, Patsiomitou, 2023c
35	Virtual Froebel Gifts	**Patsiomitou, 2022d**, Patsiomitou, 2023c
36	Virtual Cuisenaire Rods	in collaboration with Prof. Scher, 2022 (online Web sketchpad version)

37	Instrumental schema	**Patsiomitou, 2025b**
38	A Fractal -based Dynamic Program (FDP)	**Patsiomitou, 2016a**, b, Patsiomitou, 2026
39	A Pedagogical Strategy for Integrating Real-World Activities	**Patsiomitou, 2016a**, b, Patsiomitou, 2026

As noted above, Table 1 presents the introduced terms, along with the corresponding publications in which they were first introduced; the relevant publications are indicated in bold. The majority of these notions are grounded in the concept of transformations, understood as the set of operations that a user may enact upon dynamic mathematical objects within the digital environment. Transformations used by the students in the DGS environment can be distinguished through the following (Patsiomitou, 2014, p.30, Patsiomitou, 2022a):

- *Transformation generated* from the reflection, dilation, rotation, or translation of the object. Dragging on rotated (dilated, reflected, or translated) objects maintain the congruency and structural relationship between the elements of the construction.
- *Transformations generated from the utilization of the action buttons* tools (for example, the hide/show action button, the link button, the movement button, or animation).
- *Transformations generated from the annotation of the dynamic diagram* (for example, use of colours, formulations, and the trace tool). Moreover, the combination of transformations (e.g., the trace tool and dragging tool, the calculations and the dragging of the geometrical object's points).
- *Transformations generated from the application of the custom tools*. The application of custom tools reorganizes the external representation. The application of a custom tool (or the repetition of the application of a custom tool) is accomplished in a sequence of steps directly perceived by the user. Consequently, custom tools operate as a referent point for *organizing, pursuing, and retrieving information.*
- *Transformations generated from the synthesis of the dynamic diagram*.
- *Transformations generated from the reconfiguration of the dynamic representation*.
- *Combinations of transformations due to the synthesis of the software's interaction techniques* (Sedig & Sumner, 2006).

I have also discussed several kinds of transformations and transformational results that ensue from implementing dragging on screen (Patsiomitou, 2019b, p. 43-44):

- *Dragging and tracing of a geometric object (for example, a point, segment or line)*

Dragging a point on screen results in the transformation of its position and the simultaneous appearance of traces on screen tracking the path the point has followed or the tracks that a line passes due to dragging transformations. This action reveals in the determination of a basic property of the diagram that cannot be directly perceived from the diagram in its hybrid form, or a property of the diagram that remains stable and unaltered.

- *Dragging and measuring (or calculations) the geometric object.*

Dragging a point on the screen leads to a change in the measurements of the object, which we have chosen to display and in its calculations. In this case, the measurements change, but the calculations may do one of two things: they may remain unchanged, indicating a stability that demonstrates the validity of a theorem or general theoretical approach (a proposal or a confirmed porisma--meaning a conclusion or an inference) or they may change, allowing the user to observe and draw conclusions from empirical results.

- *Dragging and animating, or dragging, animating and tracing objects*

A point on an object is dragged--for example, the vertex points of a triangle to which a point on one side is connected with motion. The animation of the diagram and the simultaneous dragging allow us to understand a condition which is not defined during the diagram's structuring process. For example, it may make us aware of a theoretical limitation that has not been determined or established before, but which appears on the diagram when it is dragged. This condition leads to an investigation of the validity of a theorem or proposal.

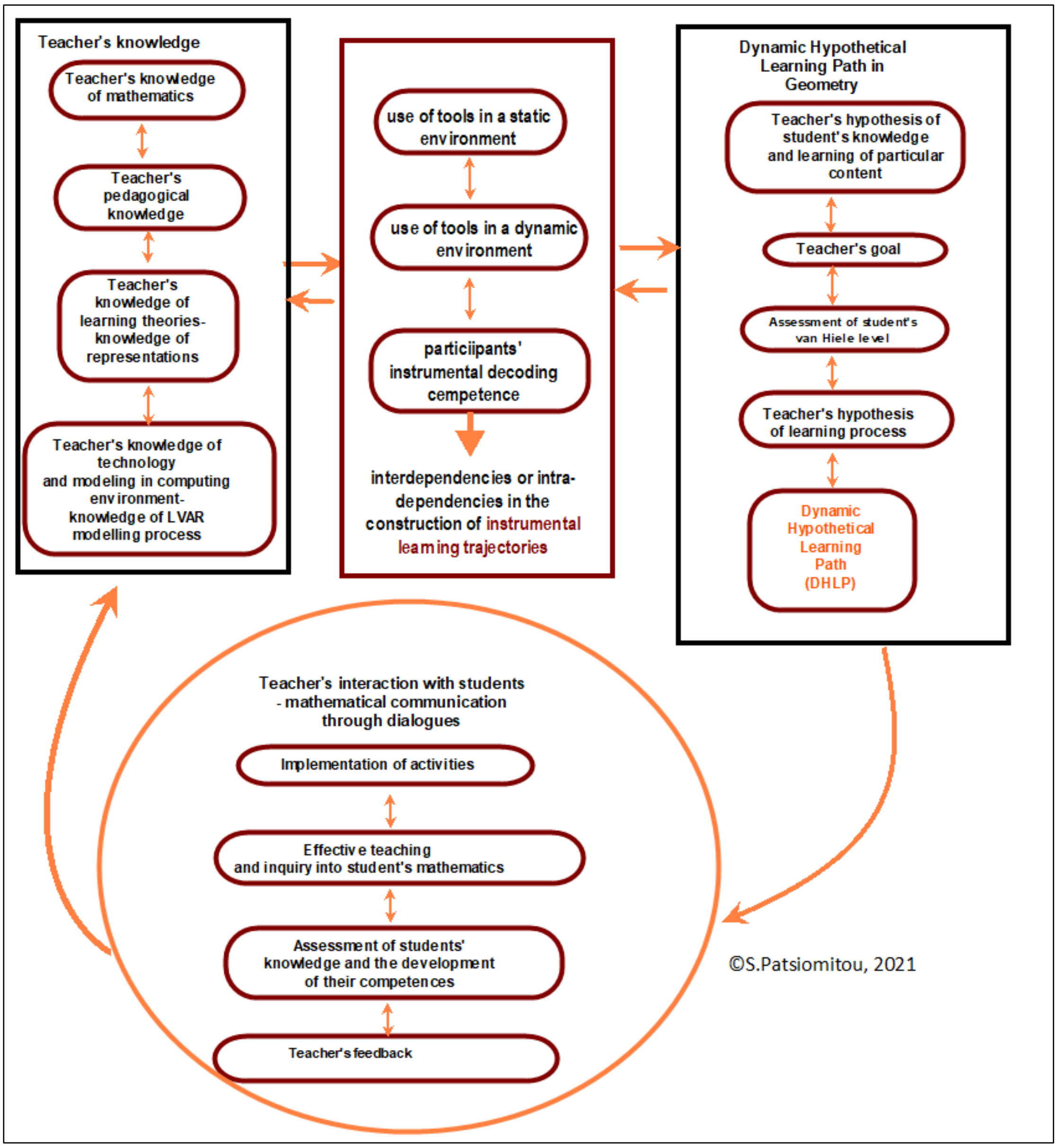


Figure 1: The Mathematics Teaching Cycle incorporating the notion of *instrumental trajectory* (Patsiomitou, 2021a, p. 94)

Didactics of Geometry supports the construction of hypothetical learning trajectories and learning progressions. In my research study *Students' learning progression through instrumental decoding of mathematical ideas* (Patsiomitou, 2014), I created an adaptation of Simon's (1995) *Mathematics Teaching Cycle,* examining the use of technology in the teaching cycle, which plays an important role in the development of mathematical discussions (Patsiomitou, 2014, p.35). In the year 2021, I enlarged the diagram (Figure 1) to include the concept of "*instrumental trajectories*" (Patsiomitou, 2021a, p. 94). In my studies, I introduced and analysed several *conceptual and instrumental learning trajectories* for the teaching of, or the conducting of research into Geometry, which employ digital dynamic means: in other words, the construction of *instrumental learning trajectories* within "Dynamic Euclidean Geometry" (Patsiomitou, 2021a, p.16)

The extended diagram by adding the intermediate column captures the role of instrumental decoding in both static and dynamic environments, as well as the ways in which the competence of participants (students and teacher) may influence the overall outcome of the learning process by generating *interdependencies and intra-dependencies during the construction of instrumental learning trajectories* (Patsiomitou, 2021a, p. 94). Moreover, I argue that: (a) the notion of LVAR is directly linked to the notion of instrumental decoding; (b) a dynamic diagram expresses the interdependencies among dynamic objects; and (c) a dynamic section represents the intra-dependencies and interdependencies between dynamic diagrams and mathematical objects.

In parallel with these instructional approaches, cognitive frameworks have been employed to support curriculum design and assessment. The taxonomy proposed by Anderson & Krathwohl (2001) is an updated version that builds upon the foundational work of Bloom *et al.* (1956). Bloom's taxonomy delineates three distinct domains: the *cognitive domain*, which emphasizes the enhancement of students' intellectual abilities, including skills such as information recall, concept evaluation, and the application of knowledge in innovative contexts; *the affective domain*, which pertains to the cultivation of students' attitudes, values, and interests; and the *psychomotor domain*, which is primarily concerned with the processing of sensory information and physical movement. The revised taxonomy modifies categories and incorporates action verbs linked to each of the six cognitive dimensions. It presents six facets of learning—*Remembering, Understanding, Applying, Analyzing, Evaluating, and Creating*—along with concise descriptions of the cognitive processes involved and indicative verbs that educators can use to encourage engagement at each level. This revised taxonomy encompasses two primary dimensions. The first is the knowledge domain, which categorizes knowledge into four types (Anderson & Krathwohl, 2001): *factual, conceptual, procedural, and metacognitive*. *Factual knowledge* refers to the essential elements students must know to engage with a discipline or solve problems within it. *Conceptual knowledge* concerns the relationships among these elements, enabling them to function coherently within a broader structure. *Procedural knowledge* involves knowing how to perform tasks, including methods of inquiry and criteria for applying skills, algorithms, techniques, and procedures. Finally, *metacognitive knowledge* involves awareness and understanding of cognition in general, as well as insight into one's own cognitive processes.

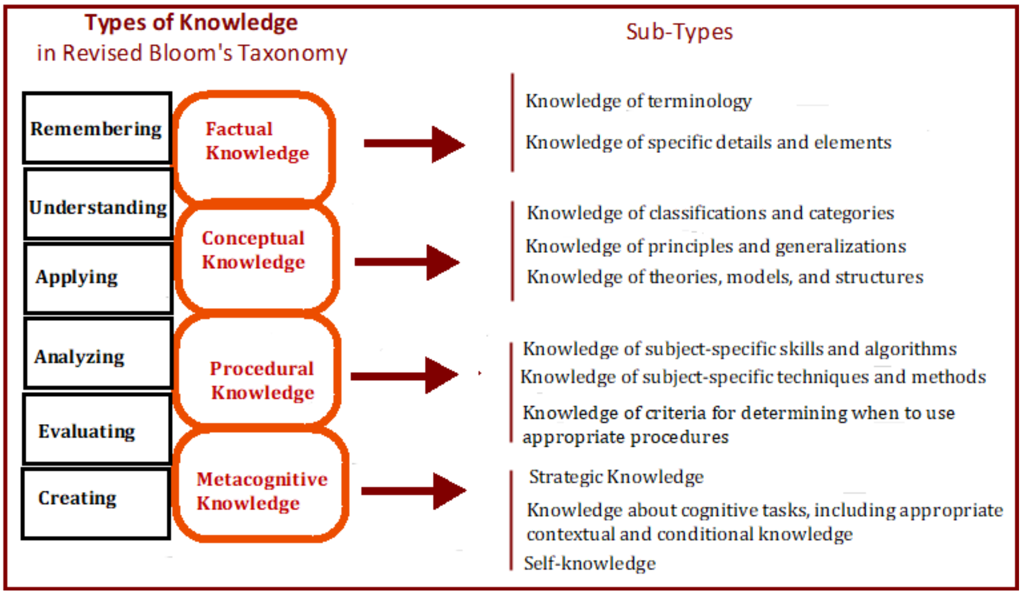


Figure 2: Types and subtypes of knowledge described in the revised Bloom's Taxonomy (adapted from Anderson & Krathwohl, 2001, in Patsiomitou, 2024b, p. 198)

Marzano and Kendall (2007, Preface, xi) articulate that Bloom's Taxonomy encompasses several distinct purposes: (1) it provides a framework for the design and classification of educational objectives, (2) it offers a structure for the designing of assessments, (3) it acts as a resource for enhancing the utility of state standards for educators, (4) it serves as a structure for curriculum design, and (5) it underpins a curriculum focused on thinking skills.

My further aim is to inculcate in teachers of mathematics a greater awareness of the theory and research into the Didactics of Mathematics, taking into account the impact representational technological environments have had on mathematics learning and teaching. As part of the learning process, we have to understand what the mathematical objects are, how to use them and how to represent them in static or dynamic means. Language also plays a crucial role in the teaching and learning process. Do we learn alone as individuals, or with others in a social context? Do we learn using traditional means, or through e-learning and computer software? Both are important for students. As teachers, we have to choose the learning trajectory our students will follow, by using a *thought experiment* to construct a *dynamic hypothetical learning path* (DHLP) (Patsiomitou, 2012b) that predicts their progress and their thought development. I know from my classroom experience that students find it difficult to translate a formal Euclidean proposition into a figure on screen, which is to say they encounter difficulties translating between different systems of representation. If the student has developed an interaction between the *external and the internal representation* (e.g., Goldin & Kaput, 1996) of the concept, then s/he has developed the level of understanding of this concept.

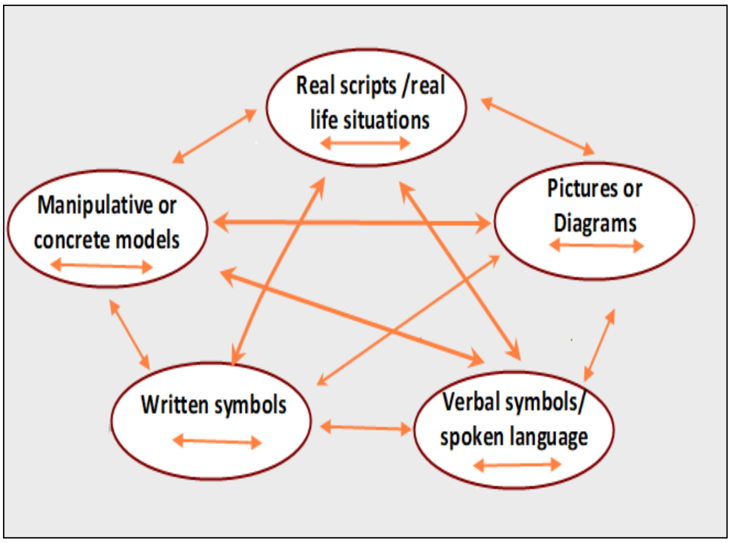


Figure 3: The Lesh (1979) multiple representation.
Translation model, adapted from Lesh, Post, and Behr (1987, p.34)

Lesh, Post and Behr (1987) proposed a multiple representation model in which they suggest a student understands a concept if s/he has the competence to translate between different modes of representation of the concept (see, for example, Patsiomitou, 2019c, pp. 42-46). Duval (2002), in Figure 4, clarifies what he means by the notions of *treatment* and *conversion* between different semiotic representations. Treatments and conversions express connections or links between different modes of representation. As a case in point, I will discuss in the following section my efforts related to the systematic translation of assessment tasks from the Item Bank of Graded Difficulty (IBGD)—which is published and maintained by the Hellenic Institute of Educational Policy (IEP)—into the GeoGebra dynamic geometry environment. Through my GeoGebra profile, I have created dynamic representations of assessment tasks, which foster visual and experiential learning while guaranteeing open access and reusability for both educators and learners. This approach facilitates the active reconstruction of knowledge and motivates teachers to enhance their own instructional resources.

This idea and proposal were initially introduced and supported during discussions held at a workshop that I organized on 24 April 2024 in the Athens C Directorate. Nonetheless, in the absence of the earlier training seminars provided to teachers and educators, it would not have been possible to clearly identify their needs regarding the usefulness and the effectiveness of translating the tasks and problems of IBGD. This highlights the importance of professional development seminars organized by Education Advisors, as well as their crucial role in shaping, developing and formulating educational ideas. All of the above can be framed within the broader context of the educational consultant’s professional identity, which will be further discussed in the following section.

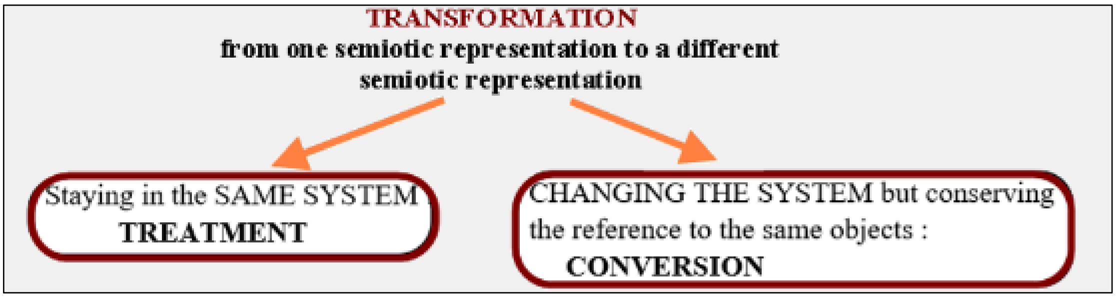


Figure 4: Types of transformations of semiotic representations
(Duval, 2002, p.3) (an adaptation for the current study)

3. The professional identity of the Education Advisor

3.1. Digital Repositories: The Translation or Conversion of IBGD into the GeoGebra Ecosystem

A key approach to addressing the challenges students encounter in learning calculus involves the *design* of activities using Dynamic Geometry Software *with the aim of fostering connections between symbolic and visual representations of calculus-related concepts, processes, and terminology.* More broadly, this approach encompasses the design of a structured learning environment or the establishment of a dedicated digital repository to support these objectives and *such instructional practices.* Tall & Vinner (1981) stated that "*we must formulate a distinction between the mathematical concepts as formally defined and the cognitive processes by which they are conceived*" (p. 151).

GeoGebra profiles function as personalized repositories within the platform's ecosystem, *hosting user-generated, shared, and curated mathematical resources*. Their primary purpose is to foster a collaborative community among educators and students by facilitating the systematic organization and distribution of interactive learning materials. As mentioned above, a characteristic example of my professional work is the initiative to converse or translate problems from the Item Bank of Graded Difficulty (IBGD) - uploaded by Hellenic Institute of Educational Policy (IEP)- into the GeoGebra environment. The IBGD [website 8] serves as a valuable resource for educators aiming to adopt innovative, student-centered curricula. It provides a range of tasks across *various subject areas,* including Mathematics, Physics, Biology, etc. (i.e., lessons in which students are assessed in Panhellenic *examinations), designed to support the development of skills and competencies through differentiated instructional approaches and problem-solving methodologies.* Within the contemporary pedagogical framework of digital education, the integration of digital tools is a crucial factor for supporting, enhancing, and enriching Mathematics instruction. Examples of these actions include the creation on my part of a dynamic profile on GeoGebra (website 9), a digital repository aimed at utilizing Geogebra tools in Mathematics teaching. More broadly, during the discussions that took place in the aforementioned workshop, I emphasized the necessity of *translating* or *conversing* IBGD problems and tasks into digital environments, such as the dynamic geometry software GeoGebra. The translation and conversion of tasks from IBGD into a digital and dynamic format have been a central concern since assuming my role as a Mathematics Education

Advisor. This initiative was grounded in the belief that teachers could effectively integrate online resources into their classroom practice, while students would, in turn, be able to further engage with this material independently at home. Although the process of translating these tasks had begun earlier, their systematic online publication commenced around April 2024. I continue to implement this process systematically in response to emerging needs. During my presentations, I have also introduced and demonstrated a large number of geometry problems and constructions modelled using The Geometer's Sketchpadv4; however, only screen-recorded video versions of the gsp files were uploaded, as direct export or upload in HTML format is not supported.

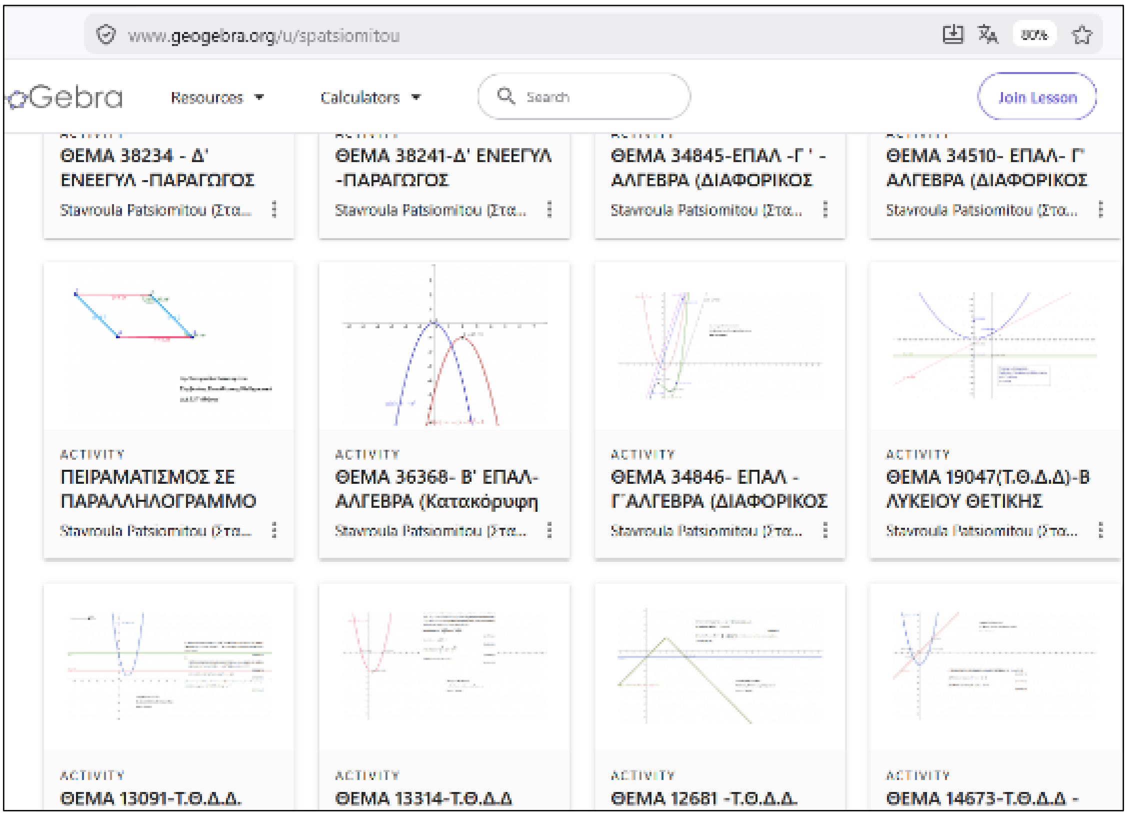


Figure 5: Screenshot of an illustrative excerpt from
my interactive profile within the GeoGebra ecosystem

In particular, I *translated* problems and tasks that had been uploaded to the IBGD into interactive, dynamic representations using GeoGebra, thereby establishing visual and experiential learning opportunities. These Geogebra files are open to everyone [teacher of mathematics or student], guaranteeing accessibility and reusability in their teaching practices. This material is freely available and is already being used by many teachers in classroom practice. More importantly, however, this practice is not reproduced passively; rather, it serves as a starting point for the creation of new educational materials by the teachers themselves.

A significant development in this effort was the use of the GeoGebra Calculator to design *interconnected linking pages* that support conceptual understanding through sequential constructional steps. The overall process is further enhanced by the use of distinct color schemes for different forms of representation, as illustrated in the figure below (Figure 6). These pages can link the constructive steps of a theorem or even a *family of representations* of the same function. The design of these interconnected pages is based on my research on "linking visual active representations," which aims to integrate symbolic, graphical, and dynamic representations in a coherent and interactive learning framework. Subsequently, the symbolic representations are dynamically linked to their graphical counterparts, facilitating a deeper and more coherent comprehension of mathematical concepts. What I propose is the systematic transformation of all IBGD problems and tasks into software-based representations, so that mathematics teachers in classroom settings can have access to both static and dynamic representations of mathematical tasks. This raises the question of potentially developing an IBGD also for lower secondary education (Gymnasium), not necessarily with the same level of formal rigidity as in upper secondary education (Lyceum), but in a way that enables teachers to more easily select and adapt tasks for the teaching, learning, and assessment of mathematical concepts. In this way, we move beyond traditional forms of professional development towards the creation of a dynamic community of learning and knowledge production.

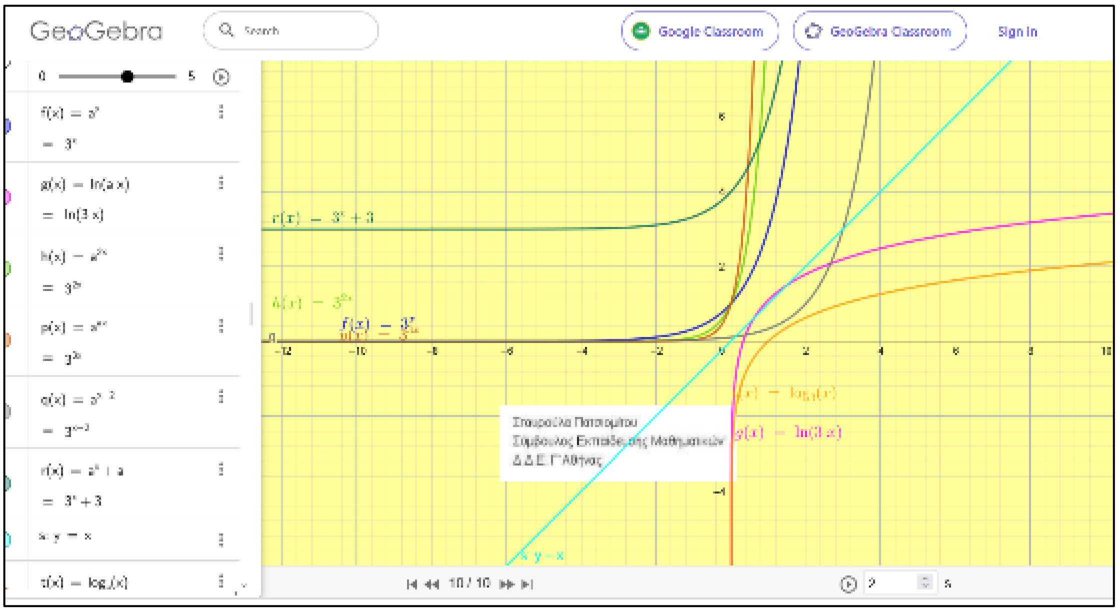


Figure 6: Screenshot depicting a dynamic family
of linking, interconnected exponential functions

Furthermore, my website *Mathematics with Dynamic and Static Tools* [website 7] constitutes a comprehensive and contemporary support resource for Mathematics Teachers, offering rich teaching and learning material tailored to the needs of mathematics education. With the aim of enhancing the quality of instruction and promoting active learning, I organized and structured the website content into thematic sections that cover all major domains of mathematics education. Ranging from core mathematical subject areas and didactical approaches to the integration of digital tools and technologies such as GeoGebra, Geometer's Sketchpad, and WebSketchpad, the material is designed to support both everyday classroom practice and teachers' professional development.

The website also includes up-to-date and comprehensive guidelines for the curricula across all types of upper secondary schools, a repository of examination topics, as well as proposed materials for diagnostic and nationwide assessments and exams. In parallel, it provides resources for lower and upper secondary school students, with a strong emphasis on differentiated and personalized learning, thereby contributing to the holistic development of mathematical competencies. The systematic documentation and organization of timetables and instructional scenarios facilitate teachers' lesson planning and implementation, while access to university-level and scholarly material strengthens their theoretical grounding. The continuous updating and enrichment of the website with new pedagogical approaches, technological applications, and research-based evidence render this initiative a valuable ally in the effort to improve the quality of mathematics teaching and assessment (e.g. Patsiomitou, 2024a), always guided by the pursuit of optimal learning experiences and the promotion of creativity, critical thinking, and collaboration within the classroom.

Over the 20 years I have been using various software environments, I have employed them for many different educational /instructional purposes and functions, which I have published (papers/articles or monographs in peer-reviewed international or panhellenic conferences and journals) (e.g., Patsiomitou, 2005-2026). In my opinion, a DGS software can play a fruitful and crucial role in the process of creating and evaluating conjectures which promote student creativity, and in so doing, greatly contribute to developing mathematical reasoning.

Furthermore, a key dimension of my work involves the design and structure of digital learning resources that extend classroom practice and promote teacher-generated innovation. As part of my responsibilities, I have developed and implemented multidimensional actions aimed at strengthening the educational process through the promotion of innovative practices, scientific documentation, and close collaboration with all members of the educational community.

By organizing seminars and scientific symposia in Mathematics, I have enhanced the scientific and pedagogical support available to teachers. Collaboration with fellow Education Advisors, school leaders, in-service teachers, pre-service teachers, school principals and higher education institutions constitutes a core component of my work, fostering a sustained ecosystem of pedagogical development and educational innovation. I also maintain continuous and constructive cooperation with preservice Mathematics teachers and higher education institutions.

Overall, the study seeks to demonstrate how well-designed professional development seminars and workshops can contribute to meaningful and sustainable improvement in mathematics teaching and learning.

3.2. Design, Organization and Coordination of Mathematics Seminars and Workshops: An overview

The design of the professional development workshops was based on an analysis of teachers' training needs and grounded in theoretical frameworks from Didactics and the Psychology of Mathematics Education. Each training activity that I organised through my own initiative, but always in collaboration with the Department's administration, included the formulation of the thematic focus and programme, continuous reflection and revision up to the final implementation stage, to ensure timely communication and invitation of teachers at school units. The preparation process was characterised by coordinated actions and multi-level communication with university academics, schools, teachers, and administrative bodies, ensuring the quality and functionality of the activities, including technical checks of venues and equipment prior to each event.

Within the scope of my role, I have developed an extensive programme of professional development activities, implemented both in-person and online through the use of my digital websites. These activities are not limited to monologic presentations but are enriched through the participation and active engagement of teachers from the schools under my scientific responsibility, who are given the opportunity to present their work, thereby further enhancing their professional development. From May 2023 to April 2026, I designed, organized, and co-organized a large number of professional development events for mathematics teachers. These included workshops, one-day and two-day seminars, interdisciplinary symposia, and online seminars. The topics addressed were diverse, focusing on the implementation of the mathematics curricula, assessment strategies, the application of dynamic geometry software (e.g., mathematical modeling through Linking Visual Active Representations), STEAM methodologies, and the use of manipulatives. Additionally, emphasis was placed on the integration of digital tools and artificial intelligence in mathematics education, equipping teachers with resources to foster active learning and representational exploration. Each event was meticulously structured to include focused presentations, engaging sessions, and practical experiences, with timeframes designed for effective implementation. Throughout this process, I ensured comprehensive scientific coordination and academic supervision, maintaining consistency, rigor, and adherence to contemporary educational standards. Several seminars were held in partnership with colleagues, promoting interdisciplinary and cross-institutional collaboration. Specifically, for each seminar, I adhered to the following procedures:

A. ***Communication and Preparation***

The seminars presented below were grounded in the theoretical framework developed in my Ph.D. thesis, monographs, and publications in both Greek and international contexts (e.g., Patsiomitou, 2008a, 2012a, 2020a, b, c). The preparation of each seminar required extensive coordination and multi-level communication processes, which I systematically designed and implemented.

- *Communication with speakers*
  - Conducted face-to-face meetings, telephone communication, and email correspondence with the invited speakers. In all seminars, the selection of speakers was made directly by me in accordance with the seminar topic.
  - Collaborated with school principals and relevant authorities to secure the use of school multipurpose halls or other appropriate venues for the implementation of the seminars.
- *Administrative and operational support*
  - Communicated with the Director of Education to ensure the timely preparation and distribution of official invitation documents to schools.
  - Conducted on-site technical inspections at the seminar venues to ensure the proper functioning of all technical equipment.

B. *Administrative documentation and preparation of invitation materials*

- Designed the seminar topic and programme and supervised their development throughout all stages leading to the implementation of the seminar.
- Drafted the official invitation documents addressed to school units, which were submitted for approval to the Director of Education, as mentioned.
- *Management of participants*
  - Created and distributed an electronic registration form (Google Form) for recording participant teachers' attendance.
  - Collected and organized participant teachers' data and maintained Excel records containing the details of participating teachers, including those who had not completed the registration form during enrolment.
- *Issuance of certificates*
  - Prepared and distributed certificates of participation for teachers to the Directorate of Education, accompanied by the complete attendance list.
  - Prepared corresponding certificates for the seminar speakers.

C. *Scientific contribution*

- Participated as a speaker in the seminars, delivering presentations grounded in the principles of the math curricula, as well as in the use of contemporary digital tools and artificial intelligence technologies in educational practice.
- Uploaded my presentations, along with presentations delivered by invited trainers and related educational material, to my personal website, making them available to all interested teachers and educators.

D. *Seminar coordination*

- I was responsible for coordinating the seminars during their implementation, ensuring the smooth flow of the programme, effective time management, and facilitation of communication among participants.

These seminars have substantially contributed to the professional growth of mathematics teachers, supporting the adoption of contemporary pedagogical and digital practices in the classroom. In summary, these seminars and symposia constitute a core component of my advisory responsibilities, demonstrating both scientific leadership and a sustained commitment to enhancing mathematics education at the secondary level. Below are briefly listed several important seminars that I designed, organized, and conducted from May 2023 to April 2026. For each seminar, I provide the Official Reference Number (O.R.N.), the Seminar Title (T.S.), and the title of my presentation. The duration of my presentations varied between 45 minutes and 1 hour. In all cases, I was responsible for the overall coordination and the academic supervision. The presentations reported here incorporate key developments in the Didactics of Geometry, as reported in my research studies. My presentations function as a fertile space for scientific dialogue and reflection on the teaching and learning of mathematics in the contemporary classroom. By offering diverse yet complementary perspectives, the seminars aim to strengthen teachers' professional knowledge and to foster shared reflections and proposals that meaningfully support teaching practice and students' learning trajectories.

I. One-day Seminar (O.R.N.: 11174/16 May 2023), which I designed, organized and coordinated, with T. S.: *Graduation Examinations in Lower Secondary School (Gymnasium): A Review of the Relevant Legislation and Sample Examination Tasks*. This seminar was held on Wednesday, 24 May 2023*, at the 1st Lower Secondary School (Gymnasium) of Agioi Anargyroi.* My presentation was entitled: *Review of the Relevant Legislation and Sample Examination Tasks – Use of Dynamic Geometry Software*.

The main goal of this workshop was to assist educators in directing students towards active participation in the process of mathematical proof. In all subsequent seminars, efforts were consistently directed toward this aim, through the appropriate use of Linking Visual Active Representations designed within the software.

My presentation included: (a) the official legislative framework regarding student assessment; (b) examples of tasks designed to foster connections among multiple forms of representation, promote the recall of definitions, theorems and the proving process, and support the development of critical thinking; (c) the modelling of algebraic expressions through the use of *structural algebraic units* (Patsiomitou, 2007e, 2008c, 2009a). From a historical perspective, the notion of *structural algebraic units* (in Greek, *algebrikes domikes monades*) was also adopted as the official translation of the term *algebra tiles* in the Greek version of The Geometer's Sketchpad. In this translation process, I was invited by the software's designer to serve in an advisory role to the translation team, and my contribution is explicitly acknowledged within the software. Since then, I have encouraged participating teachers to follow a sequence of five flexible phases during their teaching process (Patsiomitou, 2026, p. 288):

- **Phase 1:** Recall and activation of prior knowledge (e.g., Ausubel, 1968; Bransford *et al.*, 2000) through a brief review of key concepts using targeted questions (5 minutes);

- **Phase 2:** Exploration and experimentation with active student involvement, using dynamic geometry software or hands-on manipulative tools (10 minutes) (e.g., Hiebert & Carpenter, 1992; Patsiomitou, 2005-2026)
- **Phase 3:** Formulation and mathematization (e.g., Treffers, 1987; Freudenthal, 1991; Gravemeijer, 1994), whereby empirical observations are transformed into mathematical expressions and deductive arguments (10 minutes);
- **Phase 4:** Application and feedback through short problem-solving tasks (see for example the studies Patsiomitou, 2012c, 2021d) (10 minutes);
- **Phase 5:** Reflection and closure, aimed at enhancing students' metacognitive awareness (e.g., Flavell, 1979; Brown, 1987) (10 minutes).
-

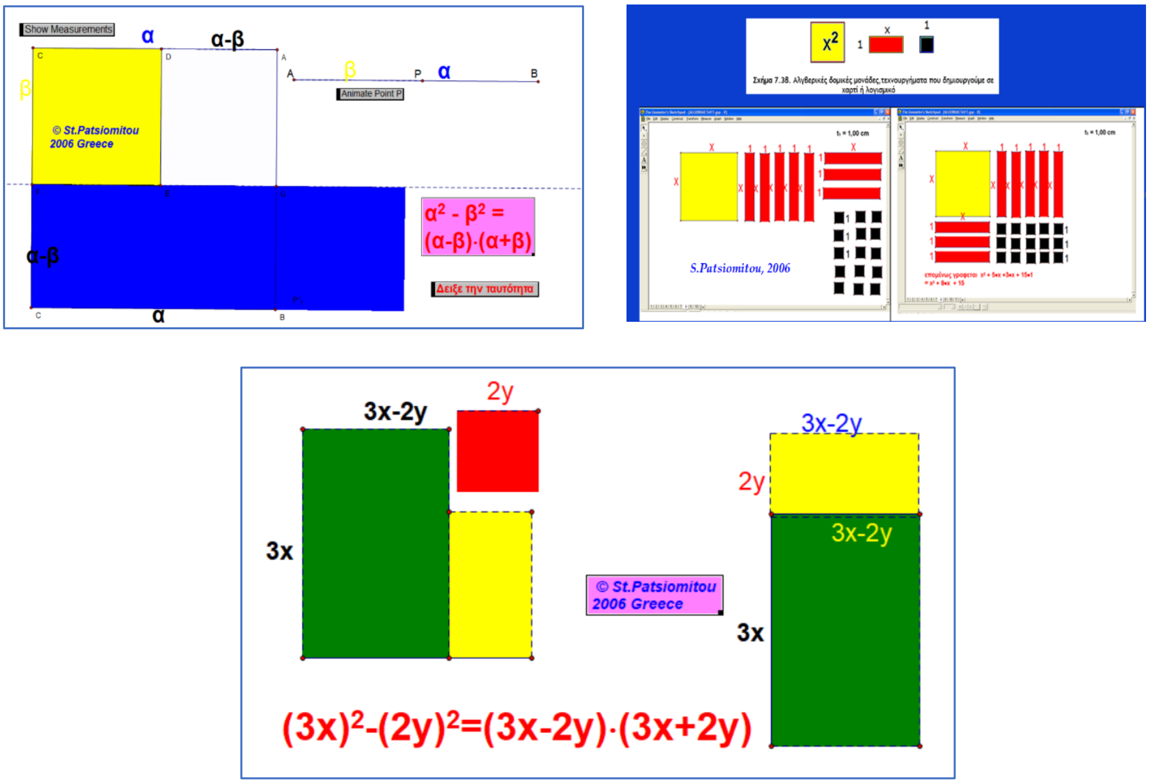


Figure 7 a, b, c.: Screenshots of the GSP file presented during the session (Patsiomitou, 2007e, 2008c, 2009a).

Particular emphasis must be placed on the observation of what are termed *critical events* (e.g., Rotem, & Ayalon, 2022; Tirosh, Tsamir, Levenson, & Barkai, 2019). Such events serve as a starting point for teacher reflection and are closely linked to the concept of *didactic transposition* (Chevallard, 1989), namely, the transformation of scientific knowledge into forms that are pedagogically accessible and meaningful for students. During the subsequent period, teachers utilized these *instructional phases* to the collaborative development of the *Evaluation Agreement* as an official document for their assessment. The *Evaluation Agreement* constitutes a form of formal document co-signed by the teacher under evaluation and the Education Advisor acting as evaluator. I think that *The Evaluation Agreement* can function as a tool *for individualized teacher professional* *development* (submitted at a local conference at April 23, 2025; however, it was subsequently withdrawn due to time constraints that prevented its presentation). The focus of this research study has been—and continues to be—the role played by the careful co-construction of the Evaluation Agreement, jointly completed by the evaluator and the teacher under evaluation, and whether it functions as a tool for individualized professional learning. I shall report briefly at the section 3.3 the way that the Evaluation Agreement supports the individualized professional development of the teacher. Furthermore, the role of seminars in shaping the theoretical framework for its completion is highlighted, as well as their contribution to the creation of a positive and supportive learning climate. This process involves, on the one hand, the construction of mathematical concepts and knowledge through active engagement, and, on the other, the cultivation of relationships both among mathematical objects and among the participants in the educational process.

The workshop in question constituted a significant milestone in my professional path, as it marked my first substantive and dynamic presence in my new role. It represented the beginning of a journey which, although initially characterized by challenges, ultimately led to both personal maturation and a strengthened commitment to the purpose and values of my educational role. That moment functioned as a point of departure not only in professional terms but also at a deeply personal level—a subtle reminder that the path of innovation and contribution is not always smooth or effortless, yet it remains worth pursuing with consistency, integrity, and perseverance.

II. One-day Seminar (O.R.N.: 12821/8 June 2023), which I designed, organized and coordinated, with T. S.: *Mathematical Modelling of Concepts – Flipped Learning – Group Self-Efficacy – Erasmus Programme.* This seminar was held on Thursday, 22 June 2023 at the 2nd Lower Secondary School (Gymnasium) of Peristeri. My presentation was entitled: *Modelling and Dynamic Modelling Using Dynamic Geometry Software: Algebraic Identities.* https://www.academia.edu/103787340/, https://www.academia.edu/3589846/

This workshop fostered a novel educational culture where technology serves as a conduit for enhancing understanding, modeling acts as a mechanism for conceptual clarity, and flipped learning functions as a method for tailoring the student learning experience. Another significant objective was for teachers /educators to combine the two fields of mathematics, Algebra and Geometry, providing tasks, problems and exercises that connect both areas. Considering the essential role of algebra in mathematics education, I concentrated on the *cognitive obstacles* that students face when learning algebraic concepts, presenting the issues through the perspective of *Action-Process-Object-Schema (APOS) theory* (e.g., Dubinsky, 1988, 1991a, b; Dubinsky & McDonald, 2001).

According to Cottrill *et al.* (1996):

> *"**An action** is any physical or mental transformation of objects to obtain other objects. It occurs as a reaction to stimuli which the individual perceives as external. It may be a single step response, such as a physical reflex, or an act of recalling some fact from memory. It*

may also be a multi-step response, by then it has the characteristic that at each step, the next step is triggered by what has come before. When the individual reflects upon an action, he or she may begin to establish conscious control over it. We would then say that the action is interiorized, and it becomes ***a process"*** (Cottrill, *et al.*, 1996, p. 171, in Davis, Tall and Thomas, 1997, p. 133). [authors italics…]

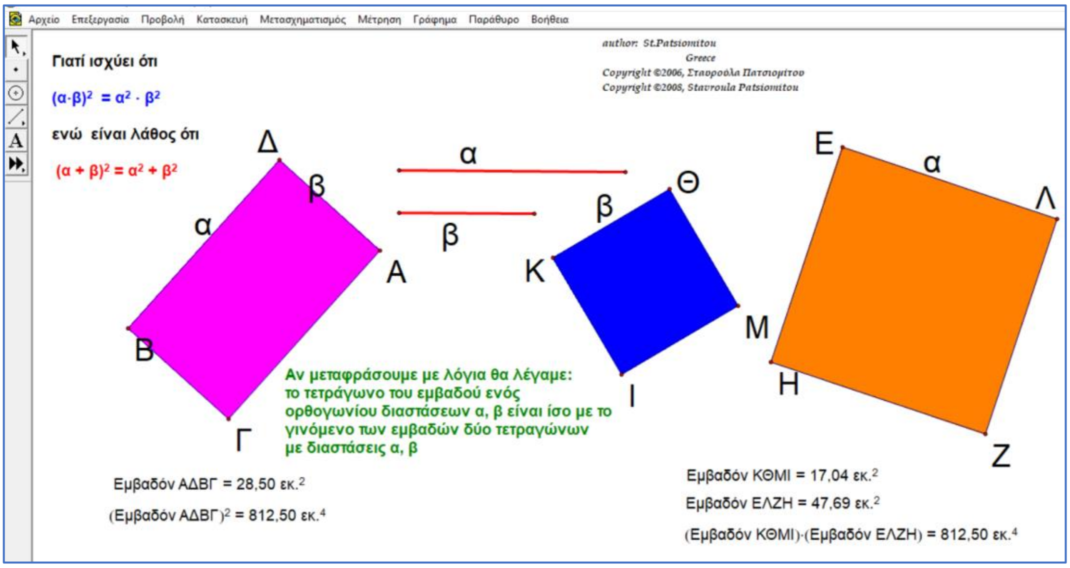


Figure 8: Experiential highlighting of the cognitive obstacle through the modelling of algebraic units

For example, the figure 8 illustrates and visualizes a cognitive obstacle that emerges during the teaching of the identity $(a+b)^2$ where students commonly make the mistake of writing $(a+b)^2 = a^2+b^2$. This error arises from an attempt to incorrectly apply the power property $(a\cdot b)^2=a^2\cdot b^2$, to the additive expression $(a+b)^2$. By calculating and comparing the areas of squares and rectangles corresponding to $(a+b)^2$, a^2, b^2, a^2+b^2, students are able to recognize the inconsistency in this mathematical expression. However, the product $a\cdot b$ represents the area of a rectangle with sides a, b. Consequently, this visual and experiential approach enables students to understand the distinction in the use of symbols and to develop a conceptual grasp of the operations of addition and multiplication of variables through visualization.

The use of dynamic geometry software, particularly in the context of algebraic identities, fosters the connection between geometry and algebra, thereby encouraging a differentiated approach. My presentation constitutes a meaningful contribution to contemporary Mathematics education, as it integrates mathematical modelling and dynamic geometry into the teaching of Algebra—an area that is traditionally regarded as abstract and often challenging for students (Patsiomitou, 2008c, 2009a). My decision to represent algebraic identities through Linking Visual Active Representations makes learning more accessible, even for students with learning difficulties.

III. One-day Seminar (O.R.N.: 16140/1 September 2023), which I designed, organized and coordinated, with T. S.: *Transforming Mathematics Education: New Curricula. This seminar was held on Wednesday, 6 September 2023.* Seminar Venue: Art Secondary School of Peristeri. My presentation was entitled: *Transforming Geometry Education: New Curricula and "Dynamic" Transformations.*

The content of my presentation is centred on a contemporary view of Geometry that moves beyond the traditional, theory-centred approach (e.g., Coxford & Usiskin, 1975). I propose an experiential and *inquiry-based mode of teaching and learning* (e.g., Jaworski, 2003), with an emphasis on interconnected representations and the cognitive transition from the concrete to the abstract. The conceptualisation of transformations as a tool for understanding provides a powerful interpretive framework within which teachers can act in a creative and reflective manner. My presentation focused on the concept of transformation, which functions not only as a mathematical concept (i.e., *geometric transformations*), but also as a metaphor for the evolution of the teaching practice itself. The term "*dynamic transformations*" refers to the shift towards interactive digital environments, with the aim of enhancing inquiry and conceptual understanding. The materials draw on chapters from my book *Learning Mathematics with Geometer's Sketchpad v4* (Patsiomitou, 2009a, 2009b), which was updated in 2022 under the title *Conceptual and Instrumental Trajectories Using Linking Visual Active Representations Created with the Geometer's Sketchpad* (Patsiomitou, 2022a). In the sequential figures I aimed to visualize geometric transformations using a generator triangle, with the purpose of highlighting the concept of self-similarity as an extension of similarity (Figure 9a).

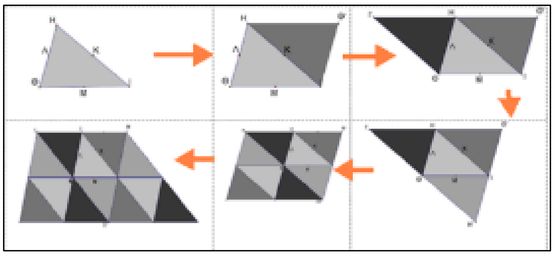


Figure 9a: Geometric transformations using a generator- triangle (Patsiomitou, 2009c, p. 26-28)

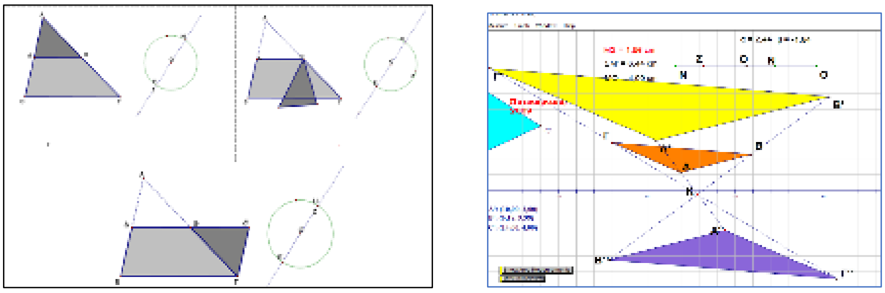


Figure 9 b, c: Geometric transformations leading
to a visual proof (Patsiomitou, 2009c, p. 38, 2022a)

I also sought to reorganize geometric configurations in order to support the visual proof of theorems and definitions (Figure 9b, c) (i.e., the midpoint -connector theorem, the similarity theorem). We can employ geometric transformations to illustrate the enlargement and reduction of figures in relation to the similarity ratio, using the dilation command of the GSP software.

IV. One-day Seminar (O.R.N.: 1125/29 September 2023), which I designed, organized and coordinated, with T. S.: *Digital Artefacts and Sociocultural Tools for the Teaching and Learning of Mathematical Concepts.* This seminar was conducted on Wednesday, 4 October 2023 at the Art Secondary School of Peristeri. My presentation was entitled: *Digital Artefacts and Sociocultural Tools for the Teaching and Learning of Mathematical Concepts.*

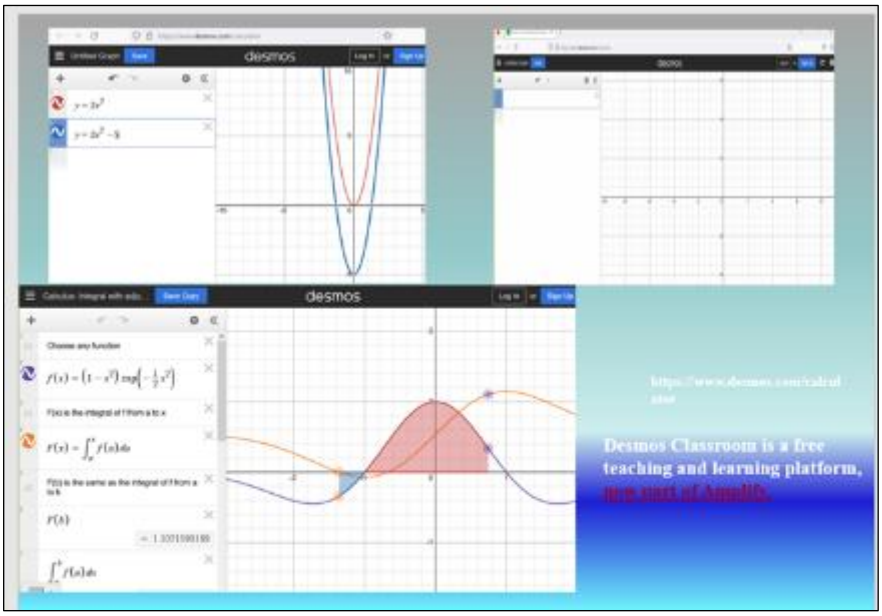


Figure 10 a, b: Linking symbolic and graphical representations using the Desmos Graphing Calculator

My presentation addresses an innovative and interdisciplinary field within Mathematics Education: the integration of technological tools (digital artefacts) with sociocultural theories (e.g., Vygotsky, 1978), which significantly enriches teachers' perspectives on the didactics of mathematics through the use of technology. Specifically, I attempted to interpret the evolution of technological instruments used for educational purposes in relation to developments in learning theories (e.g., Vygotsky, 1978; Papert, 1984; Rabardel, 1995; Trouche, 2004).

This approach supports knowledge construction through cultural practices, linking Mathematics to its broader social and cultural context, drawing on perspectives that conceptualize the computer as a *tutor, a tutee, or a tool* (Taylor, 1980), as well as on *constructionist approaches* (Papert, 1984). Particularly, Taylor (1980) outlined possible applications of the computer as (a) a tutor, where the computer provides instruction to the child, (b) a tool, which enhances the ability to tackle academic tasks, and (c) a tutee, where students acquire knowledge by programming (tutoring) the computer.

During my presentation I reported how tools exert an influence over the technical and social way in which students conduct an activity. For this, tools are considered essential to their cognitive development. According to Vygotsky (1978), tools can be considered as *external signs* and they can become tools of *semiotic mediation*. For example, DGS tools (e.g., Desmos, Geogebra, WebSketchpad) present geometric structures in an environment that emphasizes the continuous nature of Euclidean space, and thus serve as an excellent bridge between geometry and analysis [websites 10,11,12].

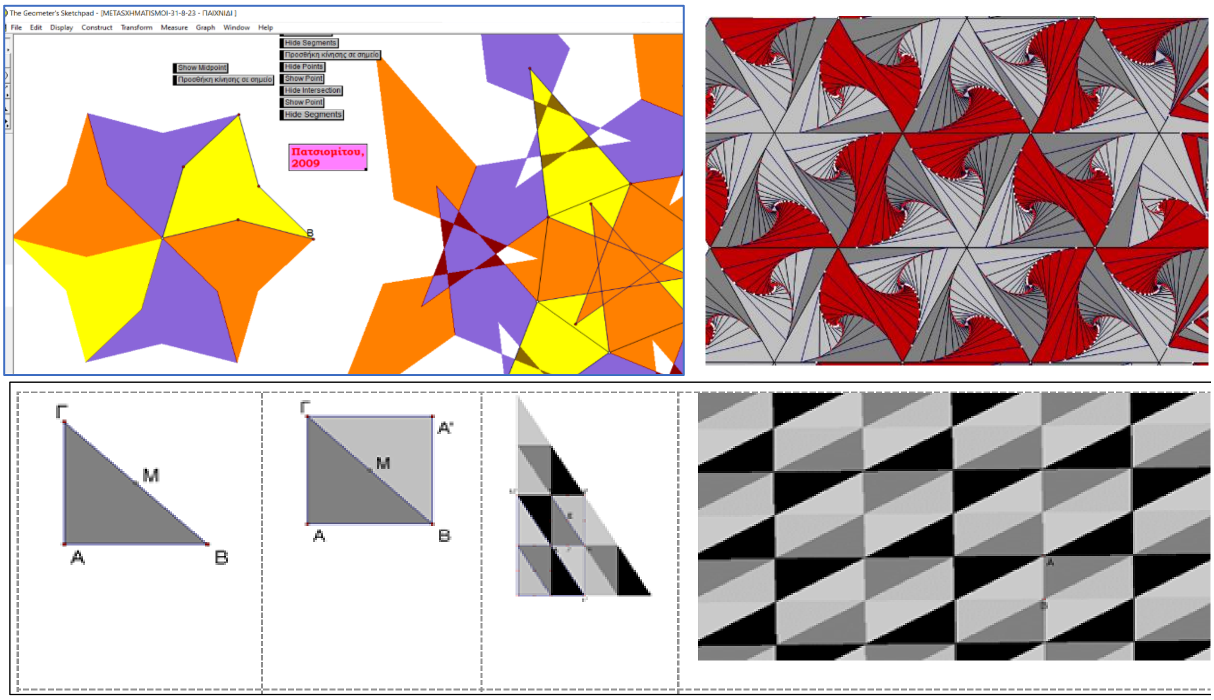
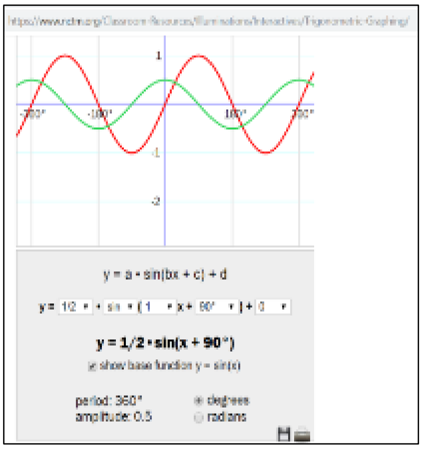


Figure 11 a, b, c, d: Various types of tessellations generated through geometric transformations in a playful, exploratory context (Patsiomitou, 2009b, 2009c)

The figures above present screenshots from the Desmos Graphing Calculator, where students can enter a symbolic expression on the left-hand side and automatically obtain its corresponding graphical representation on the right-hand side. Furthermore, I introduced the digital repository of Khan Academy [website 13] along with its YouTube channel, highlighting the instructional videos accessible on the Khan Academy platform. This resource is particularly beneficial for students' personalized learning and constitutes a valuable support for teachers in their daily instructional activities. A pedagogical action that I undertook involved a series of distinct workshops and training events, which included inviting teachers from various schools to share their effective practices that help students to *reduce mathematical anxiety*. This process served a dual pedagogical purpose: (a) it facilitated and promoted professional development by encouraging the sharing of experiences, and (b) it established a framework for role modelling, drawing on Bandura's (1977) theory of social learning, according to which the observation of effective practices can influence and shape the attitudes, beliefs, and future instructional decisions of other teachers and educators. Albert Bandura (1977) was the pioneer in introducing the term *self-efficacy*. He defined self-efficacy as an individual's belief in their capacity to perform the necessary actions to achieve specific performance objectives, or as a set of beliefs that influence how effectively a person can execute a plan of action in theoretical scenarios. Moreover, the participation of educators as presenters of effective classroom practices leads to: (a) their professional acknowledgment and sense of belonging, (b) the *enhancement of their self-esteem and perceived self-efficacy,* and (c) improved social cohesion within the educational community. In this regard, it functions as a means of pedagogical empowerment within a mutually reinforcing dynamic among participants. The screenshots in Figure 11 present images of the corresponding artifacts created using the Geometer's Sketchpad software, with the aim of integrating enjoyment and playfulness into the learning process. The primary objective is to foster positive emotions towards mathematics and to reduce mathematical anxiety. The relationship between mathematics and play may be conceptualized in two complementary ways: either as *mathematics made playful*, where mathematical content and activities are intentionally designed or experienced in a playful manner, or as *mathematising elements of play*, where aspects of play are systematically structured, interpreted, or transformed through mathematical thinking (van Oers, 1996).

V. One-day Seminar (O.R.N.: 1999/7 December 2023), which I designed, organized and coordinated, with T. S.: *Digital Scenarios in the new curricula*. This seminar was conducted on Wednesday, 13 December 2023 at the 1st Lower Secondary School (Gymnasium) of Agioi Anargyroi. My presentation was entitled: *Digital Scenarios in the New Curricula: Symbolic and Linking Visual Active Representations in Trigonometric Functions.*

The focus on digital instructional scenarios is situated at the core of contemporary Mathematics education, drawing on elements of the revised national curricula that emphasise active learning and multiple representations of mathematical concepts (e.g., Ainsworth, 2006), something I support in all my research studies. My particular emphasis on trigonometric functions, as an object of analysis through symbolic and visual active representations, promotes deeper conceptual understanding by connecting abstract mathematical ideas with visual and digital tools. Students can visualize the effect of modifying the coefficients of the trigonometric functions in the NCTM interactive diagrams [Webpage 14]. This action on interactive diagrams helps students to acquire a *direct perception* of transformations of the mathematical objects (Patsiomitou, 2006b), as well as to examine the role the coefficients play in the graphic representation of the trigonometric function.



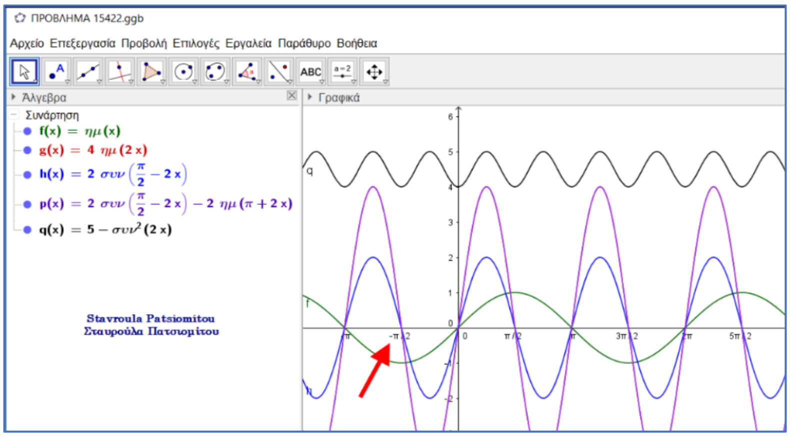
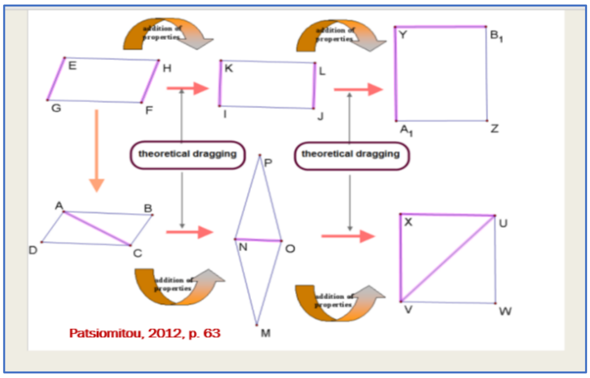


Figure 12 a, b: Trigonometric functions and their graphic representations

VI. Two-day Seminar (O.R.N.: 219/15 February 2024), which I designed, organized and coordinated, with T. S.: *Digital Mathematics: Software and Artificial Intelligence.* This seminar was conducted on Wednesday, 21 February 2024 and Thursday, 22 February 2024 at the Model Lyceum Upper Secondary School of Agioi Anargyroi. My presentations were entitled: **(**1) *Digital Learning Trajectories in Quadrilaterals: Varignon's Theorem, (2) Problem Solving in Geometry: Gamow and the Lost Treasure of Pirates.*



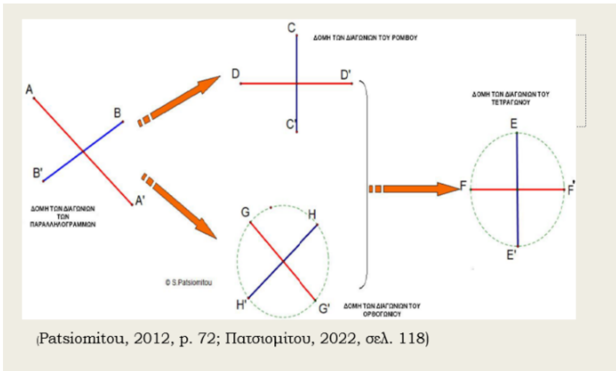


Figure 13a: LVAR pertaining to the concept of a parallelogram (Patsiomitou, 2012b, p. 63)

Figure 13b: The LVAR of the diagonals of various types of parallelograms (Patsiomitou, 2012b, p. 72)

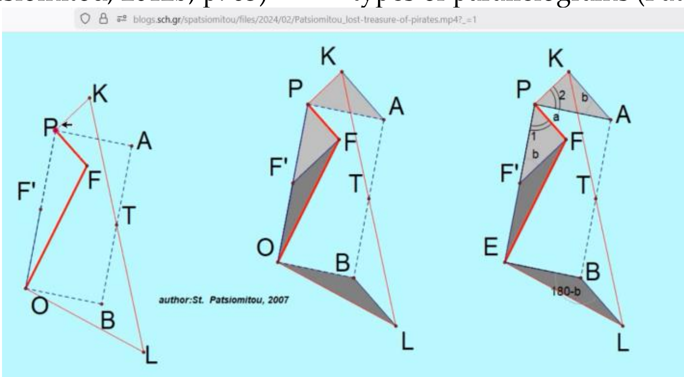


Figure 13c: LVARs (Mode C): a visual proof presented in linking diagrams (Patsiomitou, 2008a, p. 373)

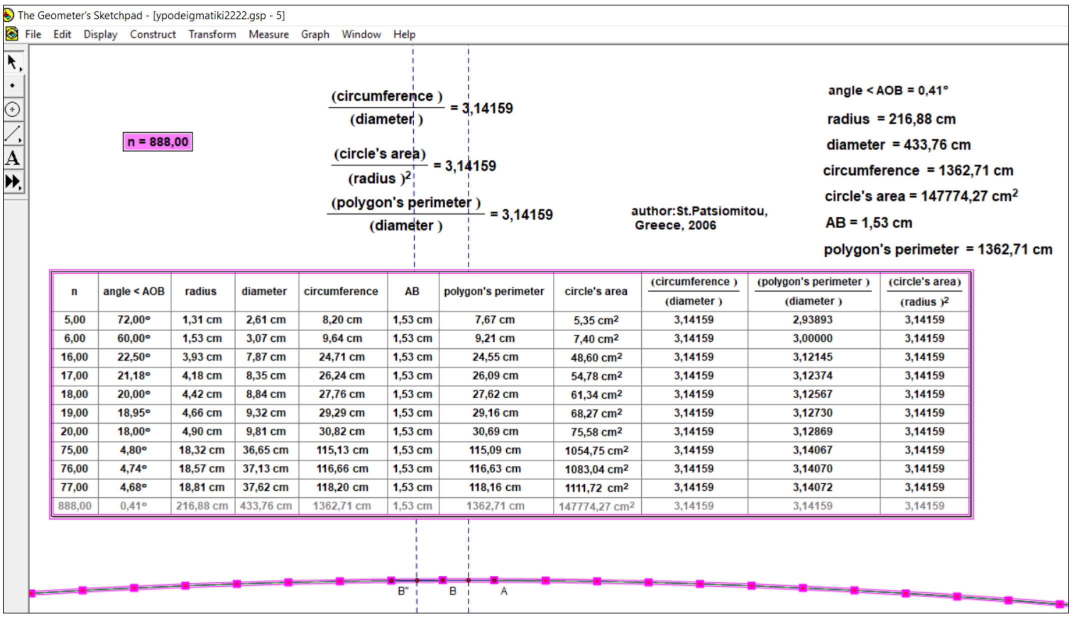
The two-day workshop focused on contemporary digital tools as a means of supporting the teaching and learning of Mathematics, incorporating hypothetical learning trajectories for the construction of meanings (e.g., the meaning of parallelogram). In my first presentation, I described a 'dynamic' hypothetical learning path (DHLP) for the learning of the concept of parallelogram in geometry, which helped the students of the experimental team of my PhD research study to raise their van Hiele levels (Patsiomitou, 2012a, 2012b). As I report in my study (Patsiomitou, 2012b, p.63) "*In the Figure 13a, we can see the linking visual active representations of the first phase. Dragging the parallelogram theoretically, we can shape a "soft" rectangle, and by dragging the rectangle theoretically, we can shape a "soft" square. If we construct a diagonal in the parallelogram, we can drag it theoretically and shape a rhombus and then a square by analyzing the figure as two sub-figures"*. In Figure 13b "*we observe the linking representations of the diagonals of different types of parallelograms. Dragging theoretically the endpoint of the diagonals of the parallelogram in order these to acquire the property of the perpendicularity leads to the structure of the rhombus diagonals (or a square's diagonals). Dragging theoretically the endpoint of the diagonals of the parallelogram in order these to acquire the property of the congruency leads to the structure of the rectangle's diagonals*" (Patsiomitou, 2012b, p. 72).

In the second presentation I focused on Vecten's theorem. From a historical perspective, in 2007 I extended my investigation of Vecten's theorem to its well-known real-world formulation, originally introduced by Gamow (1948, reprinted 1988). This work involved modelling the problem within the dynamic geometry environment of The Geometer's Sketchpad (e.g., Patsiomitou, 2008a, b). In my opinion, a teacher's investigational activity in relation to the problem posed has to be implemented at several levels of sophistication, if a teacher is to help his/her students to develop deeper understanding and coherent reasoning. Both presentations highlight two key dimensions: geometric understanding through digital environments, and the development of problem-solving skills in a playful and creative manner. The use of dynamic software provides increased interactivity, fosters active learning, and makes mathematical structures and relationships more visible.

VII. One-day Seminar (O.R.N.: 322/8 March 2024), which I designed, organized and coordinated, with T. S.: *Digital Tools in the Interpretation of Dynamic Representations and in the Problem-Solving Method.* This seminar was conducted on Thursday, 14 March 2024 at the 6th Lower Secondary School (Gymnasium) of Peristeri). My presentation was entitled: *Approaching the Number pi(π) through Dynamic Active Parametric n-gons or Riemann Integrals.*

My presentation focused on a classical mathematical concept—the approximation of the number π—through a contemporary perspective, by employing digital tools for dynamic, parametric representations and the computation of Riemann integrals (Patsiomitou, 2006f, 2007c, 2018a, p.238). Specific examples from my experimental research using dynamic active representations have been analyzed in the methodology section of my study *A dynamic active learning trajectory for the construction of number pi (π):*

transforming mathematics education (Patsiomitou, 2018a): (a) the construction of number pi as an approximation process (Patsiomitou, 2006c, 2007a, 2009). For this, I created in the Geometer's Sketchpad software the process of an inscribed or circumscribed n-gon in a circle with a view to using the tabularized measurements and calculations of a ratio in combination with the software's iteration process to lead the students to visualize the approximation process of number pi; (b) the construction of number pi through Riemann sums in a DGS environment (Patsiomitou, 2006c); (c) the construction of number pi by means of a real world problem (Patsiomitou, 2013b, 2016a, b).



n	angle < AOB	radius	diameter	circumference	AB	polygon's perimeter	circle's area	(circumference) / (diameter)	(polygon's perimeter) / (diameter)	(circle's area) / $(radius)^2$
5,00	72,00°	1,31 cm	2,61 cm	8,20 cm	1,53 cm	7,67 cm	5,35 cm^2	3,14159	2,93893	3,14159
6,00	60,00°	1,53 cm	3,07 cm	9,64 cm	1,53 cm	9,21 cm	7,40 cm^2	3,14159	3,00000	3,14159
16,00	22,50°	3,93 cm	7,87 cm	24,71 cm	1,53 cm	24,55 cm	48,60 cm^2	3,14159	3,12145	3,14159
17,00	21,18°	4,18 cm	8,35 cm	26,24 cm	1,53 cm	26,09 cm	54,78 cm^2	3,14159	3,12374	3,14159
18,00	20,00°	4,42 cm	8,84 cm	27,76 cm	1,53 cm	27,62 cm	61,34 cm^2	3,14159	3,12567	3,14159
19,00	18,95°	4,66 cm	9,32 cm	29,29 cm	1,53 cm	29,16 cm	68,27 cm^2	3,14159	3,12730	3,14159
20,00	18,00°	4,90 cm	9,81 cm	30,82 cm	1,53 cm	30,69 cm	75,58 cm^2	3,14159	3,12869	3,14159
75,00	4,80°	18,32 cm	36,65 cm	115,13 cm	1,53 cm	115,09 cm	1054,75 cm^2	3,14159	3,14067	3,14159
76,00	4,74°	18,57 cm	37,13 cm	116,66 cm	1,53 cm	116,63 cm	1083,04 cm^2	3,14159	3,14070	3,14159
77,00	4,68°	18,81 cm	37,62 cm	118,20 cm	1,53 cm	118,16 cm	1111,72 cm^2	3,14159	3,14072	3,14159
888,00	0,41°	216,88 cm	433,76 cm	1362,71 cm	1,53 cm	1362,71 cm	147774,27 cm^2	3,14159	3,14159	3,14159

Figure 14a: Generating number pi through active representations (Patsiomitou, 2018a, p.232)

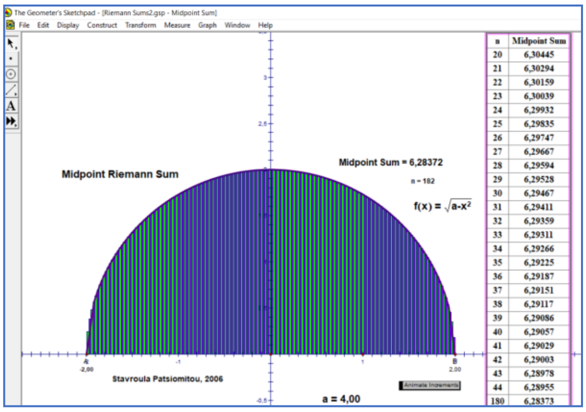


Figure 14b: Generating number pi through active representations (Patsiomitou, 2018a, p.235-238)

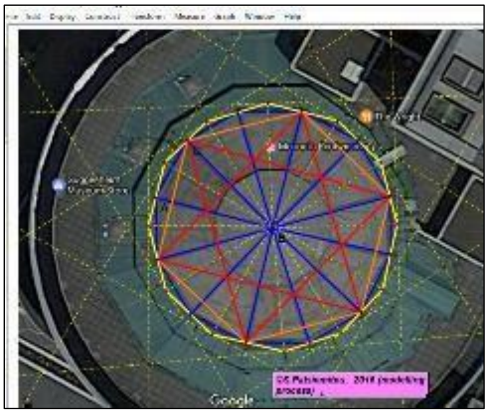


Figure 14c: Solving a real-world problem

This approach connects geometry with calculus, enabling teachers to understand and convey a multi-layered and interdisciplinary mathematical idea through the use of digital media. In Figure 14c, we observe the top view of the Guggenheim Museum alongside a representation of its form produced using the Geometer's Sketchpad. During a Google Earth exploration, I focused on the roof of the Guggenheim Museum in New York City, where I observed that its structure corresponds to a regular dodecagon (i.e., a 12-sided polygon with all sides and interior angles equal). This observation was subsequently integrated into a mathematical modelling process, in which a real-world situation was transformed into a geometrical problem and explored within a dynamic geometry environment. The figures illustrate how digital tools can support the transition from real-world observation to mathematical representation and formalisation. This type of web-based and dynamic geometry exploration aligns with an inquiry-oriented approach to teaching, in which students engage in iterative cycles of observation, conjecture, modelling, and validation, thereby enriching everyday classroom practice in mathematics education.

VIII. One-day Seminar (O.R.N.: 311/6 March 2024), which I designed, organized and coordinated in collaboration with 3 more colleagues of different subject specializations, with T. S.: *Collaboration, Empathy, Communication and Cognitive Tools in the Educational Sociocultural Context.* This seminar was conducted on Wednesday, 20 March 2024 at the 8th Lower Secondary School (Gymnasium) of Peristeri. My presentation was entitled: *Cognitive Tools in a Sociocultural Framework for the Teaching and Learning of Concepts.*

In my presentation, I sought to communicate the idea that "*engaging in mathematics [through dynamic geometry] equates to understanding mathematics*," which also serves as the title of my book (Patsiomitou, 2009 a, b). A notable example was the presentation of cube modeling, which increased the engagement of the lesson. Figure 15 presents a screenshot from the modelling of the identity $(a+b)^3$, where students recognize that it corresponds to a cube with edge length (a+b). Furthermore, the decomposition of the cube enables the visualization of the individual components of the identity. For instance, the term represents the volume of a rectangular prism with dimensions b×b×a, consisting of a square base of side b and height equal to a.

In *Didactic Approaches to Teaching Mathematics to Students with Different Learning Styles* (Patsiomitou, 2012c) [https://www.academia.edu/3517291/_], I describe a dialogue between two students in my class, one of whom has learning difficulties, as they attempt to understand the identity *$(a+b)^3$ (p. 224)* The use of dynamic tools in collaboration with physical manipulatives in this process provides immediate interaction and opportunities for experimentation, encouraging experiental learning (e.g., Kolb, 1984; Kolb & Kolb, 2005). Furthermore, my emphasis on the use of digital tools aims to engage students' interest, redirect their attention away from negative behaviours (such as violence), and enhance their motivation to learn. It reflects a deep understanding of the sociocultural challenges of school reality.

The emphasis on empathy, collaboration, and communication as key components of the educational process, combined with the use of cognitive tools, offers an innovative approach aimed at the holistic development of students and the qualitative improvement of the learning process.

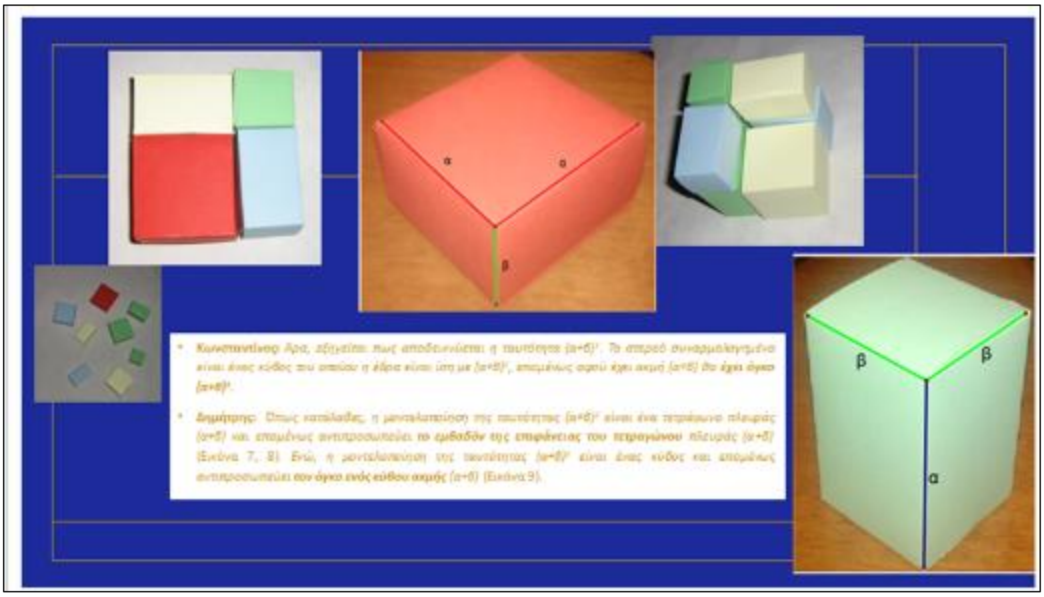


Figure 15: Screenshot of the modelling of the identity *$(a+b)^3$(Patsiomitou, 2012c, p.224-227*

IX. One-day Seminar (O.R.N.: 404/3 April 2024), which I designed, organized and coordinated, in collaboration with one more colleague of different subject specialization, with T. S.: *STEAM: Mathematics and Art in the Educational Sociocultural Context.* This seminar was conducted on Wednesday, 10 April 2024, at the Art Secondary School of Peristeri. My presentation was entitled: *STEAM Instrumental Pathways: Tessellations, Pentominoes and Rep-tiles in the New Curriculum.*

The aim of my presentation was to introduce STEAM-based instructional trajectories in an effort to foster a positive climate towards and about Mathematics. My broader objective was to present cognitive tools as a means of communication and collaboration between teachers and students. The thematic focus combines two seemingly distinct domains—Mathematics and Art—with an emphasis on STEAM (Science, Technology, Engineering, Arts, Mathematics), promoting an interdisciplinary approach that contributes to the broader pedagogical development of teachers and educators. The use of specific tools such as tessellations, pentominoes, and rep-tiles provides practical examples that connect abstract mathematical concepts with visual and creative learning artefacts, thereby enhancing the learning experience.

This approach supports teachers in developing skills that extend beyond the strictly academic domain, linking learning with creativity. In turn, this fosters students' intrinsic motivation and increases their interest in Mathematics. Moreover, the reorganisation of representations is supported by contemporary theories of mathematical representation in learning, enhancing cognitive flexibility and the active processing of knowledge. The material presented was drawn from my book *Conceptual and Instrumental Trajectories through Linking Visual Active Representations in The Geometer's Sketchpad* (Patsiomitou, 2022a).

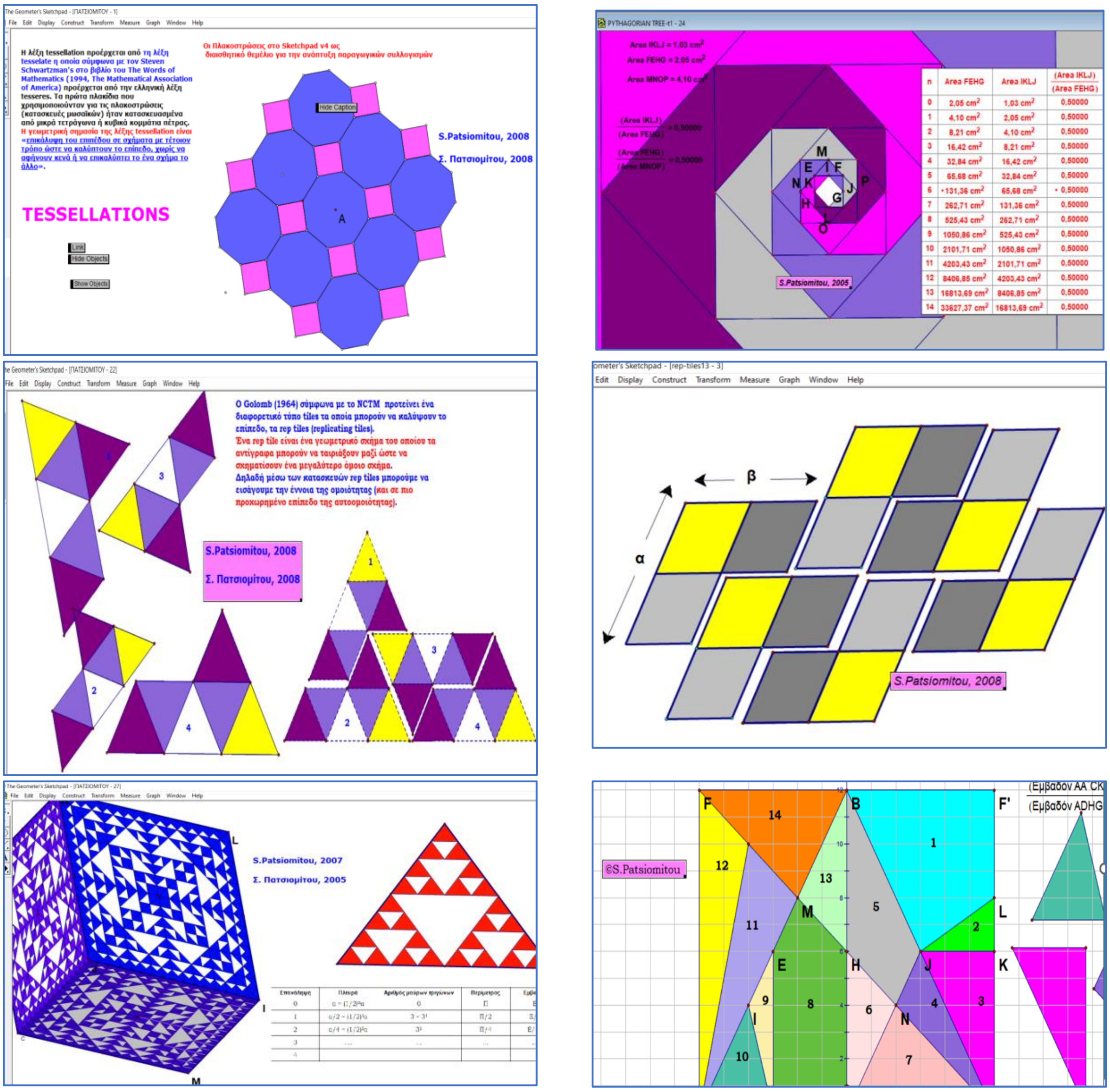


Figure 16: Screenshots illustrating the presented instructional material (Patsiomitou, 2009b, 2009c, 2022a)

X. Two-day Seminar (O.R.N.: 470/17 April 2024), which I designed, organized and coordinated, with T. S.: *Digital Technologies and Mathematics: The Role of Active Dynamic Representations in the Curriculum.* This seminar was conducted on Wednesday, 24 April 2024 and Thursday, 25 April 2024, at the Model Lyceum Upper Secondary School of Agioi Anargyroi. My presentations were entitled: (1) *Reorganisation of Representations and Visual Proof in Euclid's (2) Digital Technologies and Mathematics: "Dynamic" Translation of topics from the I.B.G.D. -Topic Bank".*

I contributed to this seminar by delivering two presentations: In my first presentation, I discussed and placed emphasis on visual proof, which constitutes a more accessible form of learning classical geometric theorems, thereby strengthening mental visualization and conceptual understanding. As an indicative example, in my presentation I demonstrated how the Pythagorean theorem can be proved through the use of *Linking Visual Active Representations* (LVAR), i.e., linking visual active representations, which I created through a carefully designed process within the software environment. These representations were connected through a vector translation transformation, whereby each successive representation constitutes a subsequent step in the proof, following a structure analogous to Euclid's original proof of the Pythagorean theorem.

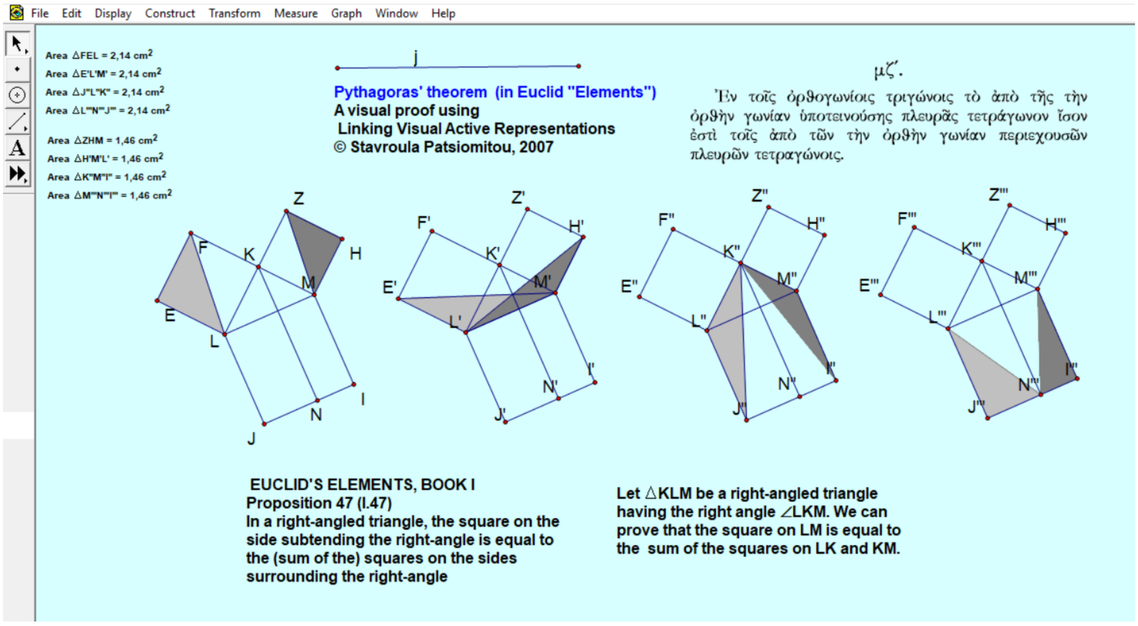


Figure 17: Proposition I.47 using LVARs (Mode C): a visual proof presented in four linking diagrams (Patsiomitou, 2019a, p. 11)

The second presentation focused on the utilization of digital technologies for the dynamic transformation of problems from the Item Bank of Graded Difficulty (IBGD), supporting both teachers and students in adapting to contemporary educational requirements. I have reported this initiative in the previous section.

XI. One-day Seminar (O.R.N.:511/13 May 2024), which I designed, organized and coordinated, with T. S.: *Designing Activities with Concrete and Digital Materials for the Teaching, Learning and Assessment of Mathematical Concepts.* This seminar was conducted on Thursday, 23 May 2024, at the Special Vocational Education and Training School (EN.E.E.GY.-L.) of Peristeri. My presentation *was entitled: Realistic Problems and End-of-Year Examinations in Lower Secondary School (Gymnasium).*

According to De Corte, Verschaffel, & Greer (2000), the implementation of mathematics to solve real-world problems can be understood as a complex process involving a number of phases: understanding the situation described; constructing a mathematical model that describes the essence of those elements and relations embedded in the situation that are relevant; working through the mathematical model to identify what follows from it; interpreting the outcome of the computational work to arrive at a solution to the practical situation that gave rise to the mathematical model; evaluating that interpreted outcome in relation to the original situation; and communicating the interpreted results (p. 1). The image below (Figure 18a) presents simulations of physical objects, aiming at the transformation of a real-world situation into a mathematical problem through the modelling process. Specifically, I designed in the DGS environment an elevator-like structure in such a way that, by dragging a point, the entire platform moves vertically. Simultaneously, the geometric configurations formed by intersecting lines change dynamically, allowing the resulting shapes to transform, creating different kinds of parallelograms, rhombi, and squares. The figures 18a, 18b, are also screenshots from image processing within the software, aimed at integrating mathematics into real-world contexts.

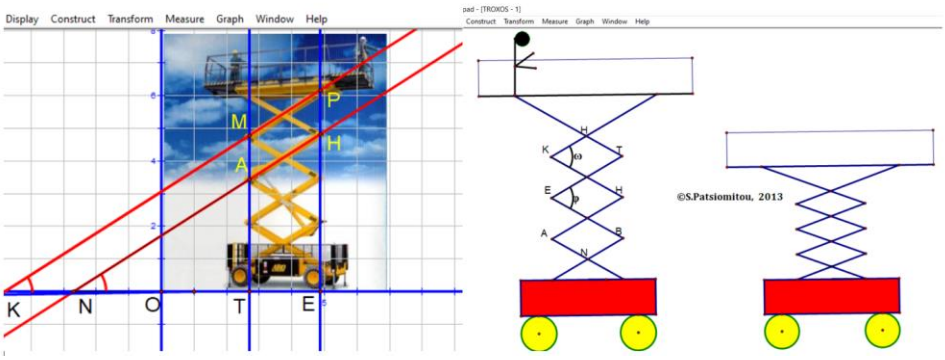


Figure 18a: Modelling a real-world problem as a simulation (Patsiomitou, 2013b)

They include a variety of problems that I have designed using real-world images, in which I completed the proofs in symbolic form, thereby interconnecting different types of representations through translation from one form to another. These problems, as well as similar ones, are discussed in my article (Patsiomitou, 2013b) and are also explained extensively in my monograph (Patsiomitou, 2020b). *In this way, all types of representations are interconnected through translation from one representation to another.* Fitzpatrick and Morrison (1971, p. 239) argue that a performance measure relevant to "real life" must be administered under conditions that reflect the stimuli and responses encountered in authentic settings. Building on this, Palm (2007, p. 40) clarifies that, when *authenticity* is understood in this way, a school task situated outside the classroom can be regarded as authentic only if it adequately mirrors a real-life situation and reproduces its key elements to a reasonable extent.

The focus on real-life problems contributes to the development of students' ability to apply mathematical concepts in everyday contexts, thereby increasing both engagement and the perceived usefulness of learning.

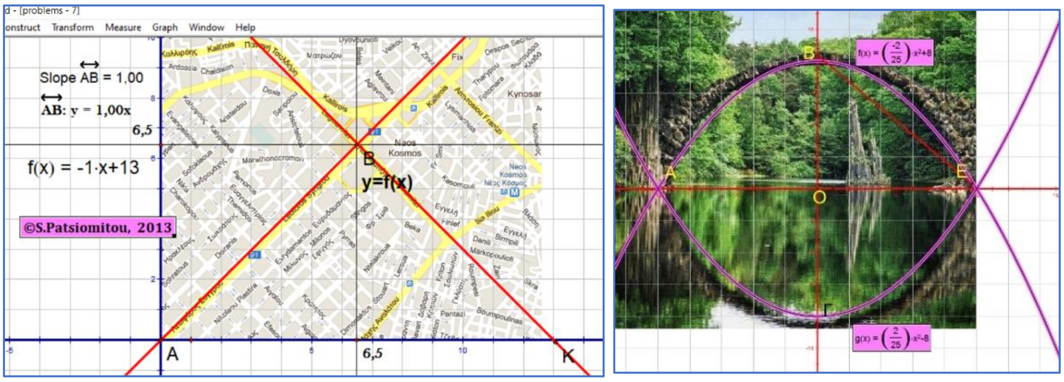


Figure 18 b, c: Simulations and modelling of real-world problems in a dynamic geometry environment (Patsiomitou, 2013b, 2020b)

XII. One-day Seminar (O.R.N.: 18623/10 September 2024), which I designed, organized and coordinated, with T. S.: *Improving Quality in Education: Interactive Programmes, Digital Media and Artificial Intelligence.* This seminar was conducted on Monday, 16 September 2024, at the "Spyros Apostolou" Cultural Centre of Agioi Anargyroi. My presentation was entitled: *The Impact of Artificial Intelligence and Digital Media on Improving the Quality of Educational Practice.*

The theme of this workshop addresses one of the most contemporary and critical issues in education today: improving the quality of educational practice through the use of interactive software, digital media, and artificial intelligence (AI) tools. The topic is highly relevant and guides teachers and educators towards understanding how new technologies and AI tools can enhance the teaching process and improve learning outcomes. My presentation focused on the pivotal role that artificial intelligence and digital tools can play in enhancing the quality of teaching and learning (Patsiomitou, 2024a). It further emphasizes the significance of digital media and Artificial Intelligence (AI) in developing innovative approaches to concept introduction. In this context, I presented an educational experiment aimed at exploring the use of AI technologies, including ChatGPT, Leonardo.AI, Lumen5.AI [websites 15, 16], in the creation of *Linking Visual Active Representations*.

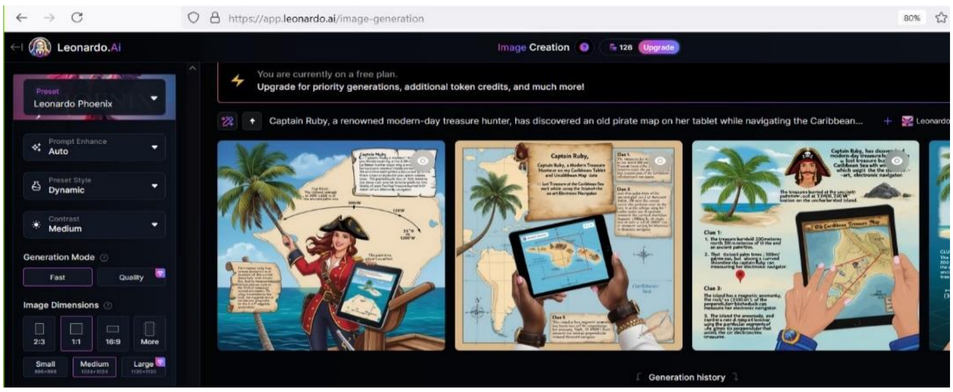


Figure 19: Creation of the "Captain Ruby" character through appropriate prompting in Leonardo and ChatGPT (Patsiomitou, 2024b, p.204)

Within this framework, the character of *'Captain Ruby'* was introduced as a female pirate who replaces the male pirate in Gamow's problem, equipped with contemporary tools such as a tablet and an electronic navigator to support exploration. The study (Patsiomitou, 2024b) highlights the potential of combining human thinking, imagination, and creativity with AI tools, suggesting that learning can become more engaging, personalized, and meaningful when approached with curiosity and an open mindset. It concludes by raising broader reflective questions about future learning possibilities, the generation of new ideas, and the transformation of knowledge into an experience that integrates play, exploration, and inspiration. In this sense, the education of the future is conceptualized as a collaborative space between humans and AI, where artificial intelligence serves as a supportive companion in the learning process. Furthermore, the presentation highlights and promotes innovation and teachers' adaptability in response to technological developments. The significance of this seminar lies in its contribution to the re-evaluation of the way Geometry is taught, particularly in an educational context that is shifting from representational to investigational and exploratory methodologies. Additionally, these methodologies prepare teachers for the integration of artificial intelligence into Mathematics education, a component expected to gain increasing importance in the coming years.

XIII. One-day Seminar (O.R.N.: 20238/24 September 2024), which I designed, organized and coordinated, with T. S.: *Indicative Proposals for Managing the Mathematics Curriculum in Lower Secondary School (Gymnasium): Teaching Scenarios with Concrete and Digital Materials.* This seminar was conducted on Wednesday, 2 October 2024, at the 1st Lower Secondary School of Agioi Anargyroi & Model Lower Secondary School of Agioi Anargyroi. My presentation was entitled: *Teaching Scenarios with Interactive Environments and Concrete Materials: Indicative Proposals for Curriculum Management.*

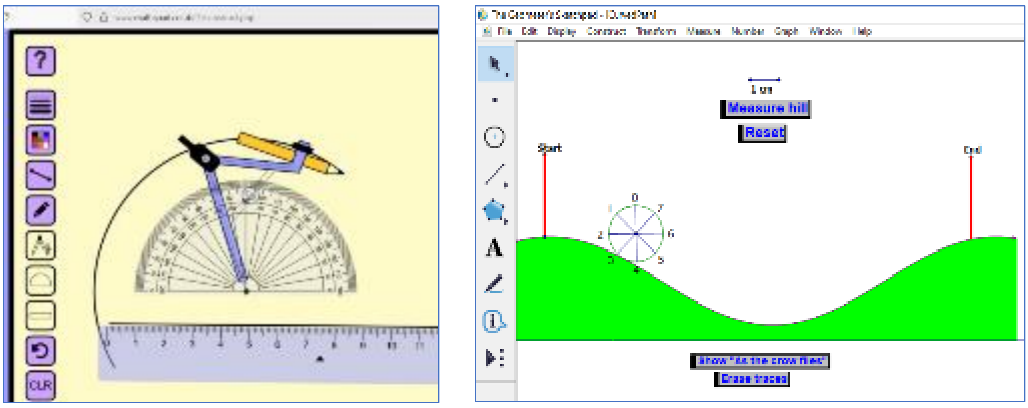


Figure 20: Presenting several websites [17, 18] and tools for experiental learning

The workshop focused on a highly important issue for the teaching of Mathematics in lower secondary education: *the organisation and management of the curriculum content* (Patsiomitou, 2025d). With an emphasis on the use of both manipulative [tangible] and digital materials, the workshop proposes innovative methods that support teachers in designing effective lessons and making learning more engaging and comprehensible for students. I presented specific examples and teaching scenarios that combine digital tools with physical materials, thereby supporting active and experiential learning. I emphasised the importance of appropriate curriculum management in order to maintain a balance between theoretical content, practical application, and the use of digital resources. Through my presentation, my aim was to help teachers understand how innovative practices can be integrated into the curriculum, thereby increasing students' interest and understanding. More generally, tools and methods were presented that facilitate the adaptation of teaching to students' needs and learning pace.

One of the key points I emphasized was the use of traditional geometric instruments (such as a ruler and compass) for constructions on the board or paper, alongside the use of interactive whiteboard tools or digital versions of geometric instruments, such as virtual compasses and rulers (e.g., website 17). Furthermore, I highlighted the difficulties students encounter in the addition of rational numbers and proposed appropriate digital tools to support their understanding (for example, calculating the distance between two points along a curved path, by counting the number of rotations of the wheel, students can calculate the distance between the two points start-end [website 18]). A particular pedagogical value was added through my proposal to involve students from several schools, who, under the guidance of their teachers, collaboratively presented an indicative teaching scenario. A specific additional educational benefit derived from this initiative lies in the development of collaborative design skills and active engagement in instructional planning. This intervention confirms the pedagogical value of inquiry-based learning and experiential approaches in Mathematics education.

XIV. One-day Seminar (O.R.N.: 25292/5 November 2024) which I designed, organized and coordinated, with T. S.: *Innovative Approaches in the Teaching and Learning of Mathematical Concepts.* This seminar was conducted on Thursday, 14 November 2024, at the Multipurpose Hall of the 6th & 8th Upper Secondary Schools of Peristeri. My presentation was entitled: *Fermat-Torricelli Theorem: Active Interactions and the Proof Process.*

The selection of the Fermat–Torricelli theorem (e.g., Hofmann, 1929, 1969) as the central theme contributes to the deepening of mathematical thinking and geometric reasoning, both of which are fundamental pillars in Mathematics education. In my presentation, the experiential demonstration of the solution of the Fermat–Torricelli theorem (Patsiomitou, 2012a, b; Patsiomitou, 2019c, p. 202) was central, as it combined theoretical discourse with investigation and experimental approach through the use of DGS software tools, as part of my PhD thesis (Patsiomitou, 2012a). The use of linking visual and active representations enhances understanding and enriches the learning experience. My presentation focused on (a) active interactions, namely a student-centred approach in which learners are actively engaged in exploring and understanding the theorem through the proving process and the development of mathematical proof, and

(b) the ways in which mathematical proofs can be made more accessible and engaging through the use of LVAR modes and appropriate digital tools. Moreover, the solution of the real-world problem during the presentation contributed to stimulating teachers' interest, making mathematical knowledge for their students more accessible and relevant to their everyday experiences. This aligns with contemporary pedagogical approaches that promote student-centred learning and active participation.

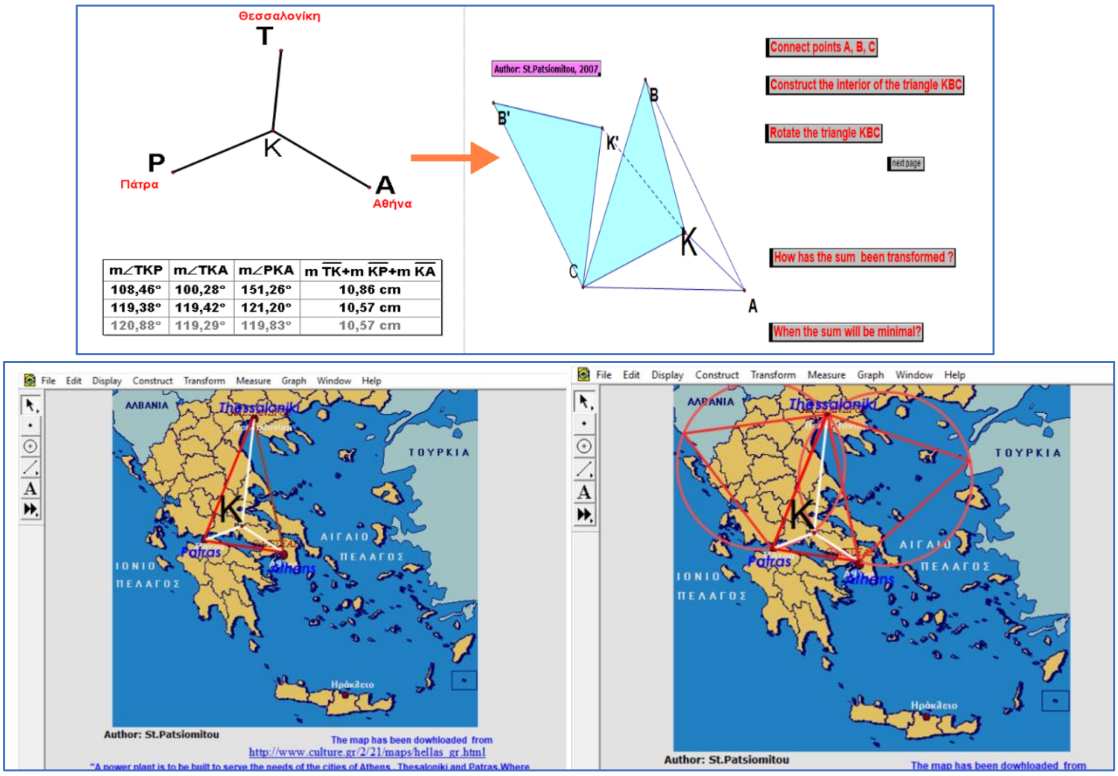


Figure 21 a, b, c, d: Use of LVARs in the modelling process of the Fermat–Torricelli theorem (Patsiomitou, 2019c, p. 202)

Colgan (2014, p. 4) emphasizes that children's discomfort with mathematics is a matter of concern and argues that fostering positive dispositions is essential, since learning depends on initial interest. She further cites Koirala (2005), who highlights the role of engagement, noting that without enjoyment, children are unlikely to commit to practice or learning, as fun supports both achievement and focus. Furthermore, the active involvement of teachers in such innovative methods facilitates the implementation of new instructional strategies in their classrooms, contributing to the renewal and improvement of Mathematics education.

XV. Two-day Seminar (O.R.N.: 29224/4 December 2024), which I designed, organized and coordinated, with T. S.: *Women and Mathematics.* This seminar was conducted on Thursday, 12 December 2024, City Hall Auditorium of Peristeri and on Friday, 13 December 2024, at the Event Hall of the Directorate of Secondary Education. My presentations were entitled: (1) *A Dynamic Trajectory in Dynamic Euclidean Geometry (2) Gender Equality in Mathematics Education: Do Men Lead and Women Teach?* [https://www.academia.edu/44760742/]

In an effort to support and empower young women teachers of Mathematics at schools, I developed the session entitled *Women and Mathematics,* which focused on increasing awareness among teachers and students regarding the significance of gender equality in STEM disciplines (e.g. Patsiomitou 2015b, 2020d). This initiative played a vital role in expanding/ broadening the educational perspective and encouraged dialogue on the empowerment of girls in Mathematics, thus facilitating the adoption of more inclusive and socially conscious curricula. Moreover, this session represents an exemplary action of civic engagement, integrated within broader gender equality initiatives in education.

I contributed in this seminar by delivering two presentations: In my first presentation, I discussed the integration of fractals into mathematics curricula (e.g., Patsiomitou, 2005a, 2005b, 2006a, 2006c, 2007a, 2007b, 2026a, 2026b, 2025b, 2026). My research on fractal geometry, which has been published in various articles and monographs, encompasses the design and creation of educational activities. Further resources can be found in my online journal focused on transformations and fractal geometry, where I offer essential theoretical foundations as well as links to materials that assist in the creation of fractals for both students and teachers of mathematics (https://schoolpress.sch.gr/testpat1/). Furthermore, during the seminar entitled *Geometry: Instructional Methods and Teaching in Two-Dimensional and Three-Dimensional Spaces*, I presented a practical approach to fractal constructions (e.g., the Sierpinski Triangle and the Pythagorean Tree), alongside experiential 3D geometry activities. In my published studies (e.g., Patsiomitou, 2016a, 2026) I explain a learning trajectory in which I propose a fractal-based dynamic program which can act as an informal curriculum (Patsiomitou, 2025c, Patsiomitou 2026).

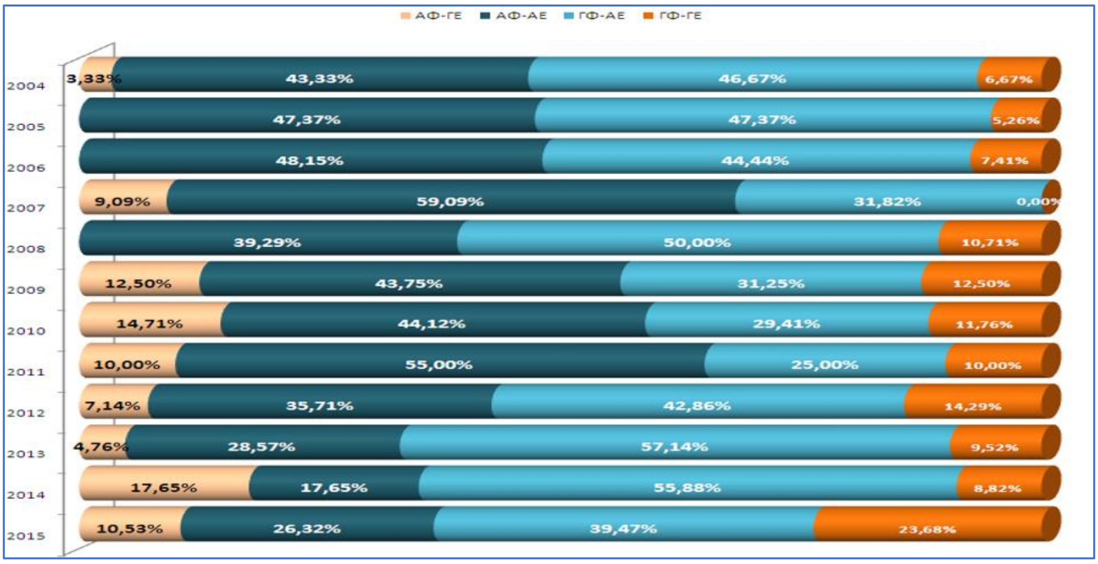


Figure 22: Indicative statistical data derived from the extended research study (Patsiomitou, 2020d)

In the second presentation, I addressed statistical information obtained from my comprehensive research (Patsiomitou, 2020d) regarding the underrepresentation of women in university Mathematics and STEM departments. The study's methodology focuses on the formation of supervisor–student pairs during the thesis writing process, considering that students had the autonomy to to independently select their supervisor. The diagram above (Figure 22) illustrates the correlation between male student–female supervisor relationships (light orange), male student–male supervisor relationships (dark blue), female student–male supervisor relationships (light blue), and female student–female supervisor relationships (dark orange) within a faculty of STEAM sciences. As is evident from the diagram, the colour blue predominates, while female supervisors are underrepresented. As I contend, it illustrates that this issue transcends personal opinion, representing a concrete, measurable, and persistent inequality. This underscores the need for targeted initiatives aimed at promoting equality and strengthening the participation of women in STEAM education. During the two-day symposium *Women and Mathematics,* many women university professors in Greece were invited to contribute as speakers.

XVI. Interdisciplinary Mathematics Symposium (O.R.N.: 2746/3 February 2025), which I designed, organized and coordinated in collaboration with *Education Advisors of different subject specialisations,* with T. S.: *Numbers and Number Sequences in Mathematics: An Interdisciplinary Approach.* This seminar was conducted on Thursday, 13 February 2025, 12:00–14:30, & Saturday, 22 February 2025, 09:00–19:00, at the City Hall Auditorium of Peristeri. My presentations were titled: (1) *Representations of Numbers: Cognitive Approach, Digital Media and Geometric Forms (Duration: 30 minutes) (2) Dynamic Sequences of Infinite Terms: π and φ.*

In an official invitation addressed to the schools under my scientific oversight, I encouraged mathematics teachers to develop problems for students related to numbers and number sequences (for instance, figurate numbers) and to subsequently present them at the aforementioned symposium. Furthermore, the development of the official invitation and the communication with university professors and fellow education advisors from the 3rd Athens Directorate constituted a considerable effort on my part, which I undertook with eagerness and a profound commitment to promoting a creative and impactful initiative. The symposium adopted an interdisciplinary perspective. Its thematic focus, which connects numbers and sequences with other scientific domains, highlights the need for a holistic and interdisciplinary approach to Mathematics teaching. This perspective enriches knowledge and makes Mathematics more accessible for both teachers and students.

Table 2

Square Years (S.Y.)	As a sequence of squares of natural numbers	Difference in years from the previous S.Y.
1849	43^2	85 years
1936	44^2	87 years
2025	45^2	89 years
2116	46^2	91 years
2209	47^2	93 years
2304	48^2	95 years

The conception of the idea for the Interdisciplinary Symposium *Numbers and Number Sequences in Mathematics: An Interdisciplinary Approach* was prompted by the observation that the year 2025 corresponds to a square number year. That is, up to the year 2025, there have been a total of 45 perfect square years. The difference in years between two consecutive square years is derived as the difference between two successive squares of natural numbers in the sequence and is given by the formula: $(n+1)^2-n^2=2n+1$. For example, $1936-1849=44^2-43^2= (44-43) \cdot (44+43) =87=86+1=2\cdot 43+1$.

My first presentation, entitled *Representations of Numbers: Cognitive Approaches, Digital Tools and Geometric Figures*, combines cognitive theory with the digital use of sequences of numbers, promoting active visual representations that facilitate the understanding of abstract mathematical concepts. During my presentation, I discussed two numbers whose history is relatively less well known: *the number 0, the concept of the "Tetractys"* and *the figurate numbers.* In particular, regarding 0, it is often noted that one of the frequently posed questions is (O'Connor, & Robertson, 2000): "*Who discovered zero?*" […] "Which genius invented it? Historical evidence suggests that zero appears in a rather "shadowy" manner, as if mathematicians were gradually approaching its discovery without initially recognizing its fundamental significance, even after encountering it. The first point that should be made about zero is that it has two distinct uses, both highly important but conceptually different. One use is to indicate an empty positional value within a numeral system […]..[website 19].

The symposium was warmly received by all invited speakers, who actively participated and contributed high-quality presentations. The most demanding aspect of the process was encouraging participants to engage in and contribute to this collective endeavour, which aimed to achieve a meaningful and positive educational impact. Recognising the importance of effective communication, I cultivated a purposeful and constructive communicative approach with both university professors and my colleagues. This strategy not only facilitated collaboration but also strengthened mutual trust and commitment. Indeed, the university professors with whom I collaborated during these events expressed their appreciation for this approach, acknowledging its positive contribution to the success of our activities. Concerning figurate numbers, Kalavasis (2018) contends that their representational constructions developed through the practical and intellectual tool of the gnomon. (p. 16). He further notes that: "*The role of representations and symbolic languages, while being crucial in mathematics, may also become an obstacle in students' interdisciplinary learning pathways within the constraints of the everyday* *school timetable, due to their differentiated uses across various disciplines. Thus, the widely studied didactical transposition is effectively enriched by the notion of praxeological transposition"* (p. 9). In Figure 23, the construction of the first five successive square numbers is illustrated through *gnomon representations*, highlighting the additive structure underlying square numbers.

The term *gnomon* [Ancient Greek, https://en.wikipedia.org/wiki/Gnomon] has two distinct meanings that are relevant in mathematical contexts. (1) In school geometry, a gnomon refers to a geometric instrument in the form of a right-angled triangle (commonly 90°–45°–45° or 90°–60°–30°), used for constructing right angles, drawing perpendicular and parallel lines, and supporting geometric constructions. In mathematics education, it is considered an essential tool for fostering students' geometric reasoning and for linking hands-on activity with formal geometric concepts. (2) In the historical and mathematical sense of ancient Greek mathematics, a gnomon refers to a letter in the form of an inverted L that is added to a square or rectangle in order to generate successive square numbers.

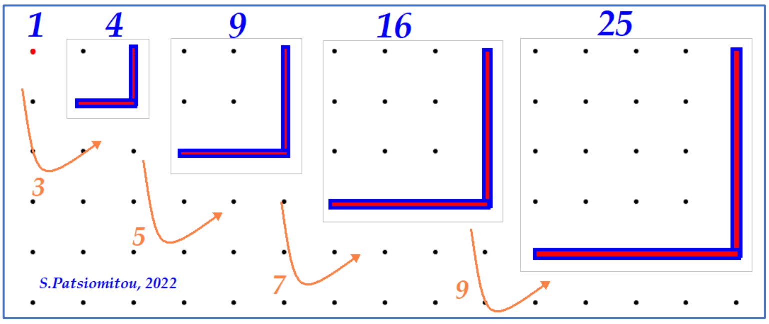


Figure 23: The first five square numbers illustrated through gnomon representations

This notion is closely related to the representation of square numbers as sums of consecutive odd numbers and highlights a structural interpretation of numerical patterns.

ναὶ μὰ τὸν ἁμετέρᾳ ψυχᾷ παραδόντα τετρακτύν, παγὰν ἀενάου φύσεως

Tetractys is (Burton, 2011, p. 91): "the *symbol on which the members of the Pythagorean community swore […] which was supposed to stand for the four elements: fire, water, air, and earth. The tetractys was represented geometrically by an equilateral triangle made up of 10 dots, and arithmetically by the number 1 +2 + 3 + 4 =10"*. The first four numbers symbolize the harmony of the spheres and the Cosmos (Universe). A prayer of the Pythagoreans shows the importance of the Tetractys [https://en.wikipedia.org/wiki/Tetractys], meaning "*Bless us, divine number, thou who generated gods and men! O holy, holy Tetractys, thou that containest the root and source of the eternally flowing creation!*

My second presentation, *Dynamic Sequences of Infinite Terms: numbers pi and fi (π and φ)*, (Patsiomitou, 2006g, 2008e, 2019b) adopts a dynamic and experimental approach to well-known mathematical concepts, enhancing the interest of both students and teachers, as it connects mathematical ideas with artistic applications. Overall, the presentations presented at the symposium support teachers in enriching their instructional materials through the adoption of pedagogical approaches that integrate the history of mathematics, thereby improving the quality of Mathematics education in secondary schools.

According to Fried (2007, p. 205): *many reasons have been given for introducing history of mathematics into mathematics education (see, for example, Fauvel, 1991) and many ways have been proposed for doing it. The former includes humanizing mathematics, making mathematics more engaging and approachable for students, providing insights into mathematical problems, techniques, and concepts.*

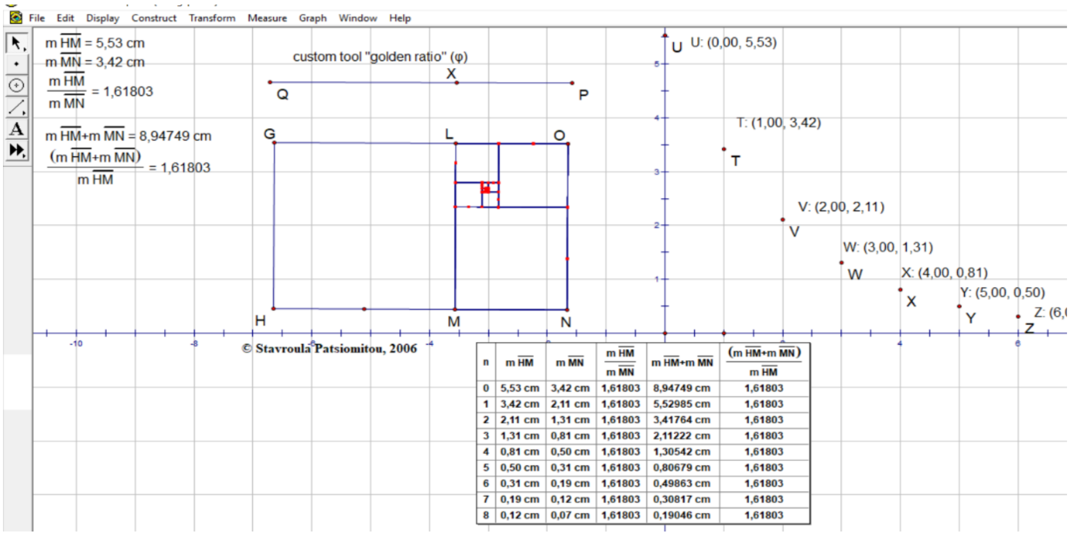


Figure 24: Dynamic linking of the tabulated measurements of the golden rectangle with the plotted points (Patsiomitou, 2006g, 2008e, 2019b)

XVII. One-day Seminar / Workshop (O.R.N.:18699/18 June 2025), which I designed, organized and coordinated, with T. S.: *Geometry: Teaching and Didactics in the Plane and in Space.* This seminar was conducted on Thursday, 26 June 2025, at the Event Hall of the 3rd Directorate of Secondary Education. My presentations and Hands-on Workshop were entitled: *Geometric Constructions in Plane and Space: Static, Digital, Dynamic, and AI-supported.*

In my presentation, I focused on a didactical approach that employs a range of digital, dynamic, and AI-based tools in order to support the understanding of geometric figures and sub-structures within figures, both in the plane and in three-dimensional space. My main objective was to explore and share practices and experiences that would make Geometry more accessible, engaging, and experiential for our students.

In the Pythagorean Tree diagram (Patsiomitou, 2009c, p.168, Patsiomitou, 2022a, p.371), my objective was to emphasize the sequential stages of the construction by consistently using the same colour for the corresponding squares, thereby making the iterative process visually explicit. Additionally, it is observed that two right-angled triangles with side length x form a square whose area is equal to half of the square with side length a. This relationship is also reflected in the sequence of terms generated directly within the software, where the underlying pattern becomes visually and dynamically apparent. Anderson (2009, p.41) states that we organize objects into units according to the *gestalt principles* of organization, after the Gestalt psychologists proposed them (e.g., Wertheimer, 1922, 1923). The use of successive identical colours plays a crucial role in supporting students' perceptual organisation of the construction, as it facilitates the tracking of structural correspondences across iterative steps. In this sense, it can be interpreted in terms of Gestalt psychology, according to which visual perception tends to organize elements into coherent wholes based on principles such as similarity and visual grouping. Thus, colour operates as a perceptual cue that enhances the recognition of mathematical structure and regularity. Moreover, the multi-layered approach to Geometry—through static, digital, and dynamic tools—promotes a holistic understanding of the subject matter. The implementation of artificial intelligence (AI) to address problem-solving in an educational setting, particularly in Geometry which is frequently regarded as an abstract subject, is both groundbreaking and supported by research. It suggests a methodological reorientation grounded in emerging technological capabilities. From a pedagogical perspective, the experiential workshop can create opportunities for active learning among teachers by providing access to alternative instructional tools. The alternation of media (tangible, digital, and dynamic) enables a multimodal approach to mathematical concepts (i.e. language, mathematical symbolism and images), thus improving understanding for learners with both theoretical and practical orientations (Gardner, 1983). Through the participation and active collaboration of all teachers, I believe that we can create a productive space for discussion, innovation, and collective action, which will support our classroom practice and contribute to the enhancement of the overall learning experience.

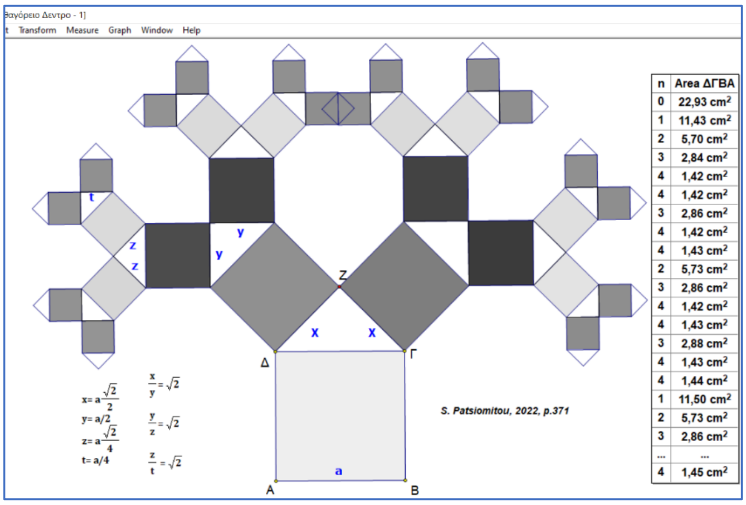


Figure 25: The Pythagorean Tree stucture
(Patsiomitou, 2009c, p.168, Patsiomitou, 2022a, p.371)

XVIII. One-day Seminar/ Workshop (O.R.N.: 21984/27 August 2025), which I designed, organized and coordinated, with T. S.: *Bridges between Mathematics, Art, Technology and*

Artificial Intelligence: Challenges and Innovations in Lower and Upper Secondary Education. This seminar was conducted on Wednesday, 3 September 2025, 10:00–13:30, at the Art Secondary School of Peristeri. My presentation was entitled: *Bridges between Mathematics, Art, Technology and Artificial Intelligence: Challenges and Innovations in Lower and Upper Secondary Education.*

The main objective of the workshop was to propose practices that foster creativity, active participation, and interdisciplinary connections between Mathematics and other fields of knowledge, building bridges with Art, Technology, and Artificial Intelligence. Furthermore, it sought to highlight teaching approaches that encourage students to develop a positive attitude towards Mathematics through enriched learning environments and innovative technologies. Recently, (28-2-2026) I discussed in the "*Synapses Academy Conference 2026*" the design, development, and real-world implementation of a Fractal-based Dynamic Program (FDP), which synthesizes findings and design principles derived from the author's prior empirical research with newly generated evidence (see also, Patsiomitou, 2026). The Fractal-based Dynamic Program is proposed as a flexible informal curriculum framework grounded in the principles of transformation geometry and fractals, and designed to support interdisciplinary, sustainability-oriented school education.

Based on conclusions drawn from a previous study (Patsiomitou, 2016c), I support: (a) the alignment of learning activities with children's biological maturation, in order to foster emotional, cognitive, and psychomotor development; (b) the promotion of children's agency through self-selection processes, enabling autonomy and personalized learning pathways; (c) the collaborative learning supported by timely pedagogical mediation, positioning educators as facilitators and designers of learning experiences; (d) the strategic planning of activities and role-playing scenarios that integrate environmental, mathematical, and artistic education within authentic contexts; and (e) the mediation through digital, dynamic, and AI-based tools that connect abstract concepts with embodied, visual, and experiential understanding.

> *"The implementation of the FDP demonstrates how fractals can function as a powerful interdisciplinary teaching context that connects mathematics and art while enhancing students' engagement and enjoyment of learning."* (Patsiomitou, 2026, p. 285)

The discussion during the seminar highlighted the high level of professionalism and dedication of the participating teachers, who continuously seek innovative practices for the benefit of their students. The atmosphere was highly constructive, and the discussion addressed key classroom concerns such as formative and summative assessments, teaching methodology, classroom activities, and original instructional ideas. I shared experiences and proposals derived from my personal endeavors related to fractals (including the Sierpiński triangle and the Baravelle spiral), (such as the Sierpiński triangle and the Baravelle spiral), using both hands-on and digital materials in order to explain how these structures can facilitate the comprehension of sequences and limits (Patsiomitou, 2005a, 2007, 2026).

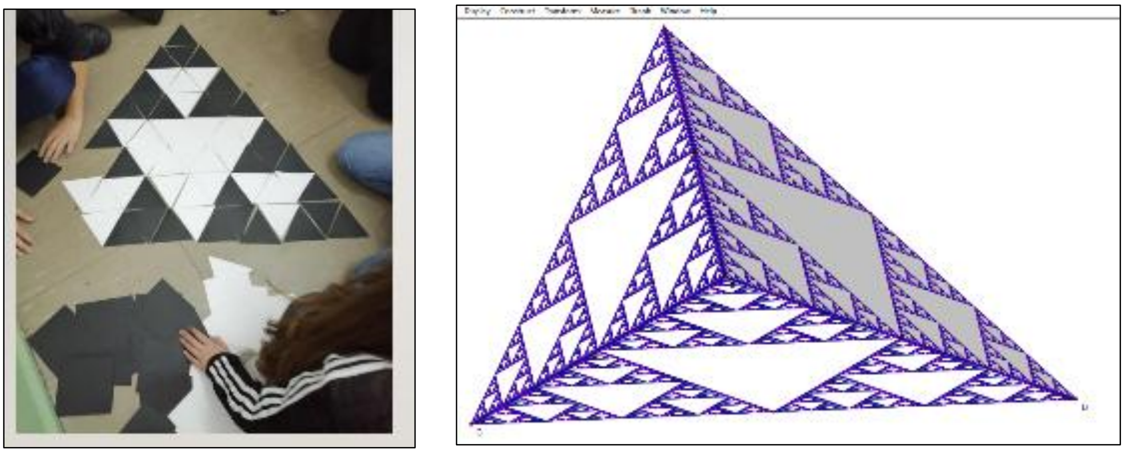


Figure 26 a, b: Construction of fractal structures in two-dimensional and three-dimensional spaces (Patsiomitou, 2005a)

By extending / broadening the geometric perspective from the 2D plane to 3D [three-dimensional] space, teachers are provided with innovative strategies to explore and emphasise the dimensions and interrelationships of geometric objects. In this way, the initiative contributes to the improvement of the quality of the learning process.

XIX. Online One-day Seminar (O.R.N.: 30556/6 November 2025), which I designed, organized and coordinated, with T. S.: *Didactic Use of Digital Environments for Mathematics in Lower and Upper Secondary Education.* This seminar was conducted on Wednesday, 12 November 2025, 16:00–19:00, via my Webex platform. My presentation was entitled: *Geometric Constructions in Linked Pages in Geometer's Sketchpad – Mathematics with Dynamic and Static Tools – Translations of the National Curriculum Items in GeoGebra.*

During my presentation, I demonstrated the following experiential activities: (a) the construction of an equilateral triangle and its connection to Euclid's Elements [Book I, Proposition 1] (Fitzpatrick, 2007); (b) the construction of the first two stages of the Sierpiński triangle, including the development of a *custom tool* for iterating the construction process; (c) the construction of a sequence through an iterative procedure, linked to the concepts of geometric progression, sequences, limits, and infinitesimals; (d) the application of this process in classroom practice (Figure 26 a, b).

A script /custom tool combines in a concrete and sequential order the steps that have been used to accomplish the construction. For example, if we construct a square, we can save the concrete construction in a custom tool which can repeat the construction in the concrete way used by the creator of the custom tool, meaning that is processes the objects in the same sequence. The dragging of the custom tool constructed on screen follows the rules that refer to the primitives and commands incorporated into the custom tool (i.e. if we have measured angles or segments, or calculated a ratio, during our construction of the tool, then the specific measurements and calculations are reiterated each time we utilize the custom tool). If we drag the tool, the measures follow the increasing or decreasing of the length of the segments and angles (e.g., Patsiomitou, 2005a, p.83). By constructing a custom tool, we can help students to extend the capacity of their working memory, since the knowledge the student must retain is reduced. As a result of the construction and application of a custom tool the direct perception of the user is attained with regards to the steps in the development of the construction pertaining to (see) (e.g., Patsiomitou, 2007a, 2014, 2018a, b, 2019a): *1) the repetitions in the measurements or calculations of the areas of initial shapes 2) the developmental way of the construction of the figure and 3) its orientation towards the sequential steps of the construction on the screen's diagram or in successive pages of the same file*.

Furthermore, I created video screen recordings to demonstrate the construction of the activities in The Geometer's Sketchpad, in order to illustrate both the procedural and conceptual aspects of the construction process. Tall *et al.* (2001) argue that the development of abstract concepts "*begins from the ability to perceive things, to act on them and to reflect upon these actions to build theories*" (p.81) (Italics by the authors).

XX. Participation as presenter in Professional Development Seminar (O.R.N.: 27123/9 October 2025, Approval Number: 23923/28 October 2025). The seminar was organised by the Education Advisors of the Directorate of Secondary and Primary Education. T. S.: *Classroom Problem Management*. My presentation, entitled *Interdisciplinary Mathematics and Happy Schools: From Solving Exercises to Solving Relations*, was delivered on Thursday, 13 November 2025, and Tuesday, 18 November 2025.

The "Happy Schools" framework (UNESCO, 2016) is implemented across all levels of education, from kindergarten to upper secondary school, and has been designed to integrate well-being and happiness as an integral part of the educational process. This framework adopts a holistic approach to learning and well-being, recognising happiness as both a means and an end of quality education. Its implementation includes, in Greece, the development of specific educational materials based on similar principles, which have been approved by the Institute of Educational Policy (IEP) and the Ministry of Education for nationwide use across all educational levels. UNESCO (2024) promotes the "Happy Schools" philosophy globally, emphasising the need for an environment that fosters joy and the overall development of students, teachers, and the wider school community.

In my presentation, I referred to the four core pillars of the *Happy Schools* framework (UNESCO, 2024)—*People, Process, Place,* and *Principles and purpose*—as well as its twelve high-level criteria guiding the transformation of learning:

- The *People* pillar focuses on interpersonal relationships, well-being, and the attitudes of key school actors, including students, teachers, parents, and the wider community, emphasizing supportive, respectful, and collaborative relationships that foster a positive pedagogical climate (UNESCO, 2024, p. 41).
- The *Process* pillar concerns the transformation of curricula, pedagogies, and assessment practices to promote joyful and meaningful learning, including

collaborative approaches, holistic assessment, student participation, and the integration of creativity, play, and extracurricular activities (p. 53).

- The *Place* pillar addresses the redesign of physical and digital environments to ensure safe, inclusive, accessible, and aesthetically supportive school spaces that function as community hubs (p. 62).
- Finally, the *Principles and purpose* pillar refers to the foundational values that underpin school communities and enable the other pillars, emphasizing ethics, social awareness, responsibility, and personal development (p. 70).

Subsequently, I related the "Happy Schools" framework to several learning theories and analysed my reflections, a topic which is beyond the scope of the present article. In brief, I particularly focused on Vygotsky's theory, which emphasises that learning is a collaborative process. Within the Happy Schools framework, this implies that students are not only individually happy but also engage in group activities where they learn from one another. Furthermore, the involvement of parents and the wider community in school life is of central importance.

XXI. One-day Conference (O.R.N.: 29916/3 November 2025, which I designed, organized and coordinated, with T. S.: *Euclidean and Non-Euclidean Geometries: Manuscripts and Digital Creations.* This seminar was conducted on Friday, 6 December 2025, at the City Hall Auditorium of Peristeri. My presentation was entitled: *Euclidean and Non-Euclidean Geometries Using Dynamic Digital Environments.*

The symposium was attended by distinguished university professors and their doctoral candidates, who participated as speakers. Among them was my University Professor in Euclidean Geometry, from whom I acquired my foundational knowledge through the study of Euclid's Elements.

.

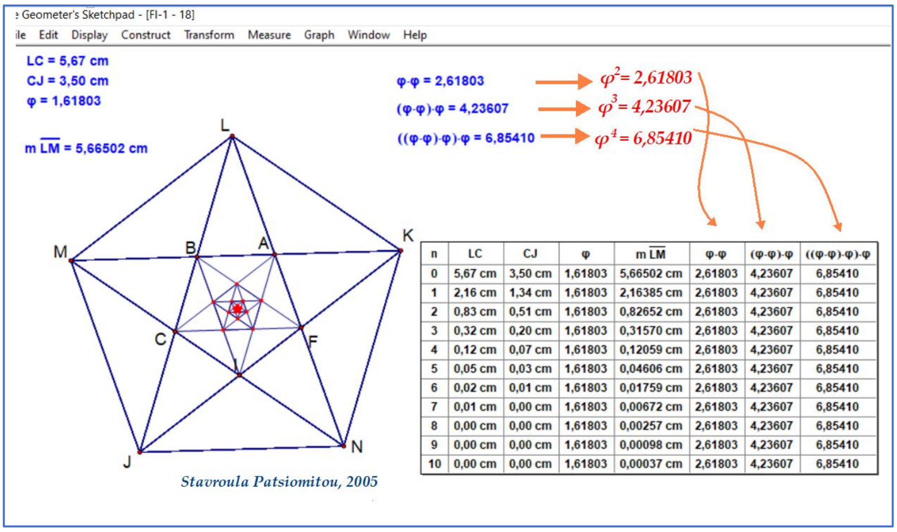


Figure 27: Creating tables for the generation of the number φ (phi) and its powers φ^2, φ^3, φ^4 (Patsiomitou, 2005a)

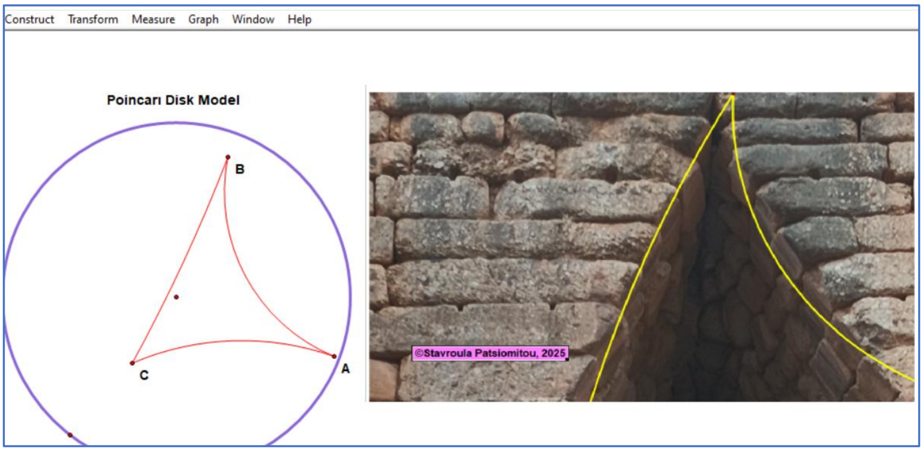


Figure 28: Comparison of the triangles: the hyperbolic triangle and the relieving triangle at Mycenae (Patsiomitou, 2025c, p. 67)

Without his guidance, I would not have decided to engage systematically with their study. In my presentation, I referred to my own work concerning the translation of propositions from Euclid's *Elements* into dynamic geometry software, as well as to a recent article of mine on the so-called "relieving triangle" at Mycenae. For example, I referred to the construction of the pentagon and the creation of iterations within its interior, as well as to the connection between the concept of the pentagon and the golden ratio φ, $φ^2$, etc. (Figure 27). Figure 28 (Patsiomitou, 2025c, p. 67) depicts an illustration of the relieving triangle, in comparison to the hyperbolic triangle generated in the Geometer's Sketchpad Poincare Disk. One side appears curved, creating the visual impression of a triangle within a hyperbolic geometric space. As I argue in my study (Patsiomitou, 2025c, p. 67), *although the Mycenaeans were unaware of Riemann's theoretical formulations, their empirical knowledge and careful observation enabled them to construct a spatial understanding that intuitively engages with concepts of non-Euclidean geometry. Further research, including precise measurements, is necessary to support a more formal evaluation of this significant monument of antiquity*. It is important to note, however, that the dome of the so-called Tomb of Clytemnestra has not collapsed, in contrast to *the tholos above the Lion Gate at Mycenae, since the triangle formed there is spherical in nature.*

In this workshop, we contributed to the dialogue on geometric concepts that have shaped, and continue to shape, our understanding of mathematical reality and the cosmos. The event featured lectures specialising in analysis and the history of mathematics, shedding light on aspects of Euclidean geometry as well as non-Euclidean theories that fundamentally transformed the foundations of classical mathematical thought.

The event aimed not only to highlight the richness of the Euclidean tradition, but also to illuminate the ways in which Geometry evolves, acquiring new forms as well as new pedagogical and research perspectives. Furthermore, the purpose of the meeting was to strengthen the dialogue between university knowledge and school practice, emphasising the enduring value of Geometry and its significance in the development of mathematical and scientific thinking.

XXII. One-day Symposium -1st Pan-Attica Conference of Mathematics Education Advisors (O.R.N.: 35009/4 December 2025), which I designed, organized and coordinated, with T. S.: *Pedagogical and Technological dimensions in the teaching of Mathematics.* This seminar was conducted on Saturday, 17 January 2026, 10:00–15:00, at the City Hall Auditorium of Peristeri. My presentation was entitled: *Linking Visual Active Representations in a DGS Environment: the birth of an idea and its application.*

The 1st Pan-Attica Conference of Mathematics Education Advisors was successfully conducted at the Amphitheatre of the Municipality of Peristeri. The conference was held in a hybrid format, enabling participation both in person and via the Webex digital platform, with many teachers and educators from school units across the Attica region taking part. I undertook the full organisational responsibility for the Symposium, in collaboration with all Mathematics Education Advisors of Attica. I also assumed full administrative and organisational responsibility for the invitations and the coordination of the conference. The event constituted a fruitful opportunity for the exchange of ideas, scholarly reflection, and professional empowerment. The presentations made a substantial contribution to enriching teaching practice and supporting the demanding and highly significant work carried out daily in the classroom. Particularly appreciated were the warm words and messages of recognition received from participating colleagues, teachers of Mathematics across all Attica schools. This acknowledgement confirms the value of collective effort and encourages us to continue our scientific and pedagogical mission with consistency and dedication. It represents a source of both emotional reward and strong motivation to pursue our work with the same sense of responsibility, collaboration, and academic integrity. With the conviction that such collective actions meaningfully strengthen the educational community, we are committed to the continuation and repetition of similar scientific processes in the future, aiming at the ongoing support and enhancement of Mathematics education.

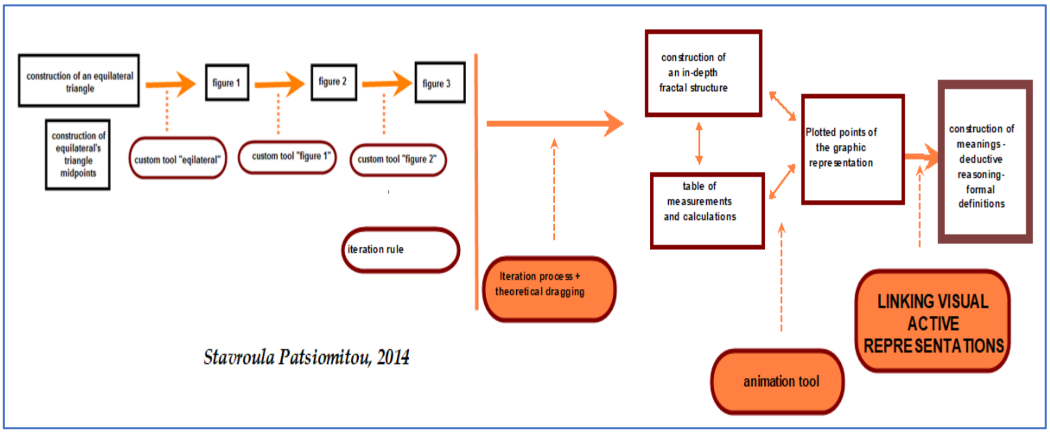


Figure 29: A Pseudo Toulmin model explaining the birth of the meaning 'Linking Visual Active Representations' (Patsiomitou, 2014, p. 30)

3.3 The Evaluation Agreement as a tool for individualized teacher professional development

As mentioned in the first reported seminar in Section 3.2, the *Evaluation Agreement* constitutes a formal document co-signed and co-completed by the teacher under evaluation and the Education Advisor acting as evaluator. In summary, the Evaluation Agreement aims at the systematic recording and analysis of the teaching process. The evaluation is based on data related to instruction and is organized into three main thematic sections:

(a) *The first section* examines the instructional approach, including teaching time, intended learning objectives (cognitive, behavioral, and psychomotor), prerequisite knowledge, as well as teaching methods and techniques, with particular emphasis on lesson structure, the use of pedagogical tools, and processes of consolidation, assessment, and reflection.

(b) *The second section* focuses on the characteristics of the student population and the learning community, addressing factors such as student heterogeneity, level of mathematical understanding, classroom pedagogical climate, and the presence of students receiving additional educational support.

(c) *The third section* addresses interdisciplinary and cultural dimensions of teaching, including connections between the subject matter, the History of Mathematics, and other scientific fields.

Particular emphasis is placed on *the instructional phases* (Patsiomitou, 2026, p. 288) recorded in the Evaluation Agreement, which may function as a tool for fostering individualized professional learning experiences for teachers under evaluation. The structured description of these phases constitutes a critical component in evaluating the teaching process, as it enables a detailed representation of pedagogical practice and an evidence-based assessment of its effectiveness. More specifically, the systematic analysis of the distinct phases of a lesson contributes to the formation of a clear and comprehensive instructional organization and learning progression. The sequential structuring of teaching—from the introduction and development of the instructional unit to consolidation and assessment—ensures lesson coherence and the effective achievement of intended learning outcomes. Furthermore, the detailed description of these phases enables the evaluation of teaching methods and pedagogical interaction, including the use of digital tools and artificial intelligence by both teachers and students. In addition, recording the duration of each phase provides valuable data on time management, a critical factor influencing learning outcomes and the optimization of teaching practice. Finally, the systematic description of instructional phases supports comparative analysis across different teaching approaches and enhances reflective practice. Furthermore, the analysis of instructional strategies, combined with the degree of active student participation, allows for assessing the extent to which teaching approaches respond to the needs and characteristics of the student population. In this way, the Evaluation Agreement emerges as an essential tool for both continuous teacher professional development and the overall improvement of the educational process.

On the other hand, I considered it important to investigate the relationship between teachers' gender, if they work at upper secondary school or lower secondary school, and the selection of the teaching unit as subject to evaluation, a process that is currently ongoing. Despite the growing body of literature concerning gender and educational evaluation, limited research has examined the relationship between the gender of evaluated teachers and the evaluation process itself, particularly within Secondary Education and the field of Mathematics Education. My study seeks to contribute to this field. The study is grounded in the theoretical frameworks of Mathematics Education and the Psychology of Mathematics Education and aims to explore whether gender-related patterns may emerge within authentic school evaluation contexts.

The methodological approach of the study is based on the systematic collection and qualitative analysis of data gathered through digital worksheets and electronic forms (Excel and Google Forms). The analysis focuses on teachers' gender, evaluation sequence, selected teaching units, and grade levels taught, while also examining potential differentiations in instructional practices, lesson planning, and assessment approaches. Through the comparative interpretation of the findings in relation to existing international literature, the study I hope to contribute to the broader discussion concerning equity in educational evaluation practices. Particular emphasis will be placed on potential variations associated with the academic and professional characteristics of teachers in relation to gender.

The issue of gender in educational evaluation has attracted increasing research interest in recent years, particularly in relation to potential biases, differentiated teaching practices, and teachers' perceptions within Mathematics Education (e.g., Copur-Gencturk *et al.*, 2020; Carlana, 2019). International studies have highlighted that gender may influence both evaluative judgments and instructional approaches, often through implicit stereotypes and socially constructed expectations regarding mathematical ability and professional competence (e.g., Felkey & Batz-Barbarich, 2021; Lindner *et al.*, 2022).

4. Discussion: Proposal for a future seminar on geometry in museum contexts

Over the course of these three years, in my role as a Mathematics Education Advisor, my goal was to build—true to the spirit of the title—a sustainable community of learning and collaboration: a "family" of Mathematics teachers and educators working collectively through structured professional development activities to improve the quality of education. The title of my latest book, self-published, *Building Geometry – Building Relationships Dynamically* (Patsiomitou, 2025d), reflects a dual approach to mathematics education: *on the one hand*, the construction of mathematical concepts and knowledge through active engagement, and *on the other*, the cultivation of relationships—both among mathematical objects and among the participants in the educational process. I hope that this work—available in printed form at the National Library—will offer a small yet meaningful step in this direction, serving as both a practical tool and a point of reference for future generations of Education Advisors who seek to serve education with purpose and to redefine the concept of quality within our educational system.

Furthermore, building upon the interdisciplinary connections between mathematics, history, and cultural heritage, a future seminar is proposed for mathematics teachers, aiming to explore the pedagogical potential of museum environments as contexts for teaching geometric concepts. The seminar will focus on the integration of architectural and artistic elements from selected international museums, where geometry is embedded in both design and structure. Within the framework of enhancing the teaching of local history, it is pedagogically appropriate to incorporate the use of museums as experiential learning environments. Particular emphasis may be placed on museums of traditional costume, which provide rich visual material. Through such exhibits, students are given the opportunity to approach geometric concepts and shapes in a more concrete and intuitive manner, addressing difficulties often encountered in the conventional teaching of mathematics.

From a personal perspective, I would like to share my impressions from a visit to the *Museum of Greek Folk Art*, (http://www.melt.gr) which I conducted with a group of students on 21 November 2011 (Patsiomitou, 2012c, pp. 30–41). The exhibits included traditional folk costumes from various regions of Greece (e.g., mountainous Epirus, Thessaly, and the Aegean islands). The students were particularly impressed by the embroidered women's costumes. Some of these featured recurring geometric motifs that were interpreted as expressions of a woman's inner desire for good fortune, happiness, and longevity. These geometric motifs can be interpreted as expressions of cultural symbolism and meaning-making processes, in line with perspectives from ethnomathematics (D'Ambrosio, 1985) and semiotic approaches to visual representation (Yeh, & Nason, 2004; Bakker, & Hoffmann, 2005). In the research process that followed (Patsiomitou, 2014, pp. 11–14), the objective was to investigate how students perceived the figures and their transformations, and whether they were able to reproduce them on paper. I found that students experienced difficulties in perceiving the geometrical figures, and only after instruction using dynamic geometry software were they able, at a metacognitive stage, to construct geometric representations simulating the forms present in folk art garments.

Furthermore, it is recommended to broaden this approach through the internationalization of the educational practice. Reference to prominent museums abroad, such as the Solomon R. Guggenheim Museum (https://www.guggenheim.org/), may serve as a bridge connecting local cultural heritage with global contexts. Indicatively, architectural structures and geometric elements found in such institutions — including the use of polygonal forms, such as the dodecagon mentioned in the 8th workshop— can serve as a starting point for an interdisciplinary approach, linking history, art, and mathematics. Furthermore, this museum provides an exemplary case of spiral geometry, offering opportunities to engage with concepts such as *curves in motion* or *spirals in motion* (e.g., Patsiomitou, 2005b). Similarly, the Louvre Museum (https://www.louvre.fr/en), through its glass pyramid, presents a clear application of 3D triangular form associated with a pyramidal architecture. In addition, the Grand Egyptian Museum (https://gem.eg/) offers a meaningful context for examining fundamental geometric structures, that may be extended—on a comparative and didactic level—to introduce fractal structures such as the Sierpiński triangle, facilitating a transition from elementary geometric forms to recursive and fractal thinking. The proposed seminar aims to: (a) enhance teachers' and students' understanding of geometry as a lived and visual experience, (b) promote interdisciplinary teaching practices, (c) support the development of innovative classroom activities based on real-world contexts, and (d) encourage the internationalization of teaching approaches through the use of globally recognized cultural institutions. Ultimately, this initiative seeks to enrich mathematics education by bridging abstract concepts with tangible cultural representations, thereby fostering deeper student engagement and conceptual understanding.

On the other hand, the question posed in the title is intentionally left open, as it extends beyond the scope of this study. It is instead offered as a point of reflection for the next generation of mathematics education advisors, who seek to critically and effectively define and enact their professional role. Future research may further explore how mathematics education advisors construct and negotiate their professional identity across the continuum of consultancy, leadership, and research; how these roles are enacted in the design and implementation of seminars and workshops; and how such practices influence teachers' professional learning and pedagogical development.

Conflict of Interest Statement

The author declares no conflicts of interest.

About the Author

Stavroula Patsiomitou is an Education Advisor for Mathematics in the C Area of the Athens region. She holds a Ph.D. in the Didactics and Psychology of Mathematics, with a focus on computational environments, from the University of Ioannina. She has served as a University Fellow and Visiting Lecturer in the Department of Mathematics and Applied Mathematics at the University of Crete. She has also collaborated for many years with the National and Kapodistrian University of Athens (NKUA), supervising preservice teachers of Mathematics during their practicum. Her research work includes numerous monographs [https://www.academia.edu/44314999/] and peer-reviewed articles published both in Greece and internationally. Since 2009, she has been a reviewer for the International Conference on the Psychology of Mathematics Education (PME). For further information, see: https://www.academia.edu/4608309/

References

Ainsworth, S. (2006). Deft: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16,* 183–198. https://doi.org/10.1016/j.learninstruc.2006.03.001

Anastasiades, P. (2023). Artificial Intelligence in Education: Pilot study to explore teachers' views in Greece. *Proceedings of the World Research Society International Conference*, October 21 – 22, 2023 Zarqa, Jordan.

Anderson, J. R. (2009). *Cognitive Psychology and Its Implications*. New York: Worth Publishers. Retrieved from https://www.academia.edu/17613920/Cognitive_Psychology_and_Its_Implications_and_Scientific_American_Explores_the_Hidden_Mind

Anderson, L., & Krathwohl, D. (Eds.). (2001). *A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives—Complete edition*. New York, NY: Addison Wesley Longman. Retrieved from https://books.google.ro/books/about/A_Taxonomy_for_Learning_Teaching_and_Ass.html?id=EMQlAQAAIAAJ&redir_esc=y

Ausubel, D.P. (1968). *Educational psychology: a cognitive view.* Holt, Rinehart and Winston: New York. Retrieved from https://ia800107.us.archive.org/21/items/in.ernet.dli.2015.112045/2015.112045.Education-Psychology-A-Cognitive-View.pdf

Bakker, A., & Hoffmann, M. H. G. (2005). Diagrammatic reasoning as the basis for developing concepts: a semiotic analysis of students' learning about statistical distribution. *Educational Studies in Mathematics*, 60, 333–358. Retrieved from https://www.jstor.org/stable/25047200

Bandura, A. (1977). Self-efficacy: toward a unifying theory of behavioral change. *Psychological review*, 84(2), 191. https://doi.org/10.1037/0033-295X.84.2.191

Battista, M. T. (2007). The development of geometric and spatial thinking. In Lester, F. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 843-908). NCTM. Reston, VA: National Council of Teachers of Mathematics. Retrieved from https://www.researchgate.net/publication/313572539_The_development_of_geometrical_and_spatial_thinking

Biehler, R., Scholz, R.W., Sträßer, R. and Winkelmann, B. (eds.). (1994). *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers, Dordrecht. Retrieved from https://www.researchgate.net/publication/257927023_Didactics_of_Mathematics_as_a_Scientific_Discipline

Bloom, B. S., Engelhart, M. D., Furst, E. J., Hill, W. H., & Krathwohl, D. R. (Eds.). (1956). *Taxonomy of educational objectivities: The classification of educational goals. Handbook I: Cognitive domain.* New York: David McKay. Retrieved from https://books.google.ro/books/about/Taxonomy_of_Educational_Objectives.html?id=hos6AAAAIAAJ&redir_esc=y

Bransford, J.D., A. L. Brown, and R.R. Cocking, eds. (2000). *How people learn: brain, mind, experience, and school.* (pp. 206–230). Washington, D.C.: National Academy Press. Retrieved from https://books.google.ro/books?id=TJMWngEACAAJ&printsec=copyright&redir_esc=y#v=onepage&q&f=false

Brown, A.L. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F.E. Weinert & R.H. Kluwe (Eds.), *Meta-cognition*, *Motivation and Understanding* (pp. 65–116). Hillsdale, NJ: Lawrence Erlbaum. Retrieved from https://www.taylorfrancis.com/chapters/edit/10.4324/9781003758167-5/metacognition-executive-control-self-regulation-mysterious-mechanisms-ann-brown

Buchbinder, O. (2013). *How novice teachers recontextualize the teaching of mathematics via reasoning and proving: A dual case study*. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the 13th Conference of the European Society for Research in Mathematics Education (CERME13)* (pp. 89–98). European Society for Research in Mathematics Education. Retrieved from https://hal.science/hal-04408285/document

Burton, D. M. (2011). *The History of Mathematics, An Introduction* (7th ed.)., McGraw-Hill. Retrieved from https://jontalle.web.engr.illinois.edu/uploads/298/HistoryMath-Burton.85.pdf

Carlana, M. (2019). Implicit stereotypes: Evidence from teachers' gender bias. *The Quarterly Journal of Economics*, 134(3), 1163–1224. https://doi.org/10.1093/qje/qjz008

Chevallard, Y. (1989). Le passage de l'arithmétique à l'algèbre dans l'enseignement des mathématiques au collège. *Petit x*, 19, 43-72. Retrieved from https://revistas.pucsp.br/emp/article/download/61803/42158/195470

Coburn, C. E. (2004). Beyond decoupling: Rethinking the relationship between the institutional environment and the classroom. *Sociology of Education*. *77*, 211–244. Retrieved from https://doi.org/10.1177/003804070407700302

Colgan, L. (2014). *Making math children will love: Building positive mathitudes to improve student achievement in mathematics.* (What Works? Research into Practice Research Monograph No. 56). Ontario Ministry of Education, Student Achievement

Division. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_Making Math.pdf

Copur-Gencturk, Y., Cimpian, J. R., Lubienski, S. T., & Thacker, I. (2020). Teachers' bias against the mathematical ability of female, Black, and Hispanic students. *Educational Researcher*, 49(1), 30–43. https://doi.org/10.3102/0013189X19890577

Cottrill, J. Dubinsky, E. Nichols, D., Schwingendorf, K., Thomas, K. & Vidakovic, D. (1996). Understanding the limit concept: Beginning with a co-ordinated process schema. *Journal of Mathematical Behavior*, 15, 167–192. Retrieved from https://www.math.kent.edu/~edd/Limit.pdf

Coxford, A. F., Usiskin Z. P. (1975). *Geometry: A Transformation Approach*, Laidlaw Brothers, Publishers. Retrieved from https://archive.org/details/geometrytransfor0000coxf

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44–38. Retrieved from https://www.jstor.org/stable/40247876

Davis, G., Tall, D., Thomas, M. (1997). What is the object of the encapsulation of a process? *Proceedings of MERGA*., Vol. 2, pp. 132–139, Rotarua, New Zealand. Retrieved from https://scholar.google.com/citations?view_op=view_citation&hl=en&user=IV1rmyEAAAAJ&citation_for_view=IV1rmyEAAAAJ:W7OEmFMy1HYC

De Corte, E., Verschaffel, L., & Greer, B. (2000). Connecting mathematics problem solving to the real world. *Proceedings of the International Conference on Mathe-matics Education into the 21st Century: Mathematics for living* (pp 66-73). Am-man, Jordan: The National Center for Human Resource Development. Retrieved from https://www.researchgate.net/publication/241145064_Connecting_mathematics_problem_solving_to_the_real_world

Dimino, J.A., Taylor, M., & Morris, J. (2015). *Professional learning communities facilitator's guide for the What Works Clearinghouse practice guide: Teaching academic content and literacy to English learners in elementary and middle grades* (REL 2015-105). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southwest.https://ies.ed.gov/rel-southwest/2025/01/professional-learning-communities-facilitators-guide

Dubinsky, E. & McDonald, M. (2001). APOS: A constructivist theory of learning. In D. Holton (Ed.) *The Teaching and Learning of Mathematics at University Level: An ICMI Study* (pp. 275–282). Dordrecht: Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47231-7_25

Dubinsky, E. (1988). On Helping Students Construct the Concept of Quantification. In *Proceedings of the 12th International Conference on the Psychology of Mathematics Education*. (Vol. 1, pp.255-262) Veszprém, Hungary. Retrieved from https://www.jstor.org/stable/40247924

Dubinsky, E. (1991a). Reflective Abstraction in Advanced Mathematical Thinking. In D. O. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 95-123). Dordrecht: Kluwer Academic Publishers. http://www.math.wisc.edu/~wilson/Courses/Math903/ReflectiveAbstraction.pdf

Dubinsky, E. (1991b). Constructive aspects of reflective abstraction in advanced mathematics. In L. P. Steffe (Ed.), *Epistemological foundations of mathematical experiences* (pp. 160–202). New York: Springer.

Duval, R. (1995). Geometrical Pictures: Kinds of representation and specific processings. In R. Sutherland and J. Mason (Eds), *Exploiting Mental Imagery with Computers in Mathematics Education.* (pp. 142-157). Berlin: Springer. https://doi.org/10.1007/978-3-642-57771-0_10

Duval, R. (2002). The cognitive analysis of problems of comprehension in the learning of mathematics. *Mediterranean Journal for Research in Mathematics Education, 1*(2), 1-16. Retrieved from https://doi.org/10.1007/s10649-006-0400-z

Edwards, L. (1998). Embodying mathematics and science: Microworls as representations. *The Journal of Mathematics Behavior,* 17(1), 53-78. https://doi.org/10.1016/S0732-3123(99)80061-3

Fauvel, J. (1991). ‘Using history in mathematics education’, *For the Learning of Mathematics* 11(2), 3–6. Retrieved from https://flm-journal.org/Articles/5B7A202B26495E83D7655D943808FF.pdf

Felkey, A. J., & Batz-Barbarich, C. (2021). Can women teach math (and be promoted)? A meta-analysis of gender differences across student evaluations of teaching. *AEA Papers and Proceedings*, 111, 184–189. https://doi.org/10.1257/pandp.20211125

Fitzpatrick, R. (2007). *Euclid's Elements of geometry*. Morrisville, NC: Lulu. Retrieved from https://farside.ph.utexas.edu/Books/Euclid/Elements.pdf

Fitzpatrick, R., & Morrison, E. J. (1971). Performance and product evaluation. In R. L. Thorndike (Ed.), *Educational measurement* (2nd ed., pp. 237–270). Washington, DC: American Council on Education. Retrieved from https://files.eric.ed.gov/fulltext/ED233088.pdf

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive–developmental inquiry. *American Psychologist,* 34 (10), 906–911. https://doi.org/10.1037/0003-066X.34.10.906

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Kluwer Acad. Publ., Dordrecht. Retrieved from https://books.google.ro/books/about/Revisiting_Mathematics_Education.html?id=gAXSBwAAQBAJ&redir_esc=y

Fried, M. N. (2007). Didactics and history of mathematics: Knowledge and self-knowledge. *Educational Studies in Mathematics, 66*(2), 203–223. https://doi.org/10.1007/s10649-006-9025-5

Fuson, K. C., Carroll, W. M., & Drueck, J. V. (2000). Achievement results for second and third graders using the *Standards*-based curriculum. *Everyday Mathematics*. *Journal for Research in Mathematics Education, 31*, 277-295. https://doi.org/10.2307/749808

Gamow, G. (1988). *One, two, three--infinity*. New York: Dover Publications. (Original work published 1947). Retrieved from https://books.google.ro/books/about/One_Two_Three_Infinity.html?id=tMQ6EQAAQBAJ&redir_esc=y

Gardner, H. (1983). *Frames of mind: The theory of multiple intelligences*. Basic Books. Retrieved from https://books.google.ro/books/about/Frames_of_Mind.html?id=ObgOAAAAQAAJ&redir_esc=y

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal*, 38(4), 915–945. Retrieved from https://www.researchgate.net/publication/237817648

Goldin, G., & Kaput J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.). *Theories of mathematical learning* (pp. 397-430). Hillsdale, NJ: Erlbaum. Retrieved from https://www.researchgate.net/publication/269407907_A_joint_perspective_on_the_idea_of_representation_in_learning_and_doing_mathematics

Govender, R. & De Villiers, M. (2004). A dynamic approach to quadrilateral definitions. *Pythagoras.* 58, pp.34-45. Retrieved from https://scholar.google.com/citations?view_op=view_citation&hl=en&user=XgHGqmwAAAAJ&citation_for_view=XgHGqmwAAAAJ:_FxGoFyzp5QC

Graumann, G. (2005). Investigating and ordering Quadrilaterals and their analogies in space-problem fields with various aspects. *Zentralblatt für Didaktik der Mathematik (ZDM),* 37(3), 190-198. https://doi.org/10.1007/s11858-005-0008-2

Gravemeijer, K. P. E. (1994). *Developing realistic mathematics education.* Utrecht, the Netherlands: CD-ß Press. Retrieved from https://www.fisme.science.uu.nl/publicaties/literatuur/1994_gravemeijer_dissertation_0_222.pdf

Gray, E. M. & Tall, D. O. (1994). Duality, Ambiguity and Flexibility: A Proceptual View of Simple Arithmetic. *Journal for Research in Mathematics Education*, 26(2),115–141. https://doi.org/10.2307/749505

Gray, E. M., & Tall D. O. (1991). Duality, Ambiguity and Flexibility in Successful Mathematical Thinking, In *Proceedings of the 13th International Conference of the Psychology of Mathematics Education* (Vol. 2, pp. 72–79). Assisi, Italy. Retrieved from https://www.researchgate.net/publication/245976490_Duality_Ambiguity_and_Flexibility_in_Successful_Mathematical_Thinking

Hershkowitz, R. (1990). Psychological aspects of learning geometry. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition*. Cambridge University Press. https://doi.org/10.1017/CBO9781139013499.006

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–100).

New York: Macmillan. Retrieved from https://psycnet.apa.org/record/1992-97586-004

Hofmann, J. E. (1929). Elementare Lösung einer Minimumsaufgabe. *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht, 60*, 22–23.

Hofmann, J. E. (1969). Über die geometrische Behandlung einer Fermatschen Extremwert-Aufgabe durch Italiener des 17. Jahrhunderts. *Sudhoffs Archiv, 53*(1), 86–99.

İpek, Z.H., Gözüm, A.İ.C., Papadakis, S., & Kallogiannakis, M. (2023). Educational Applications of the ChatGPT AI System: A Systematic Review Research. *Educational Process: International Journal*, 12(3): 26-55. Retrieved from https://www.edupij.com/index/arsiv/59/305/educational-applications-of-the-chatgpt-ai-system-a-systematic-review-research

Jackiw, N. (1991). *The Geometer's Sketchpad* [Computer Software]. Berkeley, CA: Key Curriculum Press.

Jaworski, B. (2003). Inquiry as a pervasive pedagogic process in mathematics education development. *Proceedings of the Third Conference of the European Society for Research in Mathematics Education*. Bellaria, Italy. http://www.dm.unipi.it/~didattica/CERME3

Kalavasis, F. (2018). Mathematics and the real world in a systemic perspective of the school. In B. Maj-Tatsis, K. Tatsis, & E. Swoboda (Eds.), *Mathematics in the real world*. Wydawnictwo Uniwersytetu Rzeszowskiego. Retrieved from https://www.researchgate.net/publication/334446575_MATHEMATICS_AND_THE_REAL_WORLD_IN_A_SYSTEMIC_PERSPECTIVE_OF_THE_SCHOOL

Koehler, M. J., Mishra, P., Akcaoglu, M., & Rosenberg, J. M. (2013). The technological pedagogical content knowledge framework for teachers and teacher educators. *Commonwealth Educational Media Centre for Asia*, 1–8. Retrieved from https://www.researchgate.net/publication/267028784_The_Technological_Pedagogical_Content_Knowledge_Framework_for_Teachers_and_Teacher_Educators

Koirala, H. P. (2005). The effect of mathmagic on the algebraic knowledge and skills of low-performing high school students. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 209–216). University of Melbourne. Retrieved from https://ftp.gwdg.de/pub/misc/EMIS/proceedings/PME29/PME29RRPapers/PME29Vol3KoiralaEtAl.pdf

Kolb, A. & Kolb, D. A. (2005) *The Kolb Learning Style Inventory—Version 3.1: Technical specifications*. http://learningfromexperience.com/media/2010/08/tech_spec_ls

Kolb, D. A. (1984). *Experiential learning: Experience as the source of learning and development*. New Jersey: Prentice-Hall. Retrieved from https://www.researchgate.net/publication/235701029_Experiential_Learning_Experience_As_The_Source_Of_Learning_And_Development

Laborde, J. M. (2004). Cabri 3D. Online at: http://www.cabri.com/

Laborde, J-M., Baulac, Y., & Bellemain, F. (1988) Cabri Géomètre [Software]. Grenoble, France: IMAG-CNRS, Universite Joseph Fourier

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge University Press. Retrieved from https://www.cambridge.org/highereducation/books/situated-learning/6915ABD21C8E4619F750A4D4ACA616CD#overview

Lesh, R., Post, T. & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of Representation in the Teaching and Learning of Mathematics* (pp. 33-40) Hillsdale, NJ: Lawrence Erlbaum Associates http://wayback.archive-it.org/org-121/20190122153733/http://www.cehd.umn.edu/ci/rationalnumberproject/87_5.html

Leung, A & Or C.M (2007). From construction to proof: explanations in dynamic geometry environment. In Woo, J. H., Lew, H. C., Park, K. S. & Seo, D. Y. (Eds.). *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 177-184. Seoul: PME.

Lindner, J., Makarova, E., Bernhard, D., & Brovelli, D. (2022). Toward gender equality in education—Teachers' beliefs about gender and math. *Education Sciences*, 12(6), 373. https://doi.org/10.3390/educsci12060373

Marzano, R., & Kendall, J. (2007). *The new taxonomy of educational objectives* (2nd ed.). Thousand Oaks, CA: Corwin Press. Retrieved from https://books.google.ro/books/about/The_New_Taxonomy_of_Educational_Objectiv.html?id=JT4KAgAAQBAJ&redir_esc=y

Mengel, F., Sauermann, J., & Zölitz, U. (2019). Gender bias in teaching evaluations. *Journal of the European Economic Association*, 17(2), 535–566. https://doi.org/10.1093/jeea/jvx057

Mishra, P., & Koehler, M.J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, 108(6), 1017–1054. https://doi.org/10.1111/j.1467-9620.2006.00684.x

O'Connor, J. J., & Robertson, E. F. (2000). A history of Zero. *MacTutor History of Mathematics Archive.* University of St. Andrews, Scotland, UK. https://mathshistory.st-andrews.ac.uk/HistTopics/Zero/

Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics, 67*(1), 37–58. https://doi.org/10.1007/s10649-007-9083-3

Papert, S. (1984). Microworlds: Transforming education. In *Proceedings of the ITT Key Issues Conference*. Annenberg School of Communication, University of Southern California, Los Angeles, CA. Retrieved from http://dailypapert.com/wp-content/uploads/2016/08/papert_microWorlds_chapter.pdf

Patsiomitou, S. (2005a). *Fractals as a context of comprehension of the meanings of the sequence and the limit in a Dynamic Computer Software environment* (Master's thesis). National and Kapodistrian University of Athens, Department of Mathematics, Interuniversity Postgraduate Program. Available at http://me.math.uoa.gr/dipl/dipl_patsiomitou.pdf

Patsiomitou, S. (2005b). Fractals as a context of comprehension of the meanings of the sequence and the limit in a Dynamic Software environment. In *Proceedings of the 22nd Panhellenic Conference of the Hellenic Mathematical Society* (pp. 311–323). Lamia, 18–20 November 2005. (in Greek)

Patsiomitou, S. (2006a). Transformations on mathematical objects through animation and trace of their dynamic parameters. In *Proceedings of the 5th Pan-Hellenic Conference with International Participation: Informatics and Education-ETPE* (pp. 1070–1073). Thessaloniki, 5–8 October 2006. http://www.etpe.gr/custom/pdf/etpe1213.pdf (in Greek)

Patsiomitou, S. (2006b): Conclusions from the Experimental Teaching with NCTM interactive math applets (illuminations.nctm.org): Student's perception of meanings. *Proceedings of the 5th Pan-Hellenic Conference with International Participation. Informatics and Education-ETPE*, pp. 1053-1057, Thessaloniki, 5-8 October 2006. (in Greek)

Patsiomitou, S. (2006c): The Dynamic Geometry Environment as a means for the instructional design of activities: Results of a research process in a high school. *1st ICT Conference.* University of Nicosia, Cyprus (in Greek)

Patsiomitou, S. (2006d): DGS 'custom tools/scripts' as building blocks for the formulation of theorems-in-action, leading to the proving process. *Proceedings of the 5th Pan-Hellenic Conference with International Participation "ICT in Education"* (HCICTE 2006), pp. 271-278, Thessaloniki, 5-8 October. http://www.etpe.gr/custom/pdf/etpe1102.pdf (in Greek)

Patsiomitou, S. (2006e): Aesthetics and technology, as means for students' motivation to the proving process in a dynamic geometry environment. *Proceedings of the 23rd Panhellenic Conference of the Hellenic Mathematical Society*, (pp. 515–526). Patras, 24-26 November 2006. (in Greek)

Patsiomitou, S. (2006f): An approximation process generating number π (pi) through inscribed/circumscribed parametric polygons in a circle or through Riemann integrals' approximation process: Experimentation and research in a Dynamic Geometry Software Environment. In *Proceedings of the 23rd Panhellenic Conference of the Hellenic Mathematical Society* (pp. 502–514). Patras, 24–26 November 2006. (in Greek)

Patsiomitou, S. (2006g): Dynamic geometry software as a means of investigating, verifying, and discovering new relationships of mathematical objects. *EUCLID C: Scientific journal of Hellenic Mathematical Society, 65*, 55–78. (in Greek)

Patsiomitou, S. (2007a). Fractals as a context of comprehension of the meanings of the sequence and the limit in a Dynamic Computer Software environment. In E. Milková & P. Prazák (Eds.), *Electronic Proceedings of the 8th International Conference on Technology in Mathematics Teaching (ICTMT8)* (pp. cd-rom). University of Hradec Králové. ISBN 978-80-7041-285-5

Patsiomitou, S. (2007b). Sierpinski triangle, Baravelle spiral, Pythagorean tree: The Geometer's Sketchpad v4 as a means for the construction of meanings. In *Proceedings of the 4th Pan-Hellenic ICT Conference: Exploiting Information and Communication Technologies in Educational Practices* (pp. 28–37), Greek Ministry of Education, Syros, 4–6 May 2007. (in Greek)

Patsiomitou, S. (2007c) Circle's Area and number pi: An interdisciplinary teaching approach using the Geometer's Sketchpad v4 and the history of mathematics. *Proceedings of the 4th Pan-Hellenic ICT Conference titled: Exploiting Information and Communication Technologies in Educational Practices.* (pp. 59-68), Greek Ministry of Education, Syros, 4-6 May 2007. (in Greek)

Patsiomitou, S. (2007d) The Conic Sections: A teaching approach using internet -Quick movies, Geometer's Sketchpad v4, Cabri 3D and Function Probe. *Proceedings of the 4th Pan-Hellenic ICT Conference titled: "Exploiting Information and Communication Technologies in Educational Practices.",* Greek Ministry of Education, pp. 38-47, Syros, 4-6 May 2007(in Greek)

Patsiomitou, S. (2007e): Modeling Euclid's Elements in The Geometer's Sketchpad v4 dynamic geometry software. *"Astrolavos": Scientific journal of New Technologies of the Hellenic Mathematical Society,* (8), 61-89 (in Greek)

Patsiomitou, S. (2008a). The development of students' geometrical thinking through transformational processes and interaction techniques in a dynamic geometry environment. *Issues in Informing Science and Information Technology, 5*, 353–393. https://doi.org/10.28945/1015

Patsiomitou, S. (2008b). Linking Visual Active Representations and the van Hiele model of geometrical thinking. In W.-C. Yang, M. Majewski, T. Alwis, & K. Klairiree (Eds.), *Proceedings of the 13th Asian Conference in Technology in Mathematics* (pp. 163–178). Bangkok, Thailand: Suan Shunanda Rajabhat University. http://atcm.mathandtech.org/EP2008/papers_full/2412008_14999.pdf

Patsiomitou, S. (2008c). Do geometrical constructions affect students' algebraic expressions? In Yang, W., Majewski, M., Alwis T. and Klairiree, K. (Eds.) *Proceedings of the 13th Asian Conference in Technology in Mathematics*. (pp 193-202). http://atcm.mathandtech.org/EP2008/pages/regular.html

Patsiomitou, S. (2008d) Custom tools and the iteration process as the referent point for the construction of meanings in a DGS environment. In Yang, W-C, Majewski, M., Alwis T. and Klairiree, K. (Eds.) *Proceedings of the 13th Asian Conference in Technology in Mathematics.* (pp. 179-192). http://atcm.mathandtech.org/EP2008/pages/regular.html

Patsiomitou, S. (2008e) The Geometer's Sketchpad v4 Dynamic Geometry software as a means for interpreting and designing Euclid's Elements. *Proceedings of the 1st Pan-Hellenic ICT Educational Conference, "Digital Material to support Primary and Secondary-level teachers' pedagogical work*", pp.325-333, Naoussa, 9-11 May 2008. (in Greek)

Patsiomitou, S. (2008f). The construction of the number φ and the Fibonacci sequence in *The Geometer's Sketchpad v4* Dynamic Geometry software. In *Proceedings of the 1st Pan-Hellenic ICT Educational Conference: Digital Material to support Primary and Secondary-level teachers' pedagogical work* (pp. 307–315). Naoussa, 9–11 May 2008. (in Greek)

Patsiomitou, S. (2008g). The construction of a Baravelle spiral as a means for students' intuitive understanding of ascending and descending sequences. In *Proceedings of the 1st Panhellenic ICT Educational Conference: Digital Material to support Primary and Secondary-level teachers' pedagogical work* (pp. 316–324). Naoussa, 9–11 May 2008. (in Greek)

Patsiomitou, S. (2008h) Problem solving through Linking Visual Active Representations in a Dynamic Geometry Software, leading to rigorous proof. *Proceedings of the 6th Pan-Hellenic Conference with International Participation "Information and Communication Technologies in Education"* (HCICTE 2008), University of Cyprus, pp. 81-88, Cyprus 25-28 September 2008. Http://www.etpe.gr/custom/pdf/etpe1232.pdf (in Greek)

Patsiomitou, S. (2009a) The Impact of Structural Algebraic Units on Students' Algebraic Thinking in a DGS Environment at the *Electronic Journal of Mathematics and Technology (eJMT),* 3(3), 243-260.

Patsiomitou, S. (2009b) Learning Mathematics with The Geometer's Sketchpad v4. Monograph. *Klidarithmos Publications.* Volume A. ISBN: 978-960-461-308-3) (in Greek)

Patsiomitou, S. (2009c) Learning Mathematics with The Geometer's Sketchpad v4. Monograph. *Klidarithmos Publications.* Volume B. ISBN: 978-960-461-309-0) (in Greek)

Patsiomitou, S. (2009d). Cognitive and theoretical (gnostikotheoretices) links using the Geometer's Sketchpad software's interaction techniques. *Proceedings of the 5th Pan-Hellenic ICT Conference, entitled "Exploiting Information and Communication Technologies in Didactic Practice*", Greek Ministry of Education, pp. 583-591. Syros 8, 9, 10 May 2009(in Greek)

Patsiomitou, S. (2009e) Demonstration of visual proofs through decomposition and rearrangement of equivalent figures, created in a dynamic geometry software. *Proceedings of the 5th Pan-Hellenic ICT Conference, entitled "Exploiting Information and Communication Technologies in Didactic Practice*", Greek Ministry of Education, pp. 592-600. 8, 9, 10 May 2009 Syros. (in Greek)

Patsiomitou, S. (2009f). Tessellations, Pentominos, Structural Algebraic Units, Rep-Tiles, Tangram: A proposal for a qualitative upgrading of math curricula. In *Proceedings of the 5th Pan-Hellenic ICT Conference: Exploiting Information and Communication Technologies in Didactic Practice* (pp. 601–609). Syros, 8–10 May 2009. (in Greek)

Patsiomitou, S. (2009g) Students' cognitive interactions through constructions created with the Geometer's Sketchpad v4 DG environment. *Proceedings of the 1st Educational Conference entitled "Integration and Use of ICT in the Educational Process",* pp. 129-134. Volos, 24-26 April. http://www.etpe.gr/custom/pdf/etpe1440.pdf (in Greek)

Patsiomitou, S. (2009h) Tessellations constructed using Geometer's Sketchpad v4 as an intuitive means for the development of students' deductive reasoning. In *Proceedings of the 1st Educational Conference: Integration and Use of ICT in the Educational Process, 1,* 154–160. Volos, 24–26 April. (in Greek) https://eproceedings.epublishing.ekt.gr/index.php/cetpe/article/view/6426

Patsiomitou, S. (2010). Building LVAR (Linking Visual Active Representations) modes in a DGS environment. *Electronic Journal of Mathematics and Technology (eJMT), 4*(1), 1–25. https://ejmt.mathandtech.org/Contents/eJMT_v4n1p1.pdf

Patsiomitou, S. (2011a). Theoretical dragging: A non-linguistic warrant leading to dynamic propositions. *35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 361-368. Ankara, Turkey: PME. ISBN 978-975-429-297-8. Available at https://www.researchgate.net/publication/230648462

Patsiomitou, S. (2011b). Theoretical dragging: a non-linguistic warrant leading students to develop 'dynamic' propositions. *28th Panhellenic Conference of Hellenic Mathematical Society*, pp.562-574, Department of Mathematics of the University of Athens. https://www.academia.edu/3544047

Patsiomitou, S. (2012a). The development of students' geometrical thinking through transformational processes and interaction techniques in a dynamic geometry environment: Linking Visual Active Representations (PhD thesis). University of Ioannina. https://www.didaktorika.gr/eadd/handle/10442/35816 (in Greek)

Patsiomitou, S. (2012b). A Linking Visual Active Representation DHLP for student's cognitive development. *Global Journal of Computer Science and Technology, 12*(6), 53–81. http://computerresearch.org/index.php/computer/article/view/479/479

Patsiomitou, S. (2012c) *Didactic approaches to teaching Mathematics to students with different learning styles: Mathematics in the Real World*. Self–publishing. ISBN 978-960-93-4456. (in Greek) https://www.academia.edu/2054056/ (in Greek)

Patsiomitou, S. (2012d). Building and Transforming Linking Visual Active Representations – Implementation of LVARs for the teaching of mathematics in class. *Proceedings of 8th Pan-Hellenic Conference with International Participation "ICT in Education" (HCICTE, 2012),* University of Thessaly http://www.etpe.gr/custom/pdf/etpe1895.pdf (in Greek)

Patsiomitou, S. (2013a) Students learning paths as 'dynamic encephalographs' of their cognitive development". *International journal of computers & technology* [Online], 4(3) pp. 802-806 (18 April 2013) ISSN 2277-3061, https://doi.org/10.24297/ijct.v4i3.4207, http://cirworld.com/index.php/ijct/article/view/3038/pdf

Patsiomitou, S. (2013b) Instrumental decoding of students' conceptual knowledge through the modeling of real problems in a dynamic geometry environment. *EUCLID C: Scientific journal of Hellenic Mathematical Society, 79*, 107–136. (in Greek)

Patsiomitou, S. (2014). Student's learning progression through instrumental decoding of mathematical ideas. *Global Journal of Computer Science and Technology*, *14*(1), 1–42. http://computerresearch.org/index.php/computer/article/view/41/41

Patsiomitou, S. (2015a). A Dynamic Teaching Cycle of Mathematics through Linking Visual Active Representations". *Scientific Journal "ERKYNA", PanHellenic Pedagogical Society for Secondary-level Education*. Vol.7, pp. 70-86. https://erkyna.gr/e_docs/periodiko/dimosieyseis/thet_epistimes/t07-05.pdf (in Greek)

Patsiomitou, S. (2015b) Gender equality in [STEM] education*: Why Do Women Teach and Men Manage? Proceedings of the Panhellenic conference "The New Educator (Neos Paidagogos)"*, 23-24 May, Eugenides Foundation, pp. 1267-1290(in Greek)

Patsiomitou, S. (2015c). The development of students' competence on instrumental decoding as a non-linguistic warrant for the development of their geometric thinking. *Scientific journal "The New Educator (Neos Paidagogos)".* 5th issue, pp. 29-60. (in Greek)

Patsiomitou, S. (2015d). The Open eClass platform as a means for instructional design and curriculum management. *Scientific journal "The New Educator (Neos Paidagogos)".* 6th issue, pp.211-244. (The paper has been presented at the conference of "Education in the Era of ICT", 7 November 2015, Eugenides Foundation, conference proceedings: pp. 700-738) (in Greek)

Patsiomitou, S. (2016a). Synthesis, application and evaluation of a "dynamic" curriculum: Transformations of fractals objects, parametric regular polygons and number π. Linking Visual Active Representations. A keynote speech. In *3rd Panhellenic Conference of "The New Educator (Neos Paidagogos)"* (pp. 3563–3602). Eugenides Foundation, 16–17 April. (in Greek)

Patsiomitou, S. (2016b). Linking Visual Active Representations: Synthesis, implementation and evaluation of a "dynamic" curriculum based on dynamic transformations of mathematical objects with the utilization of interaction techniques. *The New Educator (Neos Paidagogos), 7*, 315–347. (in Greek)

Patsiomitou, S. (2016c). Environment & computer environments: The role of games in the development of students' competencies and their sense for a substantial school environment. In *Proceedings of the 13th Panhellenic Conference: The Education in the era of ICT and innovation* (pp. 967–994). (in Greek)

Patsiomitou, S. (2018a). A dynamic active learning trajectory for the construction of number π: Transforming mathematics education. *International Journal of Education and Research*, *6*(8), 225–248. http://www.ijern.com/journal/2018/August-2018/18.pdf

Patsiomitou, S. (2018b). An 'alive' DGS tool for students' cognitive development. *International Journal of Progressive Sciences and Technologies (IJPSAT), 11*(1), 35–54. http://ijpsat.ijsht-journals.org/index.php/ijpsat/article/view/636

Patsiomitou, S. (2019a). From Vecten's Theorem to Gamow's Problem: Building an Empirical Classification Model for Sequential Instructional Problems in Geometry.

Journal of Education and Practice. 10(5), 1–23. https://doi.org/10.7176/JEP/10-5-01. https://iiste.org/Journals/index.php/JEP/article/view/46479

Patsiomitou, S. (2019b). Hybrid-dynamic objects: DGS environments and conceptual transformations. *International Journal for Educational and Vocational Studies. 1*(1), 31–46. pp. 31-46. DOI: https://doi.org/10.29103/ijevs.v1i1.1416. Available online at http://ojs.unimal.ac.id/index.php/ijevs

Patsiomitou, S. (2019c). *A trajectory for the teaching and learning of the didactics of mathematics [using ICT]: Linking Visual Active Representations*. Monograph. Global Journal Incorporated. ISBN 978-1-7340132-0-7. http://doi.org/10.34257/SPatTrajICT

Patsiomitou, S. (2020a). *Didactics of Mathematics I: Linking Visual Active Representations.* Monograph. Anatolikos. Athens, ISBN: 978-618-5136-46-8. https://www.academia.edu/42019703/ (in Greek)

Patsiomitou, S. (2020b). *Didactics Instruction and Assessment of Mathematics: Learning Trajectories and Curriculum.* Monograph. Anatolikos. Athens ISBN: 978-618-5136-49-9. https://www.academia.edu/43702210/(in Greek)

Patsiomitou, S. (2020c). *Didactics and Instruction of Mathematics: From theory to action using microworlds*. Monograph. Angelakis Publications. Athens. ISBN: 978-960-616-155-1. https://www.academia.edu/43795275/(in Greek)

Patsiomitou, S. (2020d). *Formulation of a gender "theory" in education: established trends.* Monograph. Angelakis Publications. Athens. ISBN: 978-960-616-171-1 , https://www.academia.edu/44760742/(in Greek)

Patsiomitou, S. (2021a). Dynamic Euclidean Geometry: pseudo-Toulmin modeling transformations and instrumental learning trajectories. International Institute for Science, Technology and Education (IISTE): E-Journals. *Journal of Education and Practice.* 12(9). pp. 80-96. https://doi.org/10.7176/JEP/12-9-09

Patsiomitou, S. (2021b). A Research Synthesis Using Instrumental Learning Trajectories: Knowing How and Knowing Why. International Institute for Science, Technology and Education (IISTE): E-Journals. *Information and Knowledge Management.* 11(3). https://doi.org/10.7176/IKM/11-3-02

Patsiomitou, S. (2021c). Instrumental learning trajectories: The case of GeoGebra. Athens: Angelakis Publications. ISBN 978-960-616-193. https://www.academia.edu/79248541/ (in Greek)

Patsiomitou, S. (2021d). Creativity and skills in mathematics. ISBN 978-618-00-3221-5. https://www.academia.edu/51047627/ (in Greek)

Patsiomitou, S. (2022a). *Conceptual and instrumental trajectories using linking visual active representations created with the Geometer's Sketchpad*. Athens: Klidarithmos Publications. ISBN 978-960-645-302-1. (in Greek)

Patsiomitou, S. (2022b). DGS Cui-Rods: Reinventing Mathematical Concepts. GPH-*International Journal of Educational Research*, 5(09), 01-11. https://doi.org/10.5281/zenodo.7036045, http://www.gphjournal.org/index.php/er/article/view/693

Patsiomitou, S. (2022c). Inquiring and learning with DGS Cui-Rods: a proposal for managing the complexity of how primary-school pupils' mathematically structure odd-even numbers". *International Journal of Scientific and Management Research*. Volume 5 Issue 8 August 2022, 143-163. http://doi.org/10.37502/IJSMR.2022.58.

Patsiomitou, S. (2022d). Digital-Concrete Materials: Revisiting Fröbel in Sketchpad Tasks. *GPH-International Journal of Educational Research*, 5(10), 01-15. https://doi.org/10.5281/zenodo.7185215

Patsiomitou, S. (2023). *A dynamic multi-level curriculum based on the central idea of Linking Visual Active Representations (LVAR): Inquiring during the years 2005–2023*. Angelakis Publications. (Original work published 2023 in Greek as: *Δυναμικό πολυεπίπεδο πρόγραμμα σπουδών βασισμένο στην ιδέα των συνδεόμενων οπτικών ενεργών αναπαραστάσεων (ΣΟΕΑ): Ερευνητικές μελέτες από το 2005–2023*. Εκδόσεις Αγγελάκη)

Patsiomitou, S. (2023a) Developing and managing knowledge through the eyes of the young learner: 'Alive' manipulatives before abstract notions. *International Journal of Scientific and Management Research*, *6*(3), 18–40. http://doi.org/10.37502/IJSMR.2023.6302

Patsiomitou, S. (2023b). A brief review on my studies: Managing the complexity of using Linking Visual Active Representations (LVAR). *International Journal of Scientific and Management Research, 6*(5), 1–33. http://doi.org/10.37502/IJSMR.2023.6501

Patsiomitou, S. (2024a). The influence of artificial intelligence and digital media on the evaluation of school units and the enhancement of educational quality. In *Proceedings of the 3rd International Conference of Educational Assessment* (Vol. II, Issue 8, pp. 351–373). https://eletea.gr/el/τεύχος-8-τόμος-ιι-2024/ (In Greek)

Patsiomitou, S. (2024b). Investigating artificial intelligence technologies in generating LVAR. *International Journal of Research in Education Humanities and Commerce, 5*(5), 194–217. https://doi.org/10.37602/IJREHC.2024.5515

Patsiomitou, S. (2024d). *Abstract, instrumental, and deductive geometric trajectories on quadrilaterals.* Monograph. Angelakis Publications.ISBN: 978-960-616-370-8. https://www.academia.edu/114452254/ (in Greek)

Patsiomitou, S. (2025a). From the concept of schema to the idea of "instrumental" social schema. *International Journal of Research in Education Humanities and Commerce. 6*(2), 262–289. https://doi.org/10.37602/IJREHC.2025.6220

Patsiomitou, S. (2025b). A proposal for a fractal-based "Dynamic" Program: The Pythagorean Tree structure generated through instrumental schemata. *International Journal of Research in Education Humanities and Commerce, 6*(2), 342–388. https://doi.org/10.37602/IJREHC.2025.6327

Patsiomitou, S. (2025c). the Treasury of Atreus relieving triangle: insights form to hyperbolic geometry. *International Journal of Research in Education Humanities and Commerce, 6*(6), 55-76. https://doi.org/10.37602/IJREHC.2025.6605

Patsiomitou, S. (2025d). *Building geometry – building relationships dynamically: Professional development material for the mathematics education advisor*. Self-published. ISBN 978-618-87884-0-4

Patsiomitou, S. (2026). Reimagining sustainable school education through a fractal-based dynamic program (FDP): An experiential and interactive approach. *International Journal of Research in Education Humanities and Commerce, 7*(2), 285–317. https://doi.org/10.37602/IJREHC.2026.7223

Patsiomitou, S. & Koleza, E. (2008). Developing students' geometrical thinking through linking representations in a dynamic geometry environment. In O. Figueras & A. Sepúlveda (Eds.), *Proceedings of the Joint Meeting of the 32nd Conference of the International Group for the Psychology of Mathematics Education and the XX North American Chapter* (Vol. 4, pp. 89–96). https://www.academia.edu/1764077/

Patsiomitou, S. and Koleza, E. (2009). The development of students' geometrical thinking through Linking Visual Active Representations. In M. Kourkoulos & Tzanakis, C. (Eds.). *Proceedings of the 5th International Colloquium on the Didactics of Mathematics* (Vol. 2, pp.157-171). Rethymnon, Greece. https://www.academia.edu/4780882/

Patsiomitou, S. and Emvalotis A. (2009a). Developing geometric thinking skills through dynamic diagram transformations. *Proceedings of MEDCONF 2009, The 6th Mediterranean Conference on Mathematics Education.* (pp.249-258). Plovdiv, Bulgaria http://www.fmi-plovdiv.org/GetResource?id=529

Patsiomitou, S. and Emvalotis A. (2009b) Does the Building and transforming on LVAR modes impact students' way of thinking? In Tzekaki, M., Kaldrimidou, M. & Sakonidis, C. (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 337-344. Thessaloniki, Greece: PME. https://www.academia.edu/17644517/

Patsiomitou, S. and Emvalotis A. (2009c) 'Economy' and 'Catachrèse' in the use of custom tools in a Dynamic geometry problem-solving process *Electronic Proceedings of the 9th International Conference on Technology in Mathematics Teaching* (ICTMT9) in Metz. https://www.academia.edu/1765045/

Patsiomitou, S. and Emvalotis A. (2009d) Composing and testing a DG research–based curriculum designed to develop students' geometrical thinking. Paper at the annual European Conference on Educational Research (ECER) Vienna, Sept. 25. - 28., 2009. https://www.academia.edu/1764260/

Patsiomitou, S., & Emvalotis, A. (2010b). The development of students' geometrical thinking through a DGS reinvention process. In M. F. Pinto & T. F. Kawasaki (Eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 33–40). BeloHorizonte, Brazil https://www.academia.edu/17644700/

Patsiomitou, S., and Emvalotis A. (2010a) Students' movement through van Hiele levels in a Dynamic geometry guided reinvention process**.** *Journal of Mathematics and Technology, 3*, 18–48. https://www.academia.edu/1764079/

Piaget, J. (1937/1971). *The construction of reality in the child* (M. Cook, Trans.). New York: Basic Books.

Punie, Y., & Redecker, C. (Eds.). (2017). *European framework for the digital competence of educators: DigCompEdu* (EUR 28775 EN). Publications Office of the European Union. https://doi.org/10.2760/159770

Rabardel, P. (1995). Les hommes et les technologies, approche cognitive des instruments contemporains. Paris: Armand Colin.

Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry, 29*(3), 315–342. https://doi.org/10.1111/0362-6784.00130

Richter-Gebert, J., & Kortenkamp, U. (1999). *User manual of the interactive geometry software Cinderella*. Springer-Verlag, Heidelberg. Retrieved from https://link.springer.com/book/10.1007/978-3-642-58318-6

Rotem, S. H., & Ayalon, M. (2022). Building a model for characterizing critical events: Noticing classroom situations using multiple dimensions. *The Journal of Mathematical Behavior*, *66*. https://doi.org/10.1016/j.jmathb.2022.100947

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114–145. https://doi.org/10.2307/749205

Strømskag, H. (2017). A methodology for instructional design in mathematics—with the generic and epistemic student at the centre. *ZDM Mathematics Education, 49*, 909–921. https://doi.org/10.1007/s11858-017-0882-0

Tall, D. O., & Vinner, S. (1981). Concept image and concept definition in mathematics, with special reference to limits and continuity. *Educational Studies in Mathematics, 12*, 151–169. https://doi.org/10.1007/BF00305619

Tall, D. O., Gray, E. M., Ali, M. B., Crowley, L. R. F., DeMarois, P., McGowen, M. C., Pitta, D., Pinto, M. M. F., Thomas, M., & Yusof, Y. B. (2001). Symbols and the bifurcation between procedural and conceptual thinking. *Canadian Journal of Science, Mathematics and Technology Education, 1*, 81–104. https://doi.org/10.1080/14926150109556452

Taylor, R. P. (1980). *The Computer in the School: Tutor, Tool, Tutee*. New York: Teacher's College Press, Columbia University. Retrieved from https://scispace.com/pdf/the-computer-in-the-school-tutor-tool-tutee-49cfkix2h6.pdf

Terwel, J. (1999). Constructivism and its implications for curriculum theory and practice. *Journal of Curriculum Studies, 31*, 195-299. https://doi.org/10.1080/002202799183223

Tirosh, D., Tsamir, P., Levenson, E., & Barkai, R. (2019). Using theories and research to analyze a case: Learning about example use. *Journal of Mathematics Teacher Education, 22*, 205–225. Retrieved from https://doi.org/10.1007/s10857-017-9386-y

Treffers, A. (1987). *Three dimensions: A model of goal and theory description in mathematics instruction* (The Wiskobas Project). D. Reidel Publ. Co., Dordrecht. Retrieved from https://www.scribd.com/document/448409201/Three-Dimensions-A-Model-of-Goal-and-Th-Adrian-Treffers-pdf

Trouche, L. (2004). Managing the complexity of the human/machine interaction in computerized learning environments: guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical* Learning *9, pp. 281-307, Kluwer academic publishers.* Retrieved from https://www.academia.edu/666692/

Tuomi, I. (2018). The impact of artificial intelligence on learning, teaching, and education. In M. Cabrera, R. Vuorikari, & Y. Punie (Eds.), *Policies for the future*. Publications Office of the European Union. https://doi.org/10.2760/12297

UNESCO. (2016). *Happy schools! A framework for learner well-being in the Asia-Pacific*. UNESCO. https://unesdoc.unesco.org/ark:/48223/pf0000244140

UNESCO. (2024). *Why the world needs happy schools: Global report on happiness in and for learning*. UNESCO Digital Library. https://unesdoc.unesco.org/ark:/48223/pf0000389119

van Oers, B. (1996). Are you sure? Stimulating mathematical thinking during young children's play. *European Early Childhood Education Research Journal, 4*(1), 71–87. https://doi.org/10.1080/13502939685207851

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press. https://doi.org/10.2307/j.ctvjf9vz4

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press. https://doi.org/10.1023/A:1023947624004

Wenger-Trayner, E., & Wenger-Trayner, B. (2015). Introduction to communities of practice: A brief overview of the concept and its uses. http://wenger-trayner.com/introduction-to-communities-of-practice

Wertheimer, M. (1922). Untersuchungen zur Lehre von der Gestalt. I. Prinzipielle Bemerkungen [Investigations on the Principles of Gestalt. I. Principal Remarks]. Psychologische Forschung, 1(1), 47-58. https://doi.org/10.1515/gth-2017-0007

Wertheimer, M. (1923). Untersuchungen zur Lehre von der Gestalt. II. [Investigations on the Principles of Gestalt. II.]. *Psychologische Forschung, 4(1),* 301-350

Yeh, A. & Nason, R. (2004) Towards a semiotic framework for using technology in mathematics education: the case of learning 3D geometry, *Paper presented at the 2004 International Conference on Computers in Education,* Melbourne, Australia. Retrieved from https://www.academia.edu/95149825/

Websites

[1] Law 4823/2021. (2021, August 3). Government Gazette A 136/03.08.2021. e-Nomothesia. https://www.e-nomothesia.gr/index.php/nomos-4823-2021-phek-136a-3-8-2021.html

[2] Cambridge University Press & Assessment. (n.d.). Cambridge Dictionary. https://dictionary.cambridge.org/

[3] Koehler, M. (2012). TPACK image [Diagram]. TPACK.ORG. https://tpack.org/tpack-image/

[4] Partovi, H., & Yongpradit, P. (2024, January 18). AI and education: Kids need AI guidance in school. But who guides the schools? World Economic Forum. https://www.weforum.org/agenda/2024/01/ai-guidance-school-responsible-use-in-education/
[5] World Economic Forum. (n.d.). Agenda: 2024 articles. Retrieved May 18, 2026, from https://www.weforum.org/agenda/2024/
[6] OpenAI. (n.d.). *ChatGPT*. https://openai.com/index/chatgpt/
[7] Spatsiomitou, S. (n.d.). Blogs.sch.gr. Retrieved May 18, 2026, from https://blogs.sch.gr/spatsiomitou/
[8] Ministry of Education and Religious Affairs. (n.d.). Retrieved May 18, 2026, from https://trapeza.iep.edu.gr/public/subjects.php
[9] Spatsiomitou, S. (n.d.). GeoGebra profile. GeoGebra. Retrieved May 18, 2026, from https://www.geogebra.org/u/spatsiomitou
[10] Desmos Graphing Calculator. (n.d.). Desmos. Retrieved May 18, 2026, from https://www.desmos.com/calculator?lang=el
[11] Scher, D., & Steketee, S. (n.d.). Teaching with Web Sketchpad. Sine of the Times. Retrieved May 18, 2026, from https://www.sineofthetimes.org/teaching-with-web-sketchpad/
[12]Forging Dynamic Connections. (n.d.). Tools. Geometric Functions. https://geometricfunctions.org/fc/tools/
[13] Khan Academy. (n.d.). Khan Academy. https://www.khanacademy.org/
[14] National Council of Teachers of Mathematics. (n.d.). Trigonometric graphing. Illuminations. https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Trigonometric-Graphing/
[15] Leonardo.AI. (n.d.). https://leonardo.ai/
[16] Lumen5. (n.d.). https://lumen5.com/
[17] MathSpad. (n.d.). Geometry construction tool. https://www.mathspad.co.uk/i2/construct.php
[18] https://www.dynamicgeometry.com/General_Resources/Classroom_Activities/KCPT/ Activities_for_Young_Learners/Sketchpad_for_Grades_3-5/Activities.html
[19] O'Connor, J. J., & Robertson, E. F. (n.d.). A history of zero. MacTutor History of Mathematics Archive, University of St Andrews. http://www-history.mcs.st-and.ac.uk/HistTopics/Zero.html