



MATHEMATICAL INVESTIGATION PROFICIENCY AMONG SENIOR HIGH SCHOOL STUDENTS: AN EXPLANATORY APPROACH

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Abstract:

This study employed an explanatory sequential mixed methods design to examine the mathematical investigation proficiency of senior high school (SHS) students in the Davao Region. In the quantitative phase, data were collected from students in using an adapted and validated mathematical investigation questionnaire, while in the qualitative phase, purposively selected participants were engaged in in-depth interviews and a focus group discussion. Descriptive and inferential statistics, including mean, standard deviation, t-test, and ANOVA, were utilized to determine the level of mathematical investigation proficiency and examine differences by sex and SHS strand. Findings revealed that students demonstrated an overall very low level of mathematical investigation proficiency, with significant differences observed across strands but not by sex. Qualitative results further explained these patterns, indicating that students' very low proficiency was associated with a reliance on formula-driven procedures, limited exposure to investigative and non-routine mathematical tasks, and challenges in articulating mathematical reasoning and justifying solutions. The integration of quantitative and qualitative findings highlighted the influence of curricular exposure, instructional emphasis, and confidence on students' performance in mathematical investigations. Based on these findings, the EXPLORE enhancement scheme was developed to provide targeted interventions aimed at improving students' mathematical investigation proficiency.

Keywords: education, mathematical investigation proficiency, senior high school, explanatory sequential design, Philippines

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1. Introduction

A mathematical investigation is an open-ended exploration of mathematical problems that requires students to identify patterns, formulate conjectures, and justify their findings. Unlike routine problem-solving, mathematical investigations encourage the understanding of mathematical facts and concepts, the analysis of mathematical problems, and the communication of mathematical ideas through mathematical symbols and notation (Nivera, 2012). Although these competencies are essential for fostering deeper levels of mathematical thinking, research and international assessment results continue to reveal students' challenges in conceptual understanding, problem-solving, and mathematical reasoning, all of which are necessary for meaningful engagement in mathematical investigations (Zorn *et al.*, 2024).

Building on the foregoing discussion, a critical concern in mathematical investigation lies in students' **foundational knowledge**, which encompasses their ability to use correct mathematical concepts, execute appropriate procedures, and avoid misconceptions. Recent studies have shown that many students demonstrate fragmented conceptual understanding and procedural inconsistencies, which hinder their ability to engage meaningfully in non-routine mathematical tasks (El Walida & Hasana, 2020; Qetrani & Achtaich, 2022). Misconceptions in fundamental mathematics topics often lead to incorrect representations and flawed solution paths, ultimately affecting the validity of the entire mathematical investigation (Kurudirek *et al.*, 2025). Furthermore, Khalid *et al.* (2019) suggest that students who rely heavily on memorized procedures without conceptual grounding struggle to transfer knowledge to unfamiliar contexts, thereby limiting their proficiency in mathematical investigations.

Another significant challenge emerges in the **investigation process**, particularly in terms of students' capacity for **analysis**, which involves exploring problems with depth and complexity and verifying solutions or conjectures. However, empirical evidence indicates that students often demonstrate limited depth in their analyses, tending to rely on single-step or superficial strategies rather than engaging in comprehensive exploration (Hadi *et al.*, 2018; Pongsakdi *et al.*, 2020). Additionally, students frequently struggle to verify the correctness of their solutions or conjectures, which is a critical component of rigorous mathematical inquiry (Kristianti *et al.*, 2020). This may mean that students may not fully understand that mathematical investigation is an ongoing process of testing and improving their solutions.

Closely related to analysis is the role of **reasoning** in mathematical investigation, particularly in terms of the validity of arguments, the quality of proofs, and the ability to establish meaningful connections across mathematical ideas. Nevertheless, studies have shown that many students struggle to produce valid and coherent justifications, often presenting incomplete or unsupported arguments (Jailani *et al.*, 2020). In addition, students tend to struggle with constructing proofs, reflecting gaps in their ability to organize logical sequences of statements (Conner and Krejci, 2022). The ability to connect problems or generalize conjectures is also frequently underdeveloped, as students often

view tasks in isolation rather than as part of a broader mathematical structure (Qurohman *et al.*, 2025). These issues indicate that weaknesses in reasoning significantly limit students' capacity to engage in authentic mathematical investigation and to produce mathematically sound conclusions.

Equally important is students' **mathematical communication**, which plays a vital role in expressing and validating the outcomes of mathematical investigations. Despite its importance, research has consistently shown that students encounter difficulties in communicating mathematical ideas with clarity and precision (Martins & Martinho, 2021; Powell *et al.*, 2017). Many students struggle with structuring their written work, leading to incomplete or disorganized presentations that obscure their reasoning processes (Wahyuni *et al.*, 2023). Furthermore, incorrect or inconsistent use of mathematical symbols and notations can result in ambiguity and misinterpretation of ideas (Guce, 2017). The inability to effectively communicate mathematical thinking not only limits students' ability to demonstrate their understanding but also undermines the overall quality of their investigations (Teledahl *et al.*, 2025). These concerns highlight the necessity of examining how students develop and demonstrate mathematical communication skills within investigation contexts.

While there is a wealth of studies on mathematical problem-solving, fewer have explored mathematical investigations in depth. Existing research (Jader, 2022; Stylianides *et al.*, 2022) has primarily examined the general benefits of engaging with mathematical investigation tasks rather than students' actual learning situations. To date, few studies have utilized mixed methods approaches to comprehensively explore the students' mathematical investigation proficiency and the qualitative dimensions of their experience. This gap is significant, as examining the open-ended nature of mathematical investigation, specifically on its components and processes, may contribute to improved mathematical understanding and reasoning, and stronger overall mathematical investigation proficiency among students. Addressing this gap, this study intends to provide social value and support to SDG 4: Quality Education by crafting an enhancement scheme aimed at fostering more responsive curriculum implementation in mathematics, and, particularly important in mathematical literacy and problem-solving, as essential 21st century competencies for participation in knowledge-driven economies.

2. Study Purpose and Research Questions

The purpose of this explanatory-sequential mixed methods study was to investigate the current state of senior high school students' mathematical investigation proficiency. This study aimed to answer the following: to identify the status of mathematical investigation proficiency among senior high school students in Davao Region, to determine if there is a significant difference in their mathematical investigation proficiency when analyzed by gender and SHS strand, to determine the standpoints of senior high school students on their mathematical investigation proficiency, to determine how qualitative data support the quantitative findings, to identify results that emerge from comparing the quantitative

and qualitative data, and to develop an enhancement scheme from the results of the study.

3. Literature Review

3.1 Mathematical Investigation

Mathematical investigation is a process that extends beyond mere calculation or the application of techniques, involving deep exploration and the generation of new understanding. In their 2009 technical report, Yeo and Yeap (2010) characterized mathematical investigation as a process involving four core cognitive activities: specializing, conjecturing, justifying, and generalizing. They distinguished between mathematical investigation as a process and as an activity, noting that the investigative process can occur even when solving problems with a closed goal and answer, whereas an investigative activity involves open tasks that include both problem posing and problem solving. Further, Cai and Cifarelli (2005) characterized mathematical investigation as a cycle of sense-making, exploring, and reflection where learners engage with mathematical ideas in a more open and creative manner. Unlike traditional problem-solving, which often has predetermined paths and solutions, mathematical investigation encourages students to pose their own questions, make conjectures, and develop personal strategies for exploration (Da Ponte, 2007). This investigative approach aligns with Polya's higher-order problem-solving processes but places greater emphasis on student autonomy and mathematical creativity.

The process of mathematical investigation is distinguished by its non-linear nature and emphasis on mathematical processes rather than products. According to Jaworski (2003), mathematical investigation involves posing questions and seeking answers, detecting patterns, making conjectures, constructing arguments to convince oneself and others, and developing a network of related ideas. This characterization highlights how investigation differs from conventional problem-solving. Researchers like Ingram et.al. (2020) indicated that investigations provide opportunities for students to experience mathematics as mathematicians do, engaging in exploratory activities where paths are not predetermined, and multiple approaches are valued.

A critical distinction between mathematical investigation and routine problem-solving lies in the cognitive demands and metacognitive processes involved. Saygili (2017) emphasizes that mathematical investigation requires higher levels of metacognition, where students must monitor their progress, evaluate strategies, and make decisions about direction without external guidance. Similarly, Mason *et al.* (2010) describe mathematical investigation as requiring mathematical thinking that involves specializing, generalizing, conjecturing, and convincing, which are processes that extend beyond the application of algorithms common in standard problem-solving tasks. This view is supported by empirical studies (Andraini et al., 2023; Tachie, 2019; Rukshana *et al.*, 2025), which demonstrate that investigation activities promote deeper conceptual

understanding and more flexible mathematical thinking compared to routine problem-solving exercises that often reinforce procedural knowledge alone.

3.2 Components of Mathematical Investigation Framework

In a study by Nivera (2012), the mathematical investigation assessment framework is composed of three key components: foundational knowledge, investigation process, and communication process. Foundational knowledge pertains to students' grasp of concepts, definitions, facts, procedures, and the identification of misconceptions, which are essential for engaging in mathematical tasks. The investigation process involves analytical and reasoning skills, including identifying patterns, formulating and verifying conjectures, and extending mathematical ideas. Lastly, the communication process emphasizes the clear and logical presentation of mathematical thinking through appropriate language, symbols, and arguments in both written and oral forms.

Mathematical investigation is a complex mental process in which students are expected to investigate, analyze, and find solutions to open-ended mathematical problems using systematic inquiry. The ability of a learner in this process largely depends on their foundational knowledge, which is the basis of the development of problem-solving capacity. As defined by PISA (2021), foundational mathematical knowledge is the knowledge that consists of the essential concepts, procedures, and reasoning skills that form the foundation of further study in mathematics and the application of mathematics in real-life situations. These are number sense, algebraic thinking, measurement, and data literacy, which help in further learning and problem-solving in a diverse range of situations. Moreover, foundational mathematical knowledge refers to the fundamental mathematical concepts and processes, including an idea of the numbers, counting, the basic operations, and patterns, without which higher-order mathematical thought cannot be developed (Sarama and Clements, 2009).

A plethora of research has increasingly emphasized the critical role that prior knowledge and conceptual understanding play in students' ability to engage in meaningful mathematical investigations (Mayasari & Habeahan, 2021; Saygili, 2017). Specifically, Mann and Enderson (2017) reported a strong and educationally significant preference of formula-based methods to real conceptual interaction in student orientations to mathematical problem solving. As their results showed, their students whose problem-solving strategies are based more on the application of rote formulas, as opposed to effective cognition of the underlying mathematical conceptualization, are systematically poorly prepared to solve non-routine or novel problems, because such problems require flexible conceptual thinking, as opposed to the automatic recall of the prescribed algorithms.

Investigative process can be defined as the way analytical and logical reasoning is used by students as they explore, analyze, and solve problems in the context of mathematical investigations. Through analysis and reasoning, students do not just attempt to obtain the right answers. They question why and how solutions have to work and develop further insight into mathematical concepts, as well as learning skills needed

in future higher-order tasks. Non-routine problems are a recurring issue in the literature of mathematical investigation. A preliminary review by Shawan *et al.* (2021) reported challenges students experience when faced with non-routine mathematical problems and found that the main issues are not computational but stem from students' failure to begin analyzing the problem. This failure at the level of analysis is consequential for downstream investigation processes; exploration, conjecturing, and reasoning are inevitably impaired in the absence of a coherent analytical representation of the problem. To apply the same result to the undergraduate level, Nasir *et al.* (2021) studied the problem-solving performance of undergraduate students on non-routine tasks and documented the prevalence of analytical deficiencies, with a significant proportion of participants failing to progress beyond initial comprehension of the problem.

Mathematics contains a special symbolic language, defined by symbols, notations, diagrams, graphs, and succinct logical arguments, that enables the expression of complicated mathematical concepts. These elements are especially essential in problem solving, where clarity, accuracy and logical approach are mandatory to read and answer mathematical relationships and solutions. Halawati and Laelasari (2022) explain this standpoint in some analytical accuracy, positing that the ability of mathematical communication is not a secondary skill that is simply added to mathematical competence, but the ability to receive, process and convey mathematical ideas in any mathematical problem. To be more exact, the ability of students to learn the meaning of mathematical terms, write solutions with proper symbols and notations, and demonstrate mathematical concepts using graphs, charts and illustrations are factors that will lead to the precise formulation of solutions and well-articulated conjectures (Nurlinda *et al.*, 2024). Skilled learners in written mathematical communication are more likely to defend their responses and develop rational arguments, thus making their mathematical arguments more transparent and reasonable (Aini *et al.*, 2020).

3.3 Mathematical Investigation Proficiency by Sex and Strand

The issue of sex differences in mathematics is a recent topic under discussion, as it has been demonstrated that male students perform better than female students on higher-cognitive tasks (Leder, 2019; Becker and Hall, 2024). The global literature supporting this assertion is substantial, but it requires considerable caution to decipher. In their study, Reilly *et al.* (2019) conducted a systematic review of gender differences in mathematics achievement and found that the difference was insignificant. They noted that gender gaps are moderated by cultural and institutional factors rather than biologically determined cognitive differences.

Additionally, Manandhar *et al.* (2022) found no statistically significant difference in students' capacity, irrespective of gender, to relate algebraic processes to underlying mathematical concepts, indicating parity in the acquisition of fundamental knowledge. Similarly, Becker and Hall (2023) documented that boys slightly outperform girls on the most complex procedural items of large-scale tests, but girls often have better conceptual

reasoning, which, nevertheless, affects overall results, which leads to the similarity of the achievements of both sexes in conceptual and procedural realms.

Increasing amounts of research indicate that there are no significant differences in the cognitive strategies that students use in mathematical reasoning along gender lines. The analysis of pattern-finding strategies in solving mathematical problems provided by Cahyadi *et al.* (2023) showed that both genders performed equally in terms of finding and using patterns, and there was no significant difference in the application of these strategies in solving mathematical problems between the two genders. This result is in line with recent studies by Garcia and Cadawas (2025) on gender differences in mathematical reasoning among junior high school students, whose findings revealed no significant difference in mathematical reasoning performance, and both male and female students had relatively high performance.

Moving to the next component, communication, Rusdi *et al.* (2020) did not find a significant difference in mathematical communication skills between male and female students, as both groups were able to explain their strategies and solutions clearly and systematically. In a similar vein, Halawati and Laelasari (2022) found that students from the low ability group had difficulty in describing the question, in using accurate mathematical symbols, and in expressing their conclusions in a logical manner.

Furthermore, the Senior High School curriculum in the Philippines is structured into separate academic and technical strands, each with prescribed emphases and depths of mathematics, and the impact of which on students' mathematical performance is well documented. For instance, a detailed competency analysis of SHS students in General Mathematics by Mamolo (2019) revealed that although all strands were at least at the basic level of operational competence, STEM students were the only ones who were uniformly competent at both the conceptual and procedural levels of the competency framework. In line with these results, Padernal and Tupas (2024) observed a significant difference in students' fundamental mathematics skills when grouped by strand. The strand-differentiated advantage is found across various content areas, but the differences are larger in the level of cognitive demand and the type of task.

A closer look at the strand-level disparity in analytical skills is provided by Cababat and Pespenan (2023), who compared numerical and analytical skills across the Academic and TVL tracks of Grade 11 students at Talisay City National High School. Academic Track (STEM and ABM) students were almost at the proficient level of analytical skills, while TVL students were at the developing level, reflecting basic but limited problem-investigation skills. Adding on, Bautista *et al.* (2021) showed that STEM graduates were significantly better at their tasks involving the construction of proofs, manipulation of variables, and the application of algebraic reasoning to novel problem structures, which are precisely those mathematical activities whose prerequisites are provided by the analytical and investigation-based activities in the STEM SHS curriculum.

The above literature collectively underscores that proficiency in mathematical investigation is a multifaceted competency that extends beyond routine problem-solving.

Effective mathematical investigation requires students to draw upon conceptual understanding and procedural fluency, engage in higher-order cognitive activities, and express their reasoning coherently through appropriate mathematical language and representations. Moreover, contextual factors such as sex and academic strand introduce additional layers of variability in students' performance in mathematical investigations. These insights collectively highlight the need for instructional approaches that deliberately nurture all dimensions of mathematical investigation proficiency across diverse learner contexts, affirming the importance and relevance of the present study.

4. Materials and Methods

This study adopted an explanatory sequential mixed-methods design, which begins with the collection and analysis of quantitative data, followed by the qualitative phase to further explain and interpret the quantitative results (Creswell, 2007). During the quantitative phase, the study utilized a descriptive-comparative research design, which aimed to describe phenomena and to compare differences between or among groups without manipulating variables (Siedlecki, 2020). Descriptive research focuses on presenting a detailed and accurate account of the characteristics of a given population, while comparative research investigates the differences and similarities among variables or groups. This method was well-suited for identifying whether significant differences exist in the mathematical investigation proficiency among senior high school students when analyzed in terms of gender and their SHS strands. In the implementation of this study, a validated mathematical investigation instrument was administered to a purposive sample of students across STEM, ABM, HUMSS, and TVL strands, and their performance was statistically analyzed to detect patterns and variations.

In the qualitative phase, the study employed a descriptive qualitative research design, which aims to present a comprehensive summary of an event or experience in the everyday terms of those events. This type of design is appropriate when researchers seek to describe participants' experiences or perspectives directly, without the imposition of theoretical frameworks or abstract interpretations (Creswell, 2007). As such, the qualitative phase involved interviews and focused group discussions with selected students whose performance in the quantitative phase was notable, either exemplary or struggling. This approach enabled a deeper understanding of the context behind the numerical findings.

For the quantitative phase, the participants were those in Grade 12 who had already taken General Mathematics during their Grade 11. Only those who were able to secure their guardians' or parents' consent were included. For the qualitative phase, six students were part of the focused group discussion (FGD), and ten students were included in the in-depth interview (IDI) based on their performance in the mathematical investigation tasks. The students who were purposefully chosen expressed their willingness to participate in interviews and were available for scheduled sessions. Students who were from public schools or schools outside Region XI, had not taken

General Mathematics in their Grade 11, did not complete the mathematical investigation task, or declined to participate in either phase of the study, were not included.

4.1 Instrument

Quantitative data were collected using the Mathematical Investigative Task questionnaire utilized by Nivera (2012). The questionnaire included three open-ended investigative tasks and an analytic rubric that manually scores three components, namely foundational knowledge, investigation process, and communication. Raw scores were converted into standardized numerical grades through a transmutation table and then interpreted using the descriptors outlined in DepEd Order 8, Series of 2015. Considering the labor-intensive nature of manually checking student outputs in the investigation tasks, the sample size for the quantitative phase ranged between 60 and 120 senior high school students. Based on the key results obtained from the initial phase, a semi-structured interview guide served as the primary instrument. The questions were designed to elicit deeper insights into students' foundational knowledge, investigation processes, and communication as demonstrated in their mathematical investigation outputs.

5. Results and Discussion

5.1 Status of Mathematical Investigation Proficiency among Senior High School Students

Shown in Table 1 is the overall mathematical investigation proficiency among senior high school students, which is 69.61, which is described as very low, indicating that students did not meet the expected level of proficiency in performing mathematical investigation tasks. Specifically, the category Foundational Knowledge has obtained a mean rating of 80.07, which is described as moderate, indicating that students' mathematical investigation proficiency in terms of their foundational knowledge is satisfactory. Notably, the component concepts, facts, and definitions reflect a mean rating of 79.79, and the component misconceptions show a mean rating of 80.47. The category Investigation Process has a mean rating of 63.65, which is very low, indicating that students did not meet the expected standards for mathematical investigation. Notably, the component analysis reflects a mean rating of 64.04, and the component reasoning shows a mean rating of 62.90. The Communication category has a mean rating of 65.86, which is described as very low, indicating that students still did not meet the expected standards for mathematical investigation. In particular, the component arguments reflect a mean rating of 61.35. Meanwhile, the component symbols and notations show a mean rating of 90.16.

Table 1: Status of Mathematical Investigation Proficiency among Senior High School

	Mean	SD	Description
Foundational Knowledge			
Concepts, Facts, and Definitions	79.79	15.47	Low
<i>Use of correct concepts, facts, and definitions</i>			
Procedures and Algorithms	79.95	14.79	Low
<i>Selection and correct performance of appropriate procedures and algorithms</i>			
Misconceptions	80.47	14.14	Moderate
<i>Absence of misconceptions</i>			
Category Mean	80.07	14.31	Moderate
Investigation Process			
Analysis			
<i>Range of depth</i>	64.95	10.47	Very Low
<i>Originality and complexity of problems investigated</i>	63.54	9.87	Very Low
<i>Systematic study of problems</i>	64.17	8.73	Very Low
<i>Verification of solution or conjecture</i>	64.95	11.57	Very Low
Reasoning			
<i>Validity and depth of reasoning</i>	67.14	12.54	Very Low
<i>Quality of the proof presented</i>	62.50	8.43	Very Low
<i>Ability to see connections</i>	59.06	6.08	Very Low
Category Mean	63.65	8.87	Very Low
Communication			
Language Criteria			
<i>Clarity of statements of problems or conjectures</i>	66.46	8.01	Very Low
<i>Clarity of written output of the investigation</i>	74.27	12.18	Very Low
Symbols and Notations			
<i>Correctness of symbols, notations and labels</i>	90.16	7.56	Very High
Arguments			
<i>Use of arguments in the written and oral report</i>	61.35	9.96	Very Low
Category Mean	65.86	9.12	Very Low
Overall Mean	69.61	9.82	Very Low

These results are consistent with the study by Septiyani and Kotimah (2026), who found that students, when asked to solve higher-order geometry questions, are able to understand problems, use and relate new to old concepts, but are still not thorough. Comparable findings were reported by Mayasari and Habeahan (2021), who concluded that students were able to provide partially correct answers to mathematical problems, but had difficulty in applying correct mathematical operations, as well as in explaining their solutions. Furthermore, this study's findings correspond with those reported by Setyahastuti *et al.* (2021), where students were given a geometry reasoning ability test; they were not able to accurately draw conclusions and check the validity of their proofs and arguments. Adding on, Ayuningtyas *et al.* (2019) revealed the same result where the students' reasoning abilities were relatively low, as students still experience difficulty in formulating conjectures, assessing mathematical arguments, and drawing logical conclusions. Relatedly, the very low performance among students in this category further

suggests that students have limited opportunities to represent, discuss, and write arguments and justifications, contributing to weaker communication despite showing better ability to recall concepts and apply procedures (Gardenia *et al.*, 2021).

5.2 Significance of the Difference in the Mathematical Investigation Proficiency When Analyzed by Sex

Presented in Table 2 is the significance of differences in mathematical investigation proficiency, grouped by sex. It is shown that the overall mean in mathematical investigation proficiency of female students (Mean = 69.10) is lower than that of the male students (Mean = 70.21), and both groups are described as very low. It further indicates that both male and female students failed to meet the expected level of mathematical investigation proficiency. Adding on, it is shown that there was no significant difference in their mathematical investigation proficiency when analyzed in terms of sex ($t = -5.48$, $df = 1$, $p > .05$).

Table 2: Significance of the Differences in Mathematical Investigation Proficiency According to Sex

	Sex	Mean	F	p-value	Remarks
Foundational Knowledge	Male	79.74	.050	.824	Not significant
	Female	80.35			
Concepts, Facts, and Definitions					
<i>Use of correct concepts, facts, and definitions</i>	Male	79.55	.092	.762	Not significant
	Female	80.00			
Procedures and Algorithms					
<i>Selection and correct performance of appropriate procedures and algorithms</i>	Male	79.66	.569	.453	Not significant
	Female	80.19			
Misconceptions					
<i>Absence of misconceptions</i>	Male	79.74	.102	.751	Not significant
	Female	80.35			
Investigation Process	Male	64.54	1.447	.232	Not significant
	Female	62.90			
Analysis					
<i>Range and depth of problems investigated</i>	Male	66.02	.299	.586	Not significant
	Female	64.04			
<i>Originality and complexity of problems investigated</i>	Male	64.55	1.097	.298	Not significant
	Female	62.69			
<i>Systematic study of problems</i>	Male	64.66	.647	.423	Not significant
	Female	63.75			
<i>Verification of solution or conjecture</i>	Male	65.91	2.041	.156	Not significant
	Female	64.13			
Reasoning					
<i>Validity and depth of reasoning</i>	Male	68.75	1.072	.303	Not significant
	Female	65.77			
<i>Quality of the proof presented</i>	Male	62.95	1.671	.199	Not significant
	Female	62.12			
<i>Ability to see connections</i>	Male	59.66	.339	.562	Not significant

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	Female	58.56			
Communication	Male	67.10	1.906	.171	Not significant
	Female	64.81			
Language					
<i>Clarity of statements of problems or conjectures</i>	Male	64.54	1.447	.232	Not significant
	Female	62.90			
<i>Clarity of the written output of the investigation</i>	Male	67.39	.419	.519	Not significant
	Female	65.57			
Symbols and Notations					
<i>Correctness of symbols, notations, and labels</i>	Male	90.91	.638	.426	Not significant
	Female	89.52			
Arguments					
<i>Use of arguments in the written and oral reports</i>	Male	62.73	4.392	.039	Significant
	Female	60.19			
Mathematical Investigation Proficiency	Male	70.21	.511	.477	Not significant
	Female	69.10			

In terms of foundational knowledge, the t-test result confirms that there is no significant difference in the foundational knowledge between the male and female students, as reflected by the p-value = .835, which is greater than the alpha value set at the .05 level of significance. Notably, it is supported by tests of significant difference in the subcomponents of concepts, facts, and definitions, procedures and algorithms, and misconceptions, with p-values ranging from .453 to .824, and are greater than the .05 level of significance. Regarding investigation process, no significant difference was observed across all subcomponents of both analysis and reasoning ($p > .05$). The empirical findings indicate no significant difference in the investigation process between male and female students ($p > .05$). Adding on, the results for communication show that in all subcomponents of language, symbols and notation, and argument, the male students consistently got higher category mean scores than the female students. Regarding statistical significance, no significant differences were observed across all subcomponents of language, and symbols and notation, except argument ($p > .05$), with all p-values exceeding the 0.05 significance level.

This study's finding aligns with those of Abletinger *et al.* (2023), who found no differences among students when their procedural knowledge was assessed, a result echoed by Lenz and Wittmann (2020), who similarly reported no significant difference in the conceptual and procedural knowledge between male and female high school students assessed on their basic knowledge in mathematical concepts. Supporting this pattern, Pina *et al.* (2021) concluded that male and female students show comparable performance when assessed for basic mathematical knowledge, reinforcing the absence of sex differences in mathematical performance. Along similar lines, Jiang (2021) noted no significant differences between male and female students in their mathematical abstraction and logical reasoning. Kadarisma *et al.* (2019) also reported that both male and female students demonstrate comparable capacities in analytical thinking when presented with mathematical problems. In addition, Firdiani and Herman (2019) showed

that students, regardless of sex, demonstrate the same ability to communicate mathematical ideas orally and in writing. Moreover, Rusdi *et al.* (2020) reported that both male and female students were equally capable of identifying what was asked, using appropriate mathematical notations and symbols in presenting their written solution and arguments.

5.3 Significance of the Difference in the Mathematical Investigation Proficiency When Analyzed by Strand

It is shown in Table 3 that the overall mean in mathematical investigation proficiency among senior high school students, when analyzed according to their SHS strand, ranges from 63.67 to 75.33. Specifically, the GAS strand reflects a mean rating of 63.67, described as very low, while the STEM track garners a mean rating of 75.33, also described as low. It also indicates that the mathematical investigation proficiency of students enrolled in the TVL, GAS, HUMSS, and ABM tracks are interpreted as not meeting the expectations, while only students from the STEM track have been determined to have fairly satisfactory mathematical investigation proficiency. Additionally, the analysis shows that there is a significant difference in the mathematical investigation proficiency among senior high school students when analyzed in terms of their strand ($F = 7.139$, $df = 4$, $p > .05$).

	Strand	Mean	F	p-value	Remarks	Post Hoc
Foundational Knowledge	TVL	70.10	11.694	.000	Significant	TVL and STEM TVL and ABM GAS and STEM GAS and ABM HUMSS and STEM HUMSS and ABM
	GAS	70.00				
	HUMSS	74.22				
	STEM	86.67				
	ABM	89.07				
Concepts, Facts, and Definitions						
<i>Use of correct concepts, facts and definitions</i>	TVL	66.18	13.116	.000	Significant	TVL and STEM TVL and ABM GAS and STEM GAS and ABM HUMSS and STEM HUMSS and ABM
	GAS	70.83				
	HUMSS	74.71				
	STEM	86.60				
	ABM	90.00				
Procedures and Algorithms						
<i>Selection and correct performance of appropriate procedures and algorithms</i>	TVL	68.23	13.100	.000	Significant	TVL and STEM TVL and ABM GAS and STEM GAS and ABM HUMSS and STEM HUMSS and ABM
	GAS	69.58				
	HUMSS	74.71				
	STEM	86.80				
	ABM	89.60				

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Misconceptions						
<i>Absence of misconceptions</i>	TVL	69.58	7.861	.000	Significant	TVL and ABM GAS and STEM GAS and ABM HUMSS and STEM HUMSS and ABM
	GAS	73.23				
	HUMSS	75.88				
	STEM	86.60				
	ABM	87.60				
Investigation Process	TVL	58.74	6.12	.000	Significant	TVL and STEM GAS and STEM HUMSS and STEM
	GAS	58.99				
	HUMSS	62.65				
	STEM	69.63				
	ABM	63.93				
Analysis						
<i>Range and depth of problems investigated</i>	TVL	60.60	3.130	.018	Significant	TVL and STEM
	GAS	60.83				
	HUMSS	63.53				
	STEM	70.20				
	ABM	65.60				
<i>Originality and complexity of problems investigated</i>	TVL	56.47	11.857	.000	Significant	TVL and STEM TVL and ABM GAS and STEM GAS and ABM HUMSS and STEM
	GAS	55.42				
	HUMSS	62.94				
	STEM	71.20				
	ABM	65.00				
<i>Systematic study of problems</i>	TVL	59.71	4.261	.003	Significant	TVL and STEM GAS and STEM
	GAS	60.42				
	HUMSS	64.12				
	STEM	69.20				
	ABM	64.00				
<i>Verification of the solution or conjecture</i>	TVL	59.41	5.593	.000	Significant	TVL and STEM GAS and STEM HUMSS and STEM
	GAS	59.17				
	HUMSS	62.35				
	STEM	72.60				
	ABM	65.60				
Reasoning						
<i>Validity and depth of reasoning</i>	TVL	60.88	5.216	.001	Significant	TVL and STEM GAS and STEM
	GAS	59.58				
	HUMSS	66.18				
	STEM	74.60				
	ABM	68.20				
<i>Quality of the proof presented</i>	TVL	57.65	5.577	.000	Significant	TVL and STEM GAS and STEM
	GAS	58.75				
	HUMSS	62.06				
	STEM	68.00				
	ABM	62.40				

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<i>Ability to see connections</i>	TVL	56.76	3.784	.007	Significant	TVL and STEM STEM and ABM
	GAS	57.80				
	HUMSS	57.94				
	STEM	62.80				
	ABM	58.75				
Communication	TVL	61.62	3.722	.007	Significant	TVL and STEM GAS and STEM
	GAS	61.98				
	HUMSS	65.96				
	STEM	70.85				
	ABM	65.56				
Language						
<i>Clarity of statements of problems or conjectures</i>	TVL	61.76	4.132	.004	Significant	TVL and STEM TVL and ABM
	GAS	64.17				
	HUMSS	64.41				
	STEM	70.00				
	ABM	68.60				
<i>Clarity of the written output of the investigation</i>	TVL	67.65	5.815	.000	Significant	TVL and STEM TVL and ABM HUMSS and STEM
	GAS	67.92				
	HUMSS	70.59				
	STEM	81.00				
	ABM	77.60				
Symbols and Notations						
<i>Correctness of symbols, notations, and labels</i>	TVL	92.94	3.146	.028	Significant	
	GAS	86.67				
	HUMSS	86.47				
	STEM	92.60				
	ABM	90.00				
Arguments						
<i>Use of arguments in the written and oral reports</i>	TVL	58.53	3.656	.008	Significant	STEM and ABM
	GAS	57.92				
	HUMSS	64.41				
	STEM	66.20				
	ABM	58.00				
Mathematical Investigation Proficiency	TVL	63.38	7.139	.000	Significant	TVL and STEM TVL and ABM GAS and STEM GAS and ABM HUMSS and STEM
	GAS	63.67				
	HUMSS	67.41				
	STEM	75.33				
	ABM	72.47				

The outcomes of this study echo the findings of several studies (Cerbito, 2020; Mamolo, 2019) establishing differences in the mathematical performance among senior high school students. Specifically, Padernal and Tupas (2024) found a significant difference in the fundamental knowledge of mathematics among students when grouped according to strand. Specifically, differences were found in estimation, pattern identification, and problem-solving, where STEM students lead in these areas. Similarly, Pespeñan and Cababat (2023) assessed the numerical and analytical skills of Grade 11 senior high school students in solving mathematical problems and reported that STEM and ABM students

were already proficient, while TVL students were generally still at the developing level. This implies that while some students possess basic computational knowledge, their conceptual understanding is still insufficiently developed to support more complex mathematical investigations, particularly among non-academic strands.

In terms of the investigation process, particularly analysis and reasoning, the results also revealed significant differences across strands. Students from STEM and ABM strands tend to demonstrate stronger analytical and reasoning skills compared to those from TVL, GAS, and HUMSS. This finding aligns with Almerino *et al.* (2020), who reported that STEM and ABM students obtained significantly higher scores in mathematical reasoning and overall mathematical capacity. Likewise, Guttierrez (2021) found that students from academic strands performed better in General Mathematics tasks involving reasoning and analytical thinking, with significant post hoc differences observed between academic and technical-vocational tracks. Supporting this, Guil *et al.* (2017) noted that students enrolled in academically rigorous strands such as STEM and ABM, exhibit greater gains in reasoning and problem-solving skills compared to those in other strands.

Regarding mathematical communication, the findings further revealed significant differences across strands. Students from STEM and ABM strands generally demonstrated better performance in articulating solutions and presenting logical arguments compared to those from TVL, GAS, and HUMSS. This is supported by Bautista *et al.* (2024), who found significant differences in students' knowledge of mathematical concepts and processes, as well as the use of mathematical communication, between STEM and non-STEM students, with STEM students obtaining higher mean scores across domains. Similarly, Baron (2022) reported significant differences in General Mathematics performance among ABM, HUMSS, and SMAW (TVL), with post hoc results showing that ABM students significantly outperformed SMAW students.

Consequently, the results of the post hoc analysis further confirm that mathematical investigation proficiency varies significantly across strands, with specific pairwise differences observed among TVL, GAS, ABM, STEM, and HUMSS. In general, students from STEM and ABM strands consistently outperformed those from TVL, GAS, and HUMSS, highlighting disparities in mathematical investigation proficiency. This finding is supported by Ragma and Cagas (2018), who reported variations in the foundational knowledge and higher-order problem-solving skills across tracks, with STEM and ABM students obtaining the highest mean scores, while HUMSS and ICT students obtained lower scores. Similarly, Parcon and Bearneza (2024) found that TVL students significantly differed from students in academic strands in terms of verbal and mathematical skills. Dela Cruz and Rivera (2022) reported that students in academic tracks, STEM and ABM, consistently outperformed their TVL counterparts in analytical problem-solving tasks. These results corroborate the present findings and suggest that students enrolled in academically intensive strands tend to develop more advanced mathematical abilities compared to those in non-academic strands.

5.4 Standpoints of the Participants on the Salient Points of Quantitative Results

Shown in Table 4 are the standpoints of the participants on the quantitative results regarding their mathematical investigation proficiency. The essential themes generated are as follows: Confirmed low level of mathematical investigation proficiency, Confirmed significance of the difference in the Mathematical Investigation Proficiency when analyzed by sex, and Confirmed Significance of the difference in the Mathematical Investigation Proficiency when analyzed according to SHS strand.

Table 3: Standpoints of the Participants on the Salient Points of Quantitative Results

Quantitative Results	Essential Themes	Typical Reasons
Level of Mathematical Investigation Proficiency	Confirmed a very low level of Mathematical Investigation Proficiency	Relying on formula-based solving rather than investigative reasoning
		Struggling with open-ended and explanation-based tasks
		Experiencing limited exposure to investigative activities
		Needing teacher scaffolding and illustrative examples
		Having challenges in comprehension and sustained focus
Significance of the difference in the Mathematical Investigation Proficiency when analyzed by sex	Disconfirmed significance of the difference in the Mathematical Investigation Proficiency when analyzed by sex	Demonstrating similar exposure to mathematics instruction
		Experiencing similar struggles in higher-order reasoning tasks
		Showing performance differences linked to study habits rather than gender
		Attributing confidence levels to motivation and effort
		Observing classroom parity in participation and task completion
Significance of the difference in the Mathematical Investigation Proficiency when analyzed by SHS strand	Confirmed Significance of the difference in the Mathematical Investigation Proficiency when analyzed by the SHS strand	Receiving different levels of analytical training across strands
		Experiencing curriculum emphasis differences
		Showing variation in confidence based on strand orientation

	Preferring structured instruction over exploratory tasks
	Having differences in prior math preparation and academic exposure
	Encountering difficulty in translating reasoning into written explanations
	Demonstrating dependence on teacher-led explanation

5.4.1 Confirmed Very Low Level of Mathematical Investigation Proficiency

The respondents confirmed the very low mathematical proficiency of senior high school students, hence validating the quantitative results. The participants shared their experiences and reported preferring formula-based problem-solving strategies to open-ended tasks. Their narratives revealed their exposure to procedural solving and formula application, which shaped how they approached unfamiliar mathematical investigative tasks. The findings align with recent studies indicating that students exposed to predominantly procedural instruction often struggle with non-routine and open-ended mathematical tasks (Quetrani & Achtaich, 2022; Kurniawan *et al.*, 2023). Adding on, Bingölbali and Bingölbali (2021) revealed that lower performances were observed in students when asked to answer open-ended questions, as students could not completely provide their arguments and generalizations. Relatedly, Wahab *et al.* (2024) pointed out that students who were less exposed to problem-solving activities show weak mathematical communication, more specifically in the devising of the plan stage, and in modeling and organizing mathematical ideas.

5.4.2 Disconfirmed Significance of the Difference in the Mathematical Investigation Proficiency when Analyzed by Sex

Participants confirmed the quantitative finding that differences exist, but noted that these differences were not inherently attributed to sex itself. Instead, both male and female students reported similar learning experiences and difficulties with higher-order mathematical tasks, suggesting that proficiency differences may stem from factors beyond sex. Hyde and Mertz (2009) demonstrated through a large-scale analysis that gender differences in general mathematics performance were nearly negligible, with no observable differences in conceptual understanding at any age level. Complementing this, Kersey *et al.* (2019) provided neuroimaging evidence showing that boys and girls exhibit statistically equivalent levels of brain development underlying mathematical reasoning, suggesting that the foundational cognitive processes for mathematics are shared across genders. Further reinforcing this, Wienclaw (2023) in her study on gender and mathematical abilities concluded that observed differences, where they exist, are more likely a reflection of socialization and learning environment rather than inherent differences in their proficiency.

5.4.3 Confirmed Significance of the Difference in the Mathematical Investigation Proficiency when Analyzed by SHS Strand

The participants affirmed that students' level of mathematical investigation proficiency varies across academic strands. Their statements consistently pointed out differences in learning experiences, context exposures, and instructional emphasis on their specific strands. This finding aligns with the study of Aguba and Villacruel (2023), who pointed out that the curriculum emphases and learning experiences students get in each of their respective strands can determine how students handle higher-order mathematical tasks. Multiple studies have documented that students who had higher exposure to mathematics-related content were found to be more confident when solving cognitively challenging tasks involving analysis and reasoning (Granada & Bacus, 2025; Sibaen, 2022). In a similar vein, Iringan (2019) reiterated that students participating in mathematics-intensive strands are more likely to have stronger skills in analytical thinking because they typically solve complex mathematical tasks on a more frequent basis. Equally, the qualitative findings reinforce the quantitative findings from international assessments, demonstrating that students who are actively involved in performing higher-order thinking tasks perform better in solving problems involving analysis (Mullis *et al.*, 2020).

5.5 Joint Display of Quantitative and Qualitative Results

Illustrated in Table 4 is the joint display of quantitative and qualitative results. This contains the research area, quantitative and qualitative results, and the nature of integration. In the research area column, the status of the variable of the study, the significant difference of the results when analyzed in terms of strand and gender. The qualitative results column displays the participants' responses as core ideas from Table 3, which show confirmation of the quantitative results column. The nature of the data integration column indicates the type of mixing used in the study, which reveals that the qualitative findings align with and support the quantitative findings.

Table 4: Joint Display of Quantitative and Qualitative Results

Research Area	Quantitative results	Qualitative Results	Nature of Integration
Mathematical Investigation Proficiency	The overall result of the status of Mathematical Investigation Proficiency among Senior High School Students in Davao Region is 69.6, described as very low.	The participants confirmed that mathematical investigation proficiency among senior high school students is very low.	Merging-Confirmation
Significance of the difference in the Mathematical Investigation Proficiency when analyzed by sex	The results show that there is no significant difference in the Mathematical Investigation Proficiency among Senior High School students when analyzed by sex.	The participants confirmed that both the male and the female senior high school students perform the same level in	Merging-Confirmation

		mathematical investigation tasks.	
Significance of the difference in the Mathematical Investigation Proficiency when analyzed according to SHS Track/Strand	The results show that there is a significant difference in the Mathematical Investigation Proficiency among Senior High School students when analyzed by SHS track. Post hoc analysis using Tukey indicated that significant differences exist between the following strand pairs: TVL and GAS, TVL and ABM, GAS and STEM, GAS and ABM, and HUMSS and STEM ($p < .05$), highlighting variations in proficiency across strands.	The participants confirmed that students' mathematical investigation proficiency varies across strands, with some strands having greater exposure to analytical subjects, demonstrating higher proficiency.	Merging-Confirmation

5.5.1 Merging-Confirmation of the Status of Mathematical Investigation Proficiency among Senior High Schools

The combined presentation shows a merging-confirmation integration of the quantitative and qualitative facts about the overall level of skill of mathematical investigation. Based on the quantitative data, the mathematical investigation proficiency of senior high school pupils is very low. This outcome indicates that, in most cases, the level of proficiency in carrying out mathematical investigations was below the expected level, especially on those tasks that involved the application of foundational knowledge, investigative process and mathematical communication. The qualitative data were supported by the fact that the participants acknowledged that they had poor mathematical investigation skills. Mathematical investigation tasks that require several steps, more substantial reasoning and succinct explanations in writing, students recognized that they were struggling. A large number of the participants claimed to use and count on formula memorized instead of mathematical justifications or exploration reasoning.

These results are in line with the research of Andal and Andrade (2022), who stated that there is still a significant number of students who are increasingly adept at procedural knowledge over investigative or conceptual reasoning. Equally, Wahab *et al.* (2024) indicated that a significant proportion of students are less accustomed to activities that actively demand them to explore, which are essential to the acquisition of more analytical thinking and investigative skills.

5.5.2 Merging-Confirmation of the Significance of the Difference in Mathematical Investigation Proficiency According to Sex

The combined display also shows a merging confirmation integration regarding the difference in mathematical investigation proficiency when analyzed in terms of sex. The qualitative data corroborated the findings that neither male nor female students met the expected skill level in mathematical investigation. Participants explained that when it

came to accomplishing mathematical problems, particularly those that required extensive reasoning and explanation, students of both sexes encountered comparable challenges.

These findings are consistent with a large body of literature indicating that gender differences in mathematics ability can decrease if equal learning opportunities and enhanced learning conditions are provided (Maamin and Iksan, 2021; Avotina *et al.*, 2023). Data from international assessments show no universal differences in mathematical performance, suggesting that the gap varies in contexts and not because of gender-related characteristics (Reilly *et al.*, 2019). Gender does not inherently determine students' ability to perform mathematical operations; rather, differences may result from differences in attitudes, self-confidence, or learning strategies (Rodriguez *et al.*, 2020). Similarly, Marifa *et al.* (2025) found that gender had no identifiable impact on students' understanding of mathematical problem-solving or their academic performance, highlighting the fact that male and female students exhibit comparable mathematical investigation abilities.

5.5.4 Merging Confirmation of the Significance of Difference in Mathematical Investigation Proficiency According to SHS Strand

The combined data support the integration of the large variation in mathematical investigation proficiency when analyzed according to their SHS strand. The participants' responses in the qualitative phase of the study show their acknowledgement of the different performance levels of students across different academic strands in mathematical investigation tasks.

The participants' narratives are in consonance with the conclusions of Szabo *et al.* (2020), who posited that the development of students' mathematical investigation skills are greatly influenced by several factors, such as academic exposure and curriculum orientation. Additionally, Siregar (2025) reiterated that the mathematical reasoning and investigative skills among students are further improved by practice and exposure to analytical and problem-based learning environments. Exposure to rich mathematical tasks and sustained practice enhances students' cognitive processes involved in investigation, thereby minimizing performance differences across groups (Liu *et al.*, 2024; Smit *et al.*, 2023)

6. Conclusions and Recommendations

The overall mathematical proficiency of senior high school students is very low, as students were unable to meet the expected level of mathematical investigation proficiency and reported difficulties with analysis, interpreting mathematical situations, and reasoning. While no significant difference in proficiency was found between sexes, with both male and female students facing similar challenges in complex mathematical tasks, suggesting that classroom experiences play a greater role than sex, a marked disparity was observed across SHS strands. Proficiency levels varied considerably across strands, with STEM students demonstrating notably higher performance. Post-hoc

analysis confirmed significant pairwise differences among strands, and qualitative data further supported that unequal exposure to mathematical concepts and analytical thinking across strands is a key factor driving these differences.

Extensive and intentional exposure of students to mathematical investigations will lead to significant improvement in the development of higher-order mathematical thinking among senior high school students. Hence, the EXPLORE enhancement scheme is proposed.

I. Title: *EXPLORE (Enhancing Proficiency through Learning Opportunities in Reasoning and Exploration)*

II. Rationale

EXPLORE is an enhancement scheme developed to foster cooperation between mathematics teachers, curriculum developers and school administrators to empower mathematics investigations competencies of students. The enhancement scheme aims to help students across all strands of senior high school gain the analytical and investigative skills they need to enter tertiary education and develop lifelong mathematical literacy.

Specifically, the EXPLORE intervention aims to:

- 1) increase the exposure of students to inquiry-based, exploratory, and non-routine mathematical investigation tasks by using structured and scaffolded instruction, and
- 2) encourage equal teaching and uphold a balanced learning environment that offers equal opportunities to both sexes and among strands to tackle mathematical investigation tasks.

III. Proposed Enhancement Scheme

Key Result	Enhancement Objective	Strategies/ Intervention	Specific Activities	Success Indicators	Persons Involved	Time Frame
<i>Low overall mathematical investigation proficiency</i>	Improve the overall mathematical investigation proficiency of SHS students, particularly in solving non-routine and investigative mathematical tasks	Strengthen exposure to inquiry-based, exploratory and non-routine mathematical tasks through structured and scaffolded instruction	Mathematical Investigation Tasks	Increased student scores in investigation tasks; improved classroom participation in exploratory activities; more students able to justify and explain solutions Improved students' skills in planning	Mathematics teachers, Master Teachers, Academic Coordinator, SHS students	1 school year, with quarterly review

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				solution steps and explaining reasoning; reduced tendency to rely solely on memorized formulas		
<i>No significant difference when analyzed by gender</i>	Sustain equitable learning opportunities for both male and female students in mathematical investigation	Maintain gender-inclusive and bias-free instructional practices	Math Trails	Comparable participation and performance trends between male and female students are maintained	Mathematics teachers, advisers, school administrators	Whole school year
<i>Significant differences in mathematical investigation proficiency across SHS strands/tracks</i>	Minimize disparities in mathematical investigation proficiency among strands by providing strand-responsive support	Differentiate instruction and intervention according to strand needs without lowering standards	Jigsaw Puzzle through Mathematical Investigation	Narrowed performance gap across strands; improved scores among lower-performing strands	Mathematics teachers, SHS coordinators, department head	Diagnostic at start of term Interventions every month Evaluation every quarter
<i>Students from strands with greater exposure to analytical subjects tend to perform better</i>	Provide comparable opportunities for analytical thinking development among non-STEM strands	Embed reasoning tasks and mathematical thinking routines in regular instruction for all strands	Situational and Logic Mathematical Questions	Increased engagement and analytical performance among non-STEM strands; improved confidence in handling mathematical investigations	Mathematics teachers, subject teachers from other learning areas, SHS coordinator	Weekly and monthly throughout school year

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