



## METHODOLOGY OF TEACHING PROBLEMS ON CALCULATING THE VOLUME OF SPHERICAL BODIES

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### Abstract:

The article investigates the problems of improving the methodology of teaching problems on calculating the volume of spherical bodies (cylinder, cone, sphere), which is one of the important sections of the geometry course in senior grades of secondary schools. The main goal of the research is to develop students' spatial thinking, form skills for applying theoretical knowledge in solving practical problems, and increase the level of preparation for final certification. The article analyzes the existing scientific and methodological literature, presents a typology of problems, and systematizes methodological approaches that can be used in the teaching process. The experimental results demonstrate the effectiveness of the proposed methodology in developing spatial thinking and improving problem-solving skills. In this article, the term "spherical bodies" is used broadly to include cylinders, cones, and spheres as solids of revolution.

**Keywords:** spherical bodies, volume, methodology of teaching geometry, spatial thinking, problem solving

### 1. Introduction

In the modern education system, teaching mathematics, especially geometry, is not limited to the transmission of formal knowledge. It also envisages the development of students' analytical and spatial thinking, and the formation of the ability to comprehend real objects and processes through mathematical models (Almubarak et al., 2025; Aliyev et al., 2021; Aliyev et al., 2023). The rapid development of technology, 3D modeling, and

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engineering-oriented fields makes the correct formation of spatial concepts even more relevant (Zhang & Wang, 2026).

From this perspective, the topic of "spherical bodies" holds a special place in the geometry course. Figures such as the cylinder, cone, and sphere are both mathematical models of objects frequently encountered in everyday life and concepts closely related to other subjects – physics, technology, and computer science (Puspita et al., 2021).

However, in school practice, one of the main difficulties during the teaching of this topic is that students master problems on volume calculation mechanically, merely at the level of memorizing formulas (Bashirova & Erboluly, 2024). Experience shows that problems related to calculating the volume of spherical bodies are considered one of the most complex topics in stereometry for students (Bashirova & Erboluly, 2024). The main reasons for this are the weak development of spatial concepts, the lack of a clear connection between the mathematical model and the real object, and the limited lesson hours (Korenova et al., 2025). As a result, students face difficulties in solving non-standard, combined problems, or those requiring logical reasoning (Zinovieiev & Manko, 2011; Aliyev et al., 2022).

The purpose of the article is to systematize scientific-methodological approaches to teaching problems on calculating the volume of spherical bodies, present a typology of problems, and justify methodological principles that increase the efficiency of the learning process. The research results have practical significance for geometry teachers, methodologists, and pedagogical researchers in secondary schools (Angelov, 2024; Aliyev et al., 2020; Aliyev et al., 2025).

## 2. Literature Review

Teaching geometry has didactic goals aimed at developing students' cognitive abilities, spatial thinking, and mathematical reasoning. Since the topic of calculating the volume of spherical bodies (cylinder, cone, sphere) is closely related to the basic concepts of stereometry, its teaching should be based on solid didactic principles (Govender & Amevor, 2025).

As the teaching of spherical bodies is closely related to planimetric knowledge, students must first have a solid understanding of the concepts of circle, triangle, circumference, and area (Hlongwana et al., 2025). The transition to stereometric concepts should be carried out gradually, starting with simple figures (e.g., the cylinder), and then moving on to more complex figures (cone, sphere).

The abstract nature of spherical bodies can pose difficulties for students. Therefore, visual aids (models, 3D images, dynamic geometry software) should be widely used in the teaching process (Aghazade & Bingöl, 2023). Physical models of the cylinder, cone, and sphere should be shown, and their cross-sections explained visually. This approach develops students' spatial imagination and helps connect abstract concepts with concrete objects (Rohendi et al., 2025).

Calculating the volume of spherical bodies should be presented not only as theoretical knowledge but also as a skill necessary for solving problems related to

practical life. Real-life situations such as calculating the capacity of a cylindrical barrel, the volume of a conical tent, or the dimensions of a spherical ball should be considered. This approach increases students' motivation and helps them understand how mathematics is applied in everyday life (Aghazade, 2014).

Problems on calculating the volume of spherical bodies create favorable opportunities for developing students' logical thinking, proof skills, and mathematical reasoning. During teaching, students should be taught not only to memorize ready-made formulas but also to understand the logic behind deriving these formulas. For instance, explaining the volume of a cylinder by analogy with a rectangular prism, and the volume of a cone by analogy with a pyramid, can be useful (Nierenberg, 1998; Aliyev et al., 2021; Aliyev et al., 2023).

The use of technology in modern education can make teaching spherical bodies more effective. Through dynamic geometry software (GeoGebra, Cabri 3D, etc.), students can interactively create spherical bodies, change their parameters, and immediately see the results (Aghazade & Bingöl, 2023).

### 2.1 Typology of problems

Problems related to calculating the volumes of spherical bodies vary in level and content characteristics:

- Elementary (standard) problems rely on direct application of formulas (e.g., calculating volume when radius and height are given).
- Problems based on interparameter dependencies require first determining functional relationships between elements (e.g., given slant height and angle, find height and radius).
- Problems involving inscribed and circumscribed spherical bodies (e.g., a sphere inscribed in a cone) require choosing the correct cross-section.
- Combined and non-standard problems present spherical bodies together with polyhedra or as unions, often requiring auxiliary constructions (Anarbekova, 2022).

### 2.2 Main methods for solving problems

Effective solving should not be limited to formula application. The method of using cross-sections reduces stereometric problems to planimetric ones (Lawrence, 2011). The auxiliary volume method relates the volume of a complex body to simpler spherical bodies (Aliyev et al., 2021). The coordinate and vector approach connects classical geometry with modern mathematical tools. For advanced students, analytical methods using integral calculus demonstrate volumes of solids of revolution.

## 3. Material and Methods

To test the effectiveness of the proposed methodological approach, a pedagogical experiment was conducted during the 2024–2025 academic year. The experiment involved 64 tenth-grade students (mean age = 16.5 years, range 16-17) from two public

secondary schools in Baku, Azerbaijan. Students were divided into a control group ( $n = 32$ ) and an experimental group ( $n = 32$ ). Groups were formed based on previous academic performance in mathematics to ensure comparability; the average mathematics grade for the control group was 78.4 (out of 100), while for the experimental group it was 78.1.

A pre-test was administered to both groups before the experiment to assess initial knowledge on calculating volumes of spherical bodies. The pre-test consisted of 10 problems: 4 elementary, 3 interparameter-based, 2 involving inscribed/circumscribed bodies, and 1 combined problem (maximum score = 20). To measure spatial thinking, a researcher-developed 15-item test was used, assessing mental rotation, spatial visualization, and spatial orientation skills (maximum score = 30). An independent samples t-test confirmed no statistically significant difference between groups prior to the experiment ( $t(62) = 0.28, p > 0.05$ ).

The teaching intervention was carried out over six weeks (two 45-minute lessons per week, total 12 lessons). Both groups covered the same curriculum content. In the control group, traditional instruction was used: teacher explained formulas, solved typical problems on the board, and students practiced individually. In the experimental group, the proposed methodological approach was implemented, including:

- 1) **Phased learning:** gradual transition from elementary to combined problems.
- 2) **Visualization:** physical models and GeoGebra software for 3D modeling and cross-sections (Aghazade & Bingöl, 2023).
- 3) **Differentiated tasks:** three levels of difficulty (basic, intermediate, advanced) based on pre-test results and formative assessment.
- 4) **Active learning methods:** group work, pair work, student presentations.
- 5) **Integration of solution methods:** explicit teaching of cross-section, auxiliary volume, and coordinate methods.

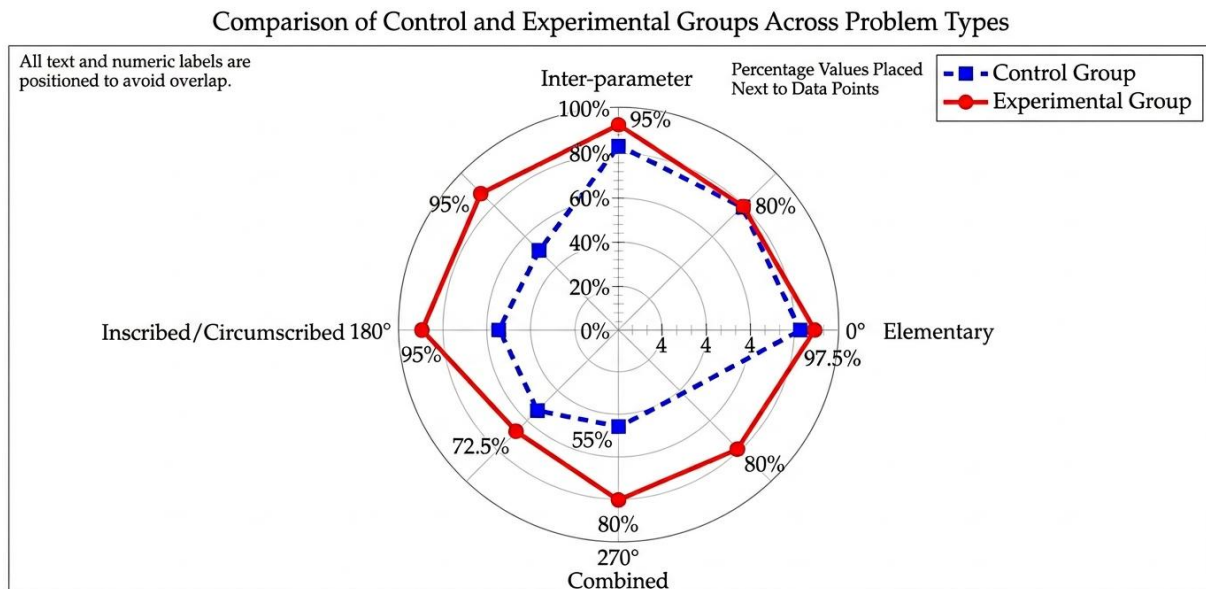
A concrete GeoGebra activity example: Students open GeoGebra's 3D Graphics view and create a cone using the "Cone" tool. They then use a slider to vary the height from 0 to 10 cm. The teacher asks: "What happens to the volume when we double the height?" Students predict, then use the "Volume" command to check. Next, students activate the cross-section tool to see the triangular axial cross-section. They measure the radius and height and verify the formula  $V = (1/3)\pi r^2 h$ .

Post-test (parallel in structure to pre-test) and the spatial thinking test were administered immediately after the intervention. A motivation questionnaire (5-point Likert scale) was given to the experimental group. Classroom observations were conducted by an independent observer.

#### 4. Results and Discussion

The post-test results revealed a notable difference between the two groups. The control group achieved a mean score of 13.8 ( $SD = 2.9$ ), while the experimental group achieved a mean score of 16.7 ( $SD = 2.4$ ). An independent samples t-test showed that this difference was statistically significant ( $t(62) = 4.41, p < 0.001$ ), with a large effect size (Cohen's  $d = 1.10$ ).

When analyzing performance by problem type, the experimental group showed particularly strong gains on problems involving inscribed and circumscribed bodies (experimental group 3.8 out of 4, control group 2.9 out of 4) and combined problems (experimental group 1.6 out of 2, control group 1.1 out of 2). On elementary problems, both groups performed similarly (experimental group 3.9 out of 4, control group 3.8 out of 4), suggesting that the main impact of the intervention was on complex, higher-order thinking tasks.



**Figure 1:** Radar chart comparing Control and Experimental groups across problem types (percentage values shown at each data point). Categories: Elementary, Inter-parameter, Inscribed/Circumscribed, Combined. Control group values: 95%, 80%, 72.5%, 55%. Experimental group values: 97.5%, 95%, 95%, 80%.

**Figure 1:** Radar chart comparing Control and Experimental groups across problem types

The spatial thinking test results also showed significant improvement in the experimental group. Before the intervention, the experimental group had a mean score of 18.2 (SD = 4.1), while the control group had 18.4 (SD = 4.0). After the intervention, the experimental group's mean score increased to 24.5 (SD = 3.5), whereas the control group's mean score increased only slightly to 19.6 (SD = 3.9). The gain in the experimental group was statistically significant ( $t(31) = 8.94, p < 0.001$ ), while the gain in the control group was not ( $t(31) = 1.52, p > 0.05$ ).

The motivation questionnaire revealed positive attitudes: "The use of models and GeoGebra helped me understand the concepts better" (mean = 4.6, SD = 0.5); "I enjoyed working in groups to solve problems" (mean = 4.4, SD = 0.7); "The differentiated tasks matched my level and helped me progress" (mean = 4.3, SD = 0.6); "I feel more confident in solving volume problems now" (mean = 4.5, SD = 0.5). Open-ended comments included: "I never understood how to find the volume of a sphere before, but now it's clear" and "Using GeoGebra made it fun to see how the cone changes when we change its height."

Classroom observations confirmed that students in the experimental group were more actively engaged, asked more questions, and spent more time discussing solution strategies with peers.

These results align with previous studies showing that visualization, differentiation, and active learning significantly improve spatial thinking and problem-solving skills (Rohendi et al., 2025; Korenova et al., 2025; Rodrigues & Oliveira, 2014).

## 5. Recommendations

Based on the findings, the following recommendations are proposed for geometry teachers, methodologists, and curriculum developers:

- **Integrate dynamic geometry software** (GeoGebra, Cabri 3D) into regular stereometry lessons, not just as demonstrations but as interactive student activities.
- **Adopt a phased problem typology** – start with elementary problems, then progress to interparameter dependencies, inscribed/circumscribed bodies, and finally combined non-standard problems.
- **Use differentiated tasks** to accommodate students with varying preparation levels. Basic-level students should receive step-by-step guided problems, while advanced students should engage in research-oriented exploratory tasks.
- **Incorporate cross-section and auxiliary volume methods** explicitly into instruction, as these help students reduce stereometric problems to planimetric ones.
- **Provide professional development** for teachers on using 3D modeling tools and formative assessment techniques for spatial thinking.
- **Future research** should investigate the long-term retention of spatial thinking skills and the applicability of this methodology to other stereometry topics (e.g., surface area, volumes of revolution).

## 6. Conclusion

Based on the conducted scientific-methodological analysis and pedagogical experiment, the following conclusions are drawn:

- The topic of calculating the volume of spherical bodies plays an important role in the development of students' spatial and analytical thinking.
- Systematic teaching based on a typology of problems (elementary, interparameter, inscribed/circumscribed, combined) helps structure students' knowledge.
- The integrated application of cross-section, auxiliary volume, and coordinate methods increases efficiency in problem-solving.
- A methodological approach based on visualization, differentiation, and active learning significantly improves learning outcomes (experimental group post-test mean 16.7 vs. control group 13.8,  $p < 0.001$ ).

- The proposed methodology also significantly improves spatial thinking abilities (experimental group gain from 18.2 to 24.5,  $p < 0.001$ ).

Thus, the proposed methodological approaches can be applied in teaching problems on calculating the volume of spherical bodies and make a real contribution to improving geometry instruction.

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### **Conflict of Interest Statement**

The authors declare no conflicts of interest.

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