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ENHANCING SUBJECT MATTER KNOWLEDGE IN PRESERVICE MATHEMATICS TEACHERS: TRANSFORMING MATHEMATICS EDUCATION THROUGH PROBLEM-POSING INTERVENTIONS

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Abstract:

Mathematics education is making a shift towards learner-centredness with the implementation of standards-based curricula, with the subtle recognition of problem-posing as a transformative strategy in most jurisdictions, and Ghana is no exception. Although problem-posing is acknowledged as promoting conceptual understanding, it is hardly a pedagogical instructional strategy recommended for use in Ghana's basic educational system. This discrepancy is worrying, given the increasing demand for inquiry-based teaching and learning. The study sought to investigate how problem-posing intervention influenced preservice teachers' subject matter knowledge within the standards-based curriculum. The study employed the pragmatic paradigm, with a quasi-mixed one-group pretest-post-test case study design utilising a mixed-methods approach grounded largely in a qualitative approach that allowed an in-depth understanding of how problem-posing influenced the 25 participants' subject matter knowledge.

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Quantitative data were collected and analysed using a survey and the SOLO taxonomy, while the qualitative data were collected through interviews and analysed using thematic categorisation from the research questions. The results showed that most participants posed low-level problems while acknowledging the transformative role of the problem-posing intervention on their subject knowledge. The study concludes with a discussion of its limitations and recommendations.

Keywords: intervention, mathematics preservice teachers, problem-posing, standards-based curriculum, subject matter knowledge

1. Introduction

In today's rapidly changing global knowledge landscape, educational systems, especially mathematics education, are shifting towards learner-centred and inquiry-based approaches that emphasise critical thinking, creativity, and problem-solving (NCTM, 2000; MoE, 2019). Problem-posing has become a transformative strategy that encourages both educators and students to formulate and explore authentic mathematical problems, moving beyond routine exercises to address real-world challenges. This approach promotes analytical reasoning and cognitive flexibility (Unlu, 2017; Lavy & Shriki, 2007). Ghana's basic school mathematics education reforms reflect this global trend by prioritising the development of strong mathematical competencies aligned with international standards (Buabeng *et al.*, 2020; Mereku, 2019). Mathematics proficiency is seen as essential for future opportunities, while deficiencies in these skills create barriers. Ensuring that every student can attain a deep understanding of mathematics is crucial (MoE, 2019; NCTM, 2000).

However, improving problem-solving ability is to be capable of problem-posing (Unlu, 2017; Abu-Elwan, 1994). For a very long time, researchers have identified that developing one's problem-posing abilities is crucial to solving good problems (Lavy & Shriki, 2007; Kilpatrick, 1987). It is established from the foregoing that the key aim of the new basic school mathematics curriculum is to equip learners with the ability to think critically, be creative, and be problem solvers (MoE, 2019). However, there cannot be problem solvers without great problem posers (Einstein & Infeld, 1938). Since there is convincing evidence that problem-posing not only influences students' achievements but also improves their capabilities of critical thinking and creativity (Leavy & Hourigan, 2020), the Ghanaian pre-service teacher should be well-resourced to pose good and insightful problems in the classroom. It is noteworthy that most preservice teachers lack the requisite subject-matter knowledge in mathematics education, going out to teach (Ibrahim & Yew, 2023), which is worrying for developing critical thinking ability in students. The study, therefore, examined the impact of problem-posing intervention in developing preservice teachers' subject-matter knowledge (SMK) in teaching the new basic mathematics curriculum.

1.1 Statement of the Problem

Although problem-posing is a vital skill that enhances both teachers' and students' mathematical understanding by encouraging deeper engagement with content (Cai & Leikin, 2020), preservice mathematics teachers often graduate with little or no exposure to problem-posing strategies and lack confidence in applying them in classroom settings (Ticha & Hošpesová, 2009). This is further exacerbated by the fact that most curricula are silent on making references to it, and therefore, little is known about how problem-posing tasks are integrated into textbooks, lesson preparations and delivery (Baumanns & Rott, 2025).

The integration of inquiry-based learning within the standards-based curricula (SBC) raises specific concerns in the context of the Ghanaian basic school mathematics curriculum. Existing literature notes some of the difficulties that preservice teachers face, especially concerning their subject matter knowledge, when trying to incorporate problem-posing into their teaching (Ibrahim & Yew, 2023). This study, therefore, aims to explore the impact of a problem-posing focused intervention on preservice teachers' knowledge of mathematics and their ability and self-confidence to use problem-posing techniques within the standards-based curriculum of the basic school mathematics. The research aims to contribute to more effective teacher preparation programmes that align with contemporary, student-centred pedagogical demands of the National Council for Curriculum and Assessment [NaCCA] (2020).

Therefore, the purpose of the study is to analyse the expressed impact of a problem-posing intervention on preservice teachers' subject-matter knowledge in teaching the basic school mathematics curriculum, concerning the inquiry-based demand of the new basic school mathematics curriculum through the preservice teachers' problem-posing experiences. And so, the potential implication of the study was to make a case for integrating problem-posing into the teaching and learning of mathematics at the basic schools and the colleges of education.

1.2 Research Questions

The overarching question was, how does a mathematical problem-posing intervention influence preservice mathematics teachers' subject-matter knowledge? The following research sub-questions guided the study in this direction:

1) How has a problem-posing intervention influenced preservice mathematics teachers' understanding and confidence in specific mathematical concepts within the SBC?
2) In what ways do problem-posing interventions reveal gaps in preservice teachers' mathematical understanding while simultaneously enhancing their subject matter knowledge?
3) What were the initial challenges preservice teachers encountered, and how has the intervention assisted them in overcoming them?

1.3 Significance of the Study

The findings would promote the development of conceptual understanding in mathematics education by identifying and addressing knowledge gaps among preservice teachers. By identifying knowledge gaps in preservice teachers, the findings would provide an opportunity to support struggling learners through structured interventions. Furthermore, the study's findings will contribute to global mathematics education research. Ultimately, the findings have strong implications for 21st-century skill development of the preservice teacher in mathematics education.

2. Literature Review

2.1 Problem-posing in Mathematics Education

Problem-posing can be seen as the process of interpreting given situations and creating meaningful mathematical problems based on the experiences of students and teachers in real-world contexts (Zhang *et al.*, 2024). This indicates that mathematical problem-posing involves creating something new from a data set that leads to authentic mathematical inquiry. Consequently, a review of the literature by Zhang *et al.* (2024) identified problem-posing in different forms – as a logical process, a cognitive activity, an instructional tool, or a learning goal that helps learners develop their understanding of tangible situations. In their write-up, Lee *et al.* (2020) consider problem-posing as an approach to engage learners in real-world and open-ended contexts to develop critical thinking, problem-solving and collaborative skills. In this study, mathematical problem-posing is regarded as a cognitive activity undertaken by preservice mathematics teachers to generate new problems or reformulate (Baumanns & Rott, 2021) existing problems through real-world contexts, while recognising the transformative influence of problem-posing interventions on their subject matter knowledge. This is necessary because, according to a review of Rosli *et al.* (2014), preservice teachers need not only to "*think mathematically but also creatively when reformulating and generating new mathematical problems*" (p. 227), which leads to personal and academic benefits.

However, challenges of problem-posing persist in its effective implementation for students and preservice teachers alike, as well as in-service teachers. In their review, Roli *et al.* (2014) identify that teachers who do not have the requisite skills find it difficult to engage in it. Since mathematics preservice teachers have to teach mathematics effectively, they need to develop the potential to pose good problems. Studies also show that despite problem-posing's significance being touted, participants often pose problems that are nonmathematical, irrelevant, unsolvable, unclear or contain errors (Joaquin, 2023; Cai & Hwang, 2002). Research shows that some of these challenges are due to preservice teachers' lack of foundational knowledge in problem-posing (Zhang *et al.*, 2024; Divrik, 2023) and subject matter knowledge; hence, this study aims to understand how problem-posing experience enhanced preservice teachers' subject matter knowledge.

Problem-posing intervention studies examine the influence of teaching mathematics with problem-posing strategies and report an improvement in students' problem-posing skills (Divrik, 2023). An experimental study on using problem-posing techniques to teach mathematics focused on the developmental impact on students and concluded that the students' ability to problem-posing improved. Kopparla *et al.* (2018) aimed to assess the growth in problem-posing and solving skills among learners from low socio-economic backgrounds. Their results showed a statistically significant improvement in both problem-posing and solving. Krawitz *et al.* (2025) investigated the effects of problem-posing on modelling performance, self-efficacy, and task values in solving modelling problems among 210 ninth- and tenth-grade students. These findings explained the influence of encouraging students to create their own problems on their self-efficacy and task value on the mathematical concepts, and this, in turn, improves their value in cognitive and non-cognitive modelling tasks. Similarly, the studies of Akay and Boz (2010) examined 82 elementary preservice teachers' attitudes towards mathematics and their mathematics self-efficacy, with findings showing that the experimental group demonstrated a more positive attitude towards mathematics. However, these studies do not consider the transformative role of problem-posing in the subject matter knowledge of preservice teachers; hence, this present study.

2.2 Subject Matter Knowledge for Preservice Mathematics Teachers

The importance of subject matter knowledge (SMK) cannot be overstated in the teacher preparation process. Subject matter knowledge, as understood from Ma's (1999) point of view, is a teacher's profound understanding of mathematical concepts, principles, procedures and the interrelatedness among them, as well as the skills of reasoning to justify these reasons. Thus, subject matter knowledge is beyond simply knowing facts, but understanding the 'why' behind the mathematical ideas and their links across various topics, and possibly their applications in real-world scenarios. This allows mathematics teachers to represent the subject effectively to learners. The study of Mae (2019) found that SMK is crucial as the foundation for pedagogical knowledge (PK). It found that while neither SMK nor teaching knowledge alone is sufficient for effective teaching, there was little evidence of PK being effective without strong proficiency in the subject matter knowledge. This attests to the belief that SMK is a critical aspect of preservice teacher knowledge that supports students' learning and achievement (Peerzada & Jabeen, 2014) and has a direct influence on how they design learning opportunities for students (Jadama, 2014). According to Mae (2019), teachers who are proficient in content and capable of solving unfamiliar, non-routine, and complex problems are significantly more likely to design higher-level tasks and accurately identify higher levels of student thinking. This means, conversely, that those with weaker subject matter knowledge design lower cognitive task demands and misinterpret higher levels of student thinking. Although the discussion presents subject matter knowledge in mathematics teaching in varied forms as in common content knowledge, specialised knowledge and horizon content knowledge (Briand-Newman *et al.*, 2012), knowledge of content and teaching, and knowledge of content and students (Lee *et al.*, 2018), and conceptual and procedural knowledge (Rittle-Johnson, 2019) the intervention centred on conceptual knowledge.

This is because proficiency in mathematics delivery requires multiple knowledge, and conceptual knowledge (subject matter knowledge) is most fundamental to all (Kilpatrick *et al.*, 2001). And since problem-posing is known to refine preservice teachers' subject matter knowledge (Lee *et al.*, 2019), the study employs a problem-posing intervention that reveals gaps in their SMK to support them in transforming their SMK to align with the standards-based curriculum.

Empirical studies on developing subject matter knowledge in preservice teachers include Ndlovu *et al.* (2017), who explored the common and specialised content knowledge of 59 final preservice teachers' written responses, categorised by competencies, on solved problems involving functions, inequalities, and the interpretation of learners' errors. The results show that while the participants demonstrated competence in solving problems, they were unable to analyse and interpret learners' errors for diagnostic purposes. Sam *et al.* (2023) investigated 79 preservice teachers' content knowledge in teaching Algebra using a quantitative descriptive survey. While the results showed their content knowledge to be superficial, the preservice teachers' low level of interest, among other factors, was said to account for this. This study investigates the self-reported experiences of how problem-posing revealed the preservice teachers' knowledge gaps and deepened their understanding of some mathematical concepts.

2.3 Problem-posing Deepens Conceptual Understanding

Engaging in problem-posing helps develop and deepen preservice teachers' understanding (Yao *et al.*, 2021; Yeo & Lee, 2022; Irshid *et al.*, 2023). According to the studies of Yao *et al.* (2021), there is a significant difference in the types of mathematical understanding exhibited by preservice teachers between the drawing task and the problem-posing task. This is because the problem-posing task more frequently elicited conceptual responses, suggesting that engaging in problem-posing encourages preservice teachers to exhibit conceptual understanding. In furtherance of this, the findings indicated problem-posing as conducive to developing preservice teachers' conceptual understanding or that preservice teachers were inclined to use that understanding, which is why engaging in it could be of significance to understanding how problem-posing can help in transforming the current study's participants' subject matter knowledge.

In deepening conceptual understanding, problem-posing emphasises justification and posing a good problem. According to Irshid *et al.* (2023), "*carrying out tasks that promote justification and problem solving*" (p. 8) encourages teachers to engage students in activities that require them to explain their reasoning and find solutions, rather than merely memorising procedures. This aligns with the ideals of NaCCA (2020) that tasks should be the primary element used to motivate learning and enable students to develop new mathematical knowledge in problem-solving. An important aspect of this is the ability of the learner to make representations and understand the concept on multiple levels that include visual, symbolic and real-life. By so doing, it would build procedural fluency from conceptual knowledge (Irshid *et al.*, 2023) to ensure that teachers could help learners apply learned knowledge flexibly and fluently in real-life contexts. The implication of their study (ibid) is that deepening understanding fosters a "community of practice" of teachers where they learn from one another, leading to improved performance of the teachers and their students.

2.4 Bridging Knowledge Gaps through Problem-posing

In understanding how problem-posing helps to transform preservice teachers' subject matter knowledge, it would serve as a valuable tool for gaining insights into preservice teachers' mathematical understanding (Yao *et al.*, 2021) that might not be revealed by other such measures. Thus, problem-posing provides an opportunity to understand preservice teachers' conceptions of mathematical ideas more fully. According to Yao *et al.* (2021), problem-posing is effective in revealing specific misunderstandings that would not be evident from just computations, stressing that while preservice teachers might be procedurally correct in their arithmetic, they may lack conceptual understanding, which problem-posing helps to uncover. Most of the preservice teachers showed proficiency in fraction division. However, 99% of them who did the fraction division lacked a conceptual understanding of it (ibid). So, problem-posing could be an effective tool for diagnosing and assessing preservice teachers' mathematical understanding (Yeo & Lee, 2022) as it offers a "*window into students' thinking*" (Yao *et al.*, 2021, p. 2) and can help teachers assess students' understanding of mathematics. This study explores their expressions of the types of incorrect problems preservice teachers pose while providing more open opportunities for expressing their thinking.

Problem-posing is effective in identifying limited understanding of mathematical processes of preservice teachers because there is a considerably less fine-grained understanding of how students and teachers go about posing mathematical problems (Tangkawsakul *et al.*, 2024), contrasting the well-framed approach of problem-solving processes. Thus, problem-posing is capable of identifying and understanding knowledge gaps since it has often been used as an assessment tool to evaluate students' and preservice teachers' understanding of mathematical concepts, rather than being investigated to understand the problem-posing process itself. It would therefore assist in investigating its role in preservice teachers' subject knowledge transformation.

To bridge this gap, Tangkawsakul *et al.*'s (2024) review on problem-posing emphasises the need for developing a problem-posing process model analogous to Polya's problem-solving model. This is identified to that effect because problem posers' knowledge and prior familiarity with the content play a significant role in the problem-posing process (ibid). This implies that pedagogical approaches must target enhancing this knowledge and understanding to bridge these gaps.

2.5 Overcoming Conceptual Challenges in Problem-posing

Problem-posing has been shown to provide opportunities for both students and preservice teachers to reveal their mathematics thinking and understanding (English,

2020) and to overcome their conceptual challenges (Irshid *et al.*, 2023). According to Otun (2022), preservice teachers frequently misunderstand key mathematical terms and phrases in word problems and face difficulties in translating verbal expressions into mathematical statements and equations. This implies that most preservice teachers lack some foundational understanding of the relatedness of mathematical content. This is therefore likely to increase procedural and arithmetic errors due to a misunderstanding of given tasks. However, such challenges can be cured through effective use of strategies like 'solve-reflect-pose' (Otun, 2022) that enhance preservice teachers' knowledge. According to Otun, this helps preservice teachers to better understand students' conceptions and misconceptions to improve their word problem analysis, the results of which are positive effects on their flexible procedural and conceptual knowledge.

Problem-posing exposes preservice teachers' lack of subject matter knowledge in how often they struggled with insufficient mathematical content knowledge, which significantly impacts their ability to formulate problems effectively. For instance, a lack of understanding of concepts like combination led to difficulties in posing mathematically valid problems (Erkan & Kar, 2022). The result is that preservice teachers tend to formulate easier and simpler problems than the given ones or change the context superficially without altering the underlying mathematical structures. Some of the conceptual challenges faced by preservice teachers include difficulty in making sense of problem-posing, limited ability to increase problem complexity and struggles with metacognitive skills (Erkan & Kar, 2022). Some of which indicate a conceptual challenge in self-regulated learning and reflection (Otun, 2022) on their problem-posing strategies.

In overcoming these challenges, Erkan and Kar (2022) identified from their studies that the majority of their preservice teachers initiated the problem formulation process by first finding the solution to the given problem. Initially, solving the problem allowed them to establish an equation, which could be adjusted by changing numbers or symbols to formulate new problems. Or associating problems with daily life contexts as a key instructional factor, as it provides ideas for using mathematical concepts and operations to enhance students' learning. One important consideration was to pose problems at the level of students to improve their understanding. This involves a steady increase in the difficulty level of the problems posed or replacing given and unknown elements to challenge students.

2.6 Standards-based Curriculum and Problem-posing Pedagogy

Mathematics problem-posing is increasingly becoming central to many mathematics standards-based curricula worldwide, as it is regarded as an essential skill all students should develop (Divrik, 2023). The new standards-based mathematics curriculum in Ghana explicitly involves learners in problem-solving (MoE, 2019) but remains somewhat silent on problem-posing. However, for decades, it has been recognised that great solutions cannot be achieved without posing appropriate problems (Einstein & Infeld, 1938). Despite this recognition and the call to incorporate problem-posing into mathematics teaching in classrooms across the United States (NCTM, 2020), problem-posing has rarely been integrated and practised within many standardised curricula, such as those in Ghana (MoE, 2019) and Germany (Bauman & Rott, 2023). The shift to the new mathematics standards-based curriculum places considerable demands on teachers in basic schools. Teachers need to be prepared to adapt to these changes, reconsider their teaching approaches, and employ methods that foster exploration and logical reasoning (Chen, 2022) rather than rote reception. However, the prevailing approach to teaching mathematics in basic schools appears to be regressing towards a more teacher-centred style than before (Nkansah, 2021), characterised by limited or no classroom discussions and interactions.

In light of the foregoing discourse, integrating problem-posing in the new mathematics standards-based curriculum appears appropriate to assist preservice teachers in promoting teaching approaches that foster the skills of problem-solving that the new curriculum sought to achieve. However, from Nkansah's (2021) studies, it is only purposefully implementing problem-posing curricula that can foster in "*students … critical thinking, and the ability to solve complex real-world problems*" (p. 177). Since the new curriculum sought to engage learners to become enquirers rather than receptors of knowledge (MoE, 2019), preservice teachers should be trained to live by it, but not to be left with the choice to implement the problem-posing. Even though Muirhead *et al.* (2025) demonstrated that teacher-intentionality could help teachers modify textbook problem-solving tasks to create new opportunities for students to engage in problem-posing, it can only yield positive results if they are given in-service training. With this, problem-posing can help teachers foster mathematical thinking and sense-making of mathematical concepts among students. This would allow students to explore problem situations, activate prior knowledge and identify mathematical features, while helping teachers to make instructional decisions on how and when these problems will be examined, discussed or solved.

Implementing problem-posing and problem-posing pedagogy in standards-based curriculum can be daunting, even so, when it has not been explicitly stated in the curriculum. In countries where problem-posing is explicitly recommended, the experience of the death of problem-posing in mathematics curriculum materials (Muirhead *et al.*, 2025; Bauman & Rott, 2023) is due to significant limitations of the scarcity of problem-posing activities in these widely used materials. Secondly, there is a disconnect between policy and practice due to misalignment between policy recommendations and actual curriculum materials (Muirhead *et al.*, 2025). It is important to understand the policy-practice gap that, while policies emphasise problem-posing, the instructional materials fall short in providing the needed support for implementation (ibid). This makes this study very important in raising awareness that problem-posing intervention is critical in transforming the preservice teachers' subject matter knowledge in implementing the standards-based curriculum by helping learners become complex problem solvers.

2.7 Theoretical Framework

Because the main aim of studying mathematics is to enable learners to examine their worldview (Gutstein, 2006), it highlights the importance of problem-posing in mathematics teaching and learning. In effect, critical pedagogy is regarded as the appropriate perspective through which the study is conducted, based on Freire's (2005) Pedagogy of the Oppressed. It is believed that mathematical learning should not be a passive activity where learners are mere recipients of knowledge, but rather active participants, making inquiry and critical thinking essential. In critical pedagogy, students are trusted to reason and contribute to the learning process, not just receive knowledge. Therefore, students are challenged through posed problems instead of being simply led into them, in order to develop their thinking skills. Consequently, preservice teachers need to possess the skills to pose insightful problems that stimulate both collaborative interactions and individualised cognition among learners.

Since critical pedagogy emphasises transformative learning and education, it tends to awaken in preservice teachers an awareness of the power of problem-posing as a tool for transformation. This is because problem-posing is a form of critical pedagogy that teachers use to identify the conditions within a problem rather than merely solving it for students (Shin & Cookes, 2005). Through this process, learners are encouraged to reflect and generate knowledge, making it necessary that, with the new standards-based curriculum, the problem-posing approach influences preservice teachers' subject matter knowledge in a transformative manner, preparing them to implement the curriculum effectively. Only by giving learners a voice can they become creative (Yulianto, 2015); they can then develop into transformed individuals who are capable of critical thinking. Therefore, by mastering problem-posing, preservice teachers will be better equipped to teach mathematics effectively, ultimately enhancing their ability to foster student engagement.

3. Methodology

3.1 Research Design

The study employed a pragmatic paradigm, focusing on practical rather than theoretical approaches (Cohen *et al.*, 2018) and thereby tried to understand the experiences of the 25 preservice mathematics teachers through a mixed-methods design grounded largely in a qualitative methodology. The qualitative aspect allowed for an in-depth understanding of how the preservice mathematics teachers develop in subject-matter knowledge. The research design was an experimental quasi-mixed one-group pretest-post-test case study design (Cohen *et al.*, 2018; Leedy & Ormrod, 2015) utilising a mixed-method (McChesney & Aldridge, 2019; Dolan *et al.*, 2023). The quasi-mixed design was one in which both qualitative and quantitative data were collected, but might not be combined in answering one particular research question (Cohen *et al.*, 2018).

3.2 Study Participants

The study involved 25 preservice teachers in their third year, specialising in mathematics education for Junior High School (JHS). Participants were chosen through a convenience purposive sampling method (Creswell, 2009) and joined voluntarily (Li *et al.*, 2020). The group comprised 13 males and 12 females, with an average age of 22.8 years, although one participant was an outlier at 15 years. All participants had completed core courses in mathematics and pedagogy and gained two years of hands-on experience through the Supported Teaching in Schools (STS) programme, equipping them with a solid grasp of the JHS curriculum.

3.3 Data Collection and Analysis

Data collection was through interventional test item activities, a self-reported survey, and was followed by interviews. The intervention occurred for one and a half hours, twice a week, for six weeks. All who failed to attend the intervention session more than four times, in addition to failing to attend the interviews, were disengaged. The participants were engaged in pre-activity tasks, which were the same as the post-activity tasks. However, they were not exposed to the tasks at the beginning and the end of the intervention to control for variability in responses. The results from the tasks form the basis for the interviews that followed to explore the emerging themes. The selected tasks were based on the mathematics teacher education programme and the basic school (SBC) mathematics curriculum to meet both interests. The tasks were adapted from research sources (Cai & Hwang, 2022, p.321; Crespo, 2003, p.254; Silver & Cai, 2005, p. 130; Singer *et al.*, 2011, p.155) and exam bodies like the West African Examination Council [WAEC] (2022, 2015), and therefore were considered to have past the piloting test. The Cronbach's reliability test of the survey instrument was found to be strong (Taber, 2018) at 0.789.

The test items from the interventional activities were analysed using Leung's (2013) classification to identify the soundness of problems posed, followed by the SOLO taxonomy (Biggs & Collis, 1982) to determine the levels at which these problems occurred (Appendix A). Semi-structured interviews were conducted with each of the participants and audio recorded with their consent. The interviews lasted between 48 minutes and 1 hour and 35 minutes. They were transcribed into a Word document using *Nota.ai*. The data analysis involved thematic and content analysis, employing both inductive and deductive techniques. To maintain anonymity, participants were identified as PST1, PST2, …, PST25. The themes were developed by first categorising the research questions and major interview questions into the main themes (Divrik *et al.*, 2019). These were complemented by themes developed by using 'dynamic software' (Ruthven, 2019, p.2) because such "*Software for analysis of qualitative data… offers significantly more flexibility in working with data than manual methods, including the capacity to ask … complex, questions of the data*" (Andrew & Halcomb, 2009, p. 98). While data from the survey were analysed using SPSS version 23 to determine how the participants reported the shift in their subject matter knowledge over time due to the intervention.

3.4 Trustworthiness and Reliability

To ensure the study's credibility, in-depth interviews lasting between thirty-five to over fifty minutes were conducted, incorporating direct quotes from participants. Major interview questions were reiterated before presenting the key themes. To enhance data dependability, expert guidance was sought from supervisors regarding the assessment instruments and feedback from colleagues on the lesson plan and evaluation methods. A thorough data description supported the transferability of findings, organising responses by emerging themes without presenting conflicting viewpoints. Interventional activities were structured according to the SOLO taxonomy and Leung's (2012) classification. Consistency was maintained through detailed explanations of the data collection and analysis procedures. To increase confirmability, researchers set aside their biases and considered input from experienced mathematics teacher educators. This collaboration occurred during the analysis of recorded lesson observations and interventional activities, resulting in an inter-rater reliability of 83.72% for the pre-test and 98.21% for the post-test. Finally, the quantitative instrument's reliability was found to be strong (Taber, 2018) at 0.789 using Cronbach's alpha.

4. Results

4.1 Demographic Analysis

The participating preservice teachers consisted of twenty-five people. Their gender balance of 13 females to 12 males can control for a common bias in educational research such as this. An added complexity in the sample was the 15-year-old participant, whose age was otherwise at the lower end, and whose "*social situation of development*" (Vygotsky, 1978) would have likely added novel insights to the data.

4.2 Responses to the Items from the Intervention

In answering the research questions, the interventional activities of the six test items (see APPENDIX B) were classified according to Leung (2012) and the SOLO taxonomy. Three of them requested participants to pose problems based on scenarios presented (items 1a, 1b, and 2b); two involved word problems to be transformed (items 1c and 2c), and one involved number investigation (2a). All these activities were selected based on the standards of the SBC. Items 1c, 2a, and 2c are indirect or implied problem posing situations. Their responses were categorised and presented in Table 1. The results from Table 2 indicated a slight improvement in the attempt of participants to properly transform the items. Number investigations were seen as the most challenging item for them. For the symbolic representation, many struggled with the task, while in the number investigation, unresolved attempts dominated, confirming the participants' expressed difficulty with it. Even though there had been a notable jump in the number of one-variable transformed story problems, the overall performance had not been huge, but the total effect of 0.84 was very impactful (Lakens, 2013) in achieving the study's objectives of how the intervention influenced their expressed views on how problem-posing could transform their subject matter knowledge.

Table 1: Pre- & Post-Interventional Responses of Posed Items [1C, 2A and 2C]

Items **Responses**		**Symbolic Representation**		**Number Investigation**		**Story Problem in One Variable**		**Total**	
		Pre-	**Post-**	**Pre-**	**Post-**	**Pre-**	**Post-**	**Pre-**	**Post-**
Resolved		1	3	0	1	4	11	5	15
Unresolved	Attempt	1	4	9	12	2	3	12	19
	Unattempt	23	18	16	12	19	11	58	41
Total		**25**		**25**		**25**		**75**	

Even though a lot more attempted to do the 'number investigation' during both the pre- and post-tests as against the 'symbolic representation' and the 'story problem in one variable', they could hardly resolve it than the others, indicating the difficulty encountered in it as against the rest. This could be that they saw only ordinary numbers rather than creative investigations of numbers.

Table 2 shows participants' responses on all six items classified according to their mathematical soundness according to Leung's (2012) classification.

Table 2: Classified Posed Problems based on Leung's (2012) Classification

Description **Activity**	**Sufficient (extraneous information)**	**Insufficient Inappropriate information**	**Impossible**	**Non-Math**	**Not a Problem**	**Total**
Pre-Activity	81(37.5%)	36(16.67)	16(7.41)	20(9.26)	63(29.16)	216
Post-Activity	133(45.24%)	37(12.59)	15(5.10)	8(2.72)	101(34.35)	294

The overall increase in the number of problems posed after the intervention suggested a heightened engagement post-intervention. The increased number in 'Not a problem' post-intervention could be attributable to the confidence participants claimed to have mustered afterwards. However, most of the items created turned out not to be appropriate. An 8%-point increase in 'Sufficient' problems indicated a promising problem posing skill gained, though.

The SOLO taxonomy was used to rate posed problems from the participants to determine the level at which the problems were posed. Tables 3 and 4 indicate the pre- and post-intervention activities of the participants, respectively.

Table 3: SOLO-Level Classified Pre-Intervention Activity [1A, 1B & 2B]

Level / Content Area	PSL	USL	MSL	RL	EAL	Total
Number Operations	9	29	12	14	4	68
Linear Equation of One Unknown	9	2	15	5	1	32
Travel Graph	6	4	5	4	0	19
Total	**24**	**35**	**32**	**23**	**5**	**119**

Note: PSL - pre-structural level, USL - unistructural level, MSL - multi-structural level, RL - relational and EAL - extended abstract levels.

Table 4: SOLO-Level Classified Post-Intervention Activity [1A, 1B & 2B]

Level / Content Area	PSL	USL	MSL	RL	EAL	Total
Number Operations	2	29	8	15	7	61
Linear Equation of One Unknown	4	10	15	7	2	38
Travel Graph	13	20	9	9	1	52
Total	**19**	**59**	**32**	**30**	**10**	**151**

Note: PSL – pre-structural level, USL – unistructural level, MSL – multi-structural level, RL – relational and EAL- extended abstract levels.

From Tables 3 and 4, it can be seen that although there was a slight improvement in higher-order level thinking (RL and EAL – 3%-points increase), low-level thinking (PSL and USL) still characterised the performance of the participants with a 2.1%-point increase. The results indicated that, for number operations, although the intervention helped, their foundational gaps persisted; performance in linear equations did not improve significantly, and the travel graph highlighted the most challenging area for attention. The results, notwithstanding, placed the participants in a good position to express their experiences with the problem-posing intervention, of how it transformed their subject matter knowledge, by tagging the intervention as very effective.

In analysing the qualitative aspect, three major themes were identified from the research questions. The first question was 'How has a problem-posing intervention influenced your mathematics understanding and confidence in specific mathematical concepts within the SBC?', with the categorised theme as 'Deepening conceptual understanding through problem-posing'. The second question was, 'In what ways do problem-posing interventions reveal gaps in your mathematical understanding while simultaneously enhancing your subject matter knowledge?' with the categorised theme as 'Identifying and bridging knowledge gaps through problem-posing'. The third being 'What were the initial challenges you encountered, and how has the intervention assisted you in overcoming them?' with its developed theme as 'Overcoming conceptual challenges in problem-posing'. These initial categorisations were followed by emerging subthemes developed using a 'dynamic software' tool (Ruthven, 2019, p.2) and summarised in the content analysis, as shown in Figures 1, 2, and 3, respectively.

4.3 Deepening Conceptual Understanding Through Problem-posing

Most preservice teachers (PSTs) reported that the problem-posing activities helped them deepen their conceptual understanding by requiring them to actively construct, analyse, and reflect on mathematical concepts such as ratio/proportion and fractions that they engaged in. *"Yeah, at first with fractions like this, I didn't know much about it, but with the problem-posing, I've been able to know how to go about it"* (PST2); and *"I have a better grasp of fractions now"* (PST18). PST6 stated that, *"the problem-posing strategy has influenced me positively, concerning ratios and proportions. Before that topic was being introduced to me, I wasn't given the opportunity to think by myself; the teacher would just come and then say, 'This is to this, this is to that'"*. PST19 expressed how problem-posing corrected misconceptions about fractions that *"back then, when I was in JHS, I thought with a fraction, when the denominator is bigger than the other denominator, let's say, one-fourth, comparing it to one-half, I thought one-fourth was bigger than the one-half because of the denominator difference, but now I'm good to understand that the figure of the denominator doesn't mean it's higher"*. This encouraged PSTs to rethink denominators.

Participants expressed how cognitive engagement through critical thinking and reflective problem analysis was a key mechanism by which problem-posing deepens their conceptual understanding. PST1 intimated that problem-posing revealed shallow prior understanding and promoted broad, flexible thinking about some concepts, the *"problem-posing intervention has influenced my understanding and concepts in mathematics in such a way that it has made me realise that I don't have a deep understanding of some concepts: rather, I have a shallow understanding, …and I've been able to think widely around most of the concepts that we've done"*. Thereby encouraging reflective thinking. PST14 illustrated how *"I was really having issues with the word problem, how to just break it down and solve it. But during the intervention, it really helped me to understand it very well"*. Breaking down word problems into mathematical sentences fosters analytical skills. The problem-posing promoted critical thinking (e.g. PST10, PST13, PST18) because as according to PST18, *"it helps me develop my critical thinking"*. The reflections showed that problem-posing facilitates cognitive engagement by allowing the preservice teachers to progress from rote memorisation and develop understanding with meaning.

The problem-posing intervention performed several roles by promoting conceptual mastery, reflective practice, and bridging theory to real-world relevance. Preservice teachers shared their experience of the transition from learners of a surface model to critical thinkers. This is supported by Table 6 on their subject matter knowledge before and after the intervention. Table 6 shows the results of the self-reported survey with a 5-point Likert scale rating as 'strongly disagree – 1', 'disagree – 2', 'undecided – 3', 'agree – 4', and 'strongly agree – 5'.

Even though Table 5 showed increases in subject matter knowledge of the PSTs after the intervention, there is still a concern about their confidence in solving and explaining complex problems relevant to the SBC curriculum to students, indicated by a marginally increased mean from 2.92 to 3.72 as against the fact that they had a strong grasp of the fundamental concepts in the SBC curriculum of an increased mean to 4.04 from 2.84.

Table 5: Survey Results on Subject Matter Knowledge (SMK)

Statement	Pre-Intervention		Post-Intervention		N
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**	
I had a strong grasp of the fundamental concepts in the JHS curriculum	2.84	1.2138	4.04	0.5385	25
I am confident in solving and explaining complex problems relevant to the JHS education	2.92	0.9539	3.72	0.6782	25
I can accurately apply mathematical theories to real-world scenarios encountered in the classroom	3.04	0.8406	4.08	0.6403	25
I consistently update my understanding of core mathematical principles aligned with the current JHS education	2.84	1.1060	3.60	0.7638	25
I can effectively identify common misconceptions in teaching basic mathematics at JHS	3.60	1.2583	4.16	0.6245	25
I can effectively address common misconceptions in basic mathematics at the JHS	2.88	1.0536	4.08	0.7024	25
I feel well-prepared to design problems that convey essential mathematical concepts to students at the basic schools	2.04	1.0198	4.00	1.0408	25

Addressing misconceptions and conceptual gaps is a fundamental way that problem-posing deepens conceptual understanding; thus, PST1 said, "*It has made me realise that I don't have a deep understanding of some concepts; rather, I have a shallow understanding.*" "*Problem posing ... made me understand ratio and proportion deeper. ... I thought I was having an immediate answer, but ... I realised that I do not know much. ... but problem posing made me compare quantities of the same unit, and I've been able to think widely around most of the concepts that we've gone through.*" PST4: "*Through problem posing, I've been able to understand ... fractions ... it's about things that are around us. ... I wasn't able to connect representing a situation by unknown variables [in algebra], but after the intervention, I can.*" PST7, "*At any time I'm given a fraction, and it ends up as a negative, I normally ignore the negative and take the positive. I've been using this for some time now, but I didn't really get the idea behind it. But during the problem-posing section, ……, [I] observed that it doesn't make sense to say a parcel of land is negative.*"

Problem-posing encourages the development of flexible thinking and mastery of conceptual language, which is crucial for deepening conceptual understanding. Participants described how, through open-ended questions, they were able to shift from rigid, closed, and static problem-solving strategies to a broader and more flexible view. For example, according to PST20*, "this is very interesting since I wasn't able to pose a problem, but through this intervention, I have been able to pose a question on a particular topic, for example, fractions, number operations and graph work (travel graph)"*, and *"The problem-posing helped me to overcome these gaps because I was asked to pose my own problems. So, I understood what I was doing better than the teacher telling me"* (PST6).

In all these, however, a unique perspective emerged from PST22 that problem-posing did not reveal gaps in their understanding but rather served to deepen existing knowledge, when they answered "No" to whether problem-posing did not revealed gaps in their knowledge. This contrasts with other participants who reported that problem-posing helped identify and address misconceptions or conceptual gaps. This insight highlights that the impact of problem-posing may vary among individuals, serving different roles depending on their prior knowledge and learning context.

4.4 Identifying and Bridging Knowledge Gaps through Problem-posing

In addressing how problem-posing impacted PSTs' subject matter knowledge, the data showed that problem-posing served as a powerful diagnostic tool that explicitly revealed participants' knowledge gaps across various mathematical topics. Participants noted that engagement in problem-posing activities helped them recognise areas where their understanding was incomplete or superficial (PST24, PST1, PST12, PST13). For example, PST1, *"… as a result of a probing question that was asked. The question made me think…. I thought I was having an immediate answer, but upon giving out the answer I was having, I realised that no… the question had a lot of things that I do not know. Instead of the immediate answer I gave, I need to go back and sit down and think about it. That's where I realised that I do not know much, and the problem-posing has opened that gap in me"*. *"But from the problem posing class, I was able to understand the main concept and able to see different approaches that I can use to solve a ratio question"* (PST15). These expressions suggested that the problem-posing actively engaged participants to recognise their knowledge limitations, making it an explicit and effective method for identifying gaps in mathematical understanding.

Again, the problem-posing facilitated bridging knowledge gaps among the PSTs by fostering reflective and critical engagement with mathematical content. It was observed that through the problem-posing, participants' ability to analyse problems was enhanced, as they could consider varying difficulty levels and connect concepts to real-life contexts. For example, PST24 described how problem-posing helped them pose problems based on scenarios and difficulty levels, scaffolding their learning, *"I've been able to understand that there are situations that we can pose problems from, based on the difficulty levels"*, and that the problem-posing *"has helped me to gain insightful knowledge on how to go about posing problems based on ratios and then how to be able to solve them accordingly"*. PST5 illustrated how careful analysis of visual problems before posing questions helped overcome challenges, *"now when I see a visual problem like that, I make sure to go through what is written first and then look at the picture very well before trying to start to pose". PST15: "I now analyse word problems by deducing equations from real-world situations."* And *"arranging* *books on the shelves and dot patterns helped me see math in daily life"* (PST5). Problem posing bridged theory and practice, especially for fractions (PST4, PST19) and algebra (PST15), fostering relevance. This expressed ability of applying mathematics to real-world contexts through problem-posing fostered contextualised learning, moving preservice teachers beyond rote procedures (e.g., PST3's algebraic representations) to meaningful applications (e.g., PST4's fractions as division of tangible objects). These expressions of problem-posing demonstrated metacognitive awareness, strategic thinking, and iterative practice, which are essential for effectively bridging knowledge gaps.

The interview results indicated that cognitive strategies such as self-questioning and problem decomposition are central to how problem-posing identifies and bridges knowledge gaps. *"But as a result of problem-posing, especially the open-ended questions, it has helped me to see that the question that I am asked, I should see that it's not always fixed, I shouldn't be static in my thinking but I should be flexible so that I can think around the questions, and that helped me to do a lot of research into almost every question that I am asked"* (PST1). PST24 described posing problems based on scenarios, problem difficulty level and scaffolding, *"it has helped me to pose problems according to the difficulty level, which is easy, moderate and then the difficult aspect"*. These strategies foster metacognitive awareness and critical thinking, enabling the preservice teachers to uncover implicit gaps and scaffold their learning effectively through problem-posing.

Further, iterative practice and collaborative problem-posing were seen to be essential in boosting the participants' confidence and effectively bridging knowledge gaps. For instance, PST8 stated that *"Initially, I struggled with it, but I solved the question before coming up with the problem myself. Through the problem-posing intervention, I gained ideas on how to formulate my own problems effectively"*.

Despite its benefits, problem-posing faced challenges and limitations in effectively bridging the knowledge gaps for some participants. For example, PST22 revealed that *"For me to pose a problem on ratio and proportion, like the scenarios given to us, it was very difficult for me to pose problems on that"*, because *"I could not understand the scenario well, so I couldn't pose problems on that"*. Some participants were challenged by their foundational misunderstanding of problem-posing and mathematical content: *"I don't get the foundation well about the problem-posing, … I mean the foundation in mathematical content"* (PST13). Thus, *"I wasn't getting the understanding to pose a problem from the graph"*, showing little or partial knowledge bridged. However, they were desirous to grow into it with time, *"but now, I've a little understanding, and as time goes on, I continue to learn, so I become fully mature to understand the problem posing well"* (PST13). In all told, participants demonstrated a shift in knowledge acquisition on problem-posing.

4.5 Overcoming Conceptual Challenges in Problem-posing

Even though the problem-posing intervention had *"helped me"* (PST25), overcoming the understanding of scenario barriers was a critical challenge to some PSTs. Participants explicitly expressed their difficulties in understanding scenarios, especially involving graphs (diagrams) and word problems, as a major obstacle to formulating meaningful problems (PST12, PST18, PST22). For instance, PST22 expressed their frustrations, saying that *"I could not understand the scenario well, so I couldn't pose problems on that"*. However, most of the participants described how problem-posing interventions helped them overcome these barriers, *"first of all, when I see a visual problem like that, I make sure to go through … and then look at the picture very well first before trying to start to pose a question about it"*. Or else, one *"[has] to sit down, look at the scenario [given], and then analyse the question" (PST19)*, calling for consistent studying of the given scenarios to be able to pose problems on it (PST4, PST15).

Participants reported bridging their conceptual understanding and application of problem-posing to overcome the challenges in posing problems. The intervention had helped them to translate their understanding of mathematical concepts into the skill of formulating meaningful problems, as in *"pose a problem based on the scenario, so that it can be in a sentence form…. it has helped me to pose problems based on the difficulty level, which is easy, moderate and then the difficulty aspect"* (PST24). Participants expressed their ability to contextualise problems (PST11, PST12). PST11*, "I was able to link the question to real-life situations and also link the question to different topics in mathematics. I linked them to ratio and proportion… to the probability aspects"*, and to clarify between *"problem-posing and problem-solving"* (PST13). The participants identified that the intervention encouraged them to deliberately do reflective thinking, *"One of the concepts that I have been finding challenging to pose problems is an algebraic expression where the questions will be put in words that you've been asked to transform into an equation like 'four times'…. I interchange them, because of problem-posing and my ability to pose problems has made me know that you don't have to rush when you are given a problem but you have to be able to think through the lines, look at the right terms like, sum, times, meaning an operation in mathematics, and an unknown variable should replace by what an unknown quantity should be replaced by an alphabet. It has helped me get the things that I need to put in to replace the specific words that are used" (*PST1*)*.

Correcting misconceptions and enhancing critical thinking are key outcomes of problem-posing interventions in overcoming conceptual challenges. For instance, PST3, *"It was quite confusing, …. But after I got to understand the concept of problem-posing and using it, frankly speaking, I didn't have any challenge with it"*. PST14*, "I was really having issues with the word problem, how to just break it down and solve it. But during the problem-posing intervention, it really helped me to understand it very well"*. PST10 said, *"You have to first understand what the question is talking about. Yes, and with the problem-posing, it really helped me to understand what exactly the content or the question is talking about, and with that, I was able to make some problems from the scenario being given"*.

Linguistic and language challenges are a notable conceptual barrier in problem-posing. Participants explicitly report that limited language skills restrict their ability to formulate moderate or difficult problems, for example, PST11 stated, *"I was only able to pose easy questions... but I couldn't write a moderate or difficult question because I didn't have more language about how to pose that question"*. This limitation highlights the importance of expanding PSTs' linguistic and conceptual repertoires to enable richer problem formulation. Problem-posing is a powerful tool to transform learners into active constructors of knowledge, correct misconceptions, enhance critical thinking, and minimise linguistic and language barriers, which could improve PSTs' skills essential for effective problem formulation.

5. Discussion

The study sought to find out how problem-posing intervention influenced the participants' mathematical subject matter knowledge. The findings indicated that most participants showed increased confidence in posing a lot more appropriate problems after the intervention, but as Joaquin (2023) and Cai and Hwang (2002) noted, they mostly posed problems that were nonmathematical, irrelevant, unsolvable, unclear or contained errors, thereby posing only an 8-point increase of 'sufficient' problems over their previous posed problems. This is suggestive that although problem-posing might be seen as a cognitively fruitful engagement (Baumanns & Rott, 2021), they still do not grasp the skills necessary to pose 'sufficient' problems for students' engagements. And since problem-posing promotes mathematical creativity that fosters a positive attitude toward mathematics (Zhang *et al.*, 2024), the mathematics preservice teachers need support in this direction.

The analysis of the levels of problems posed showed that though the intervention had some influence on the participants, their foundational gaps persisted, with interpretational problem posing (travel graph) being the most challenging for them. These difficulties, demonstrated in posing higher-order level thinking problems, were the concerns of Mae (2019) that teachers who show proficiency in content and solving nonroutine complex problems were more likely to design higher-order tasks and appropriately identify higher levels of students' thinking. Consequently, those with weaker subject matter knowledge would most probably design lower cognitive tasks and misinterpret higher levels of students' thinking. It is therefore important to carry all preservice mathematics teachers along the higher pedestal of problem-posing.

Again, the preservice teachers reported how the problem-posing intervention had deepened their subject matter knowledge and increased cognitive engagement through critical thinking and reflective problem analysis, because "*I would say, it has made me more knowledgeable about fractions*" and "*it helps me develop my critical thinking*". It was obvious that this was a key mechanism by which problem-posing deepened and refined their conceptual understanding (Lee *et al.*, 2019). The problem-posing intervention acted as a versatile tool, improving understanding and encouraging reflective practice while connecting theory to real-world applications. So, "*It has made me realise that I don't have a deep understanding of some concepts; rather, I have a shallow understanding*." Thus, helping in tackling misconceptions and filling in conceptual gaps to deepen understanding, supported by Cai and Hwang (2021) that this would help in shifting the preservice teachers' attitude towards a more active and inquiry-oriented teaching style. It therefore demonstrated how problem-posing fosters flexible thinking and mastery of conceptual vocabulary, essential for deepening understanding of mathematics. While the intervention revealed knowledge gaps in some participants, it deepened an existing knowledge of others. In all this, the intervention had a positive influence on the participants' subject matter knowledge, as shown by the survey ratings, which ranged from 2.04 in one instance to 4.16 in another, on a Likert scale of 1 to 5.

Through the intervention, problem-posing had turned out to be an incredible diagnostic tool for learning because "*from the problem-posing class, I was able to understand the main concept and able to see different approaches that I can use to solve a ratio question*." This resonated with Cai *et al.* (2020), that problem-posing enhances the teachers' abilities to formulate, evaluate, and implement viable educational assignments. Endowed with this ability, it would encourage the PSTs to engage more reflectively and critically with the mathematical content, leading to insights like, "*I now analyse word problems by deducing equations from real-world situations*." Through techniques like self-questioning and problem decomposition, preservice teachers can see how problem-posing helps identify and bridge knowledge gaps in their understanding (Zhang *et al.*, 2024).

The findings showed how the "*questions made me think" and the problem-posing "had a lot of things that I do not know*", and that is where "*I realised that I do not know much, and the problem-posing has opened that gap in me*". These expressions echo the findings of Yao *et al.* (2021) that problem-posing is, in fact, effective in revealing misunderstandings that would ordinarily not be revealed by mere computations, consequently helping the preservice teachers to "*see different approaches that I can use to solve a ratio question*". In this way, problem-posing can become an avenue to bridge knowledge gaps among teachers and learners, by helping teachers assess students' understanding in mathematics, where "*I shouldn't be static in my thinking, but I should be flexible so that I can think around the questions*".

One other impact that the intervention had on the subject matter knowledge of the PSTs was that it opened them up to confront conceptual challenges in problem-posing, as in one "*[has] to sit down, look at the scenario [given], and then analyse the question.*" This builds up the preservice teachers to understand and to pose meaningful mathematical problems (Leavy & Hourigan, 2024). However, the biggest hurdle to overcome in the art of posing problems was linguistic and language challenges, because "*I was only able to pose easy questions... but I couldn't write a moderate or difficult question because I didn't have more language about how to pose that question.*" If this is allowed to continue, creating appropriate problems will become a challenge that may lead preservice teachers to pose low-level thinking problems, which would not engage learners. And the teachers may tend to pose predominantly closed-ended questions, limiting critical analysis and reinforcing teacher control in classrooms. In effect, sound and innovative problems (Burgos *et al.*, 2024) may be sacrificed on the altar of language and linguistic difficulties.

Overcoming conceptual challenges like "*having issues with the word problem, how to just break it down and solve it*" was crucial to being transformed from holding on to "*static*" knowledge because misunderstanding key words or phrases hinders preservice teachers from translating such verbal expressions into appropriate mathematical statements (Otun, 2022). However, because of problem-posing, it has made "*me know that you don't* *have to rush when you are given a problem, but you have to be able to think through the lines"*. Such linguistic challenges only frustrate preservice teachers in formulating "*easy questions... because I didn't have more language about how to pose that question*", as Erkan and Kar (2022) also observed.

6. Conclusions

In conclusion, the findings indicated that most participants had difficulty posing problems in graphical scenarios and number investigations, as many attempted to pose problems in these areas, but very few could resolve them successfully. Even though the majority of the posed problems were considered as 'sufficient', there remained a lack of skill in posing problems, which also exposes the foundational gap in subject matter knowledge of participants. Thus, most of the problems posed were of a low thinking level (of pre- and unistructural levels).

The findings indicated the expressed influences of the intervention on the preservice teachers' conceptual understanding by identifying specific mathematical concepts where participants reported deepened comprehension, such as problem-posing as an explicit diagnostic tool for revealing knowledge, and cognitive strategies in problem-posing. The study also explored how problem-posing interventions exposed gaps in preservice teachers' mathematics knowledge and the mechanisms through which such activities facilitated remediation and enhancement of subject-matter understanding. Identifying challenging mathematical concepts faced by preservice teachers in problem-posing contexts and analysing the effectiveness of structured interventions can reveal their limited understanding of these concepts. This awareness prepares them to confront their limitations and appreciate the transformative role of problem-posing in reforming their subject matter knowledge.

7. Implications for Teaching and Teacher Education

It was revealing how the participants expressed their transformative knowledge about understanding mathematics, from "*having issues with word problems*" and having "*shallow understanding*" to thinking "*widely around most of the concepts*". This newfound understanding has "*influenced me positively*" to develop autonomy since "*I wasn't given the opportunity to think by myself*" in earlier learning. This aligns with Irshid *et al*'s (2023) claim that having such conceptual understanding ensures that teachers can assist learners in applying their knowledge flexibly and fluently in real-life situations, and further the theoretical perspective of critical pedagogy of giving learners a voice (Yulianto, 2015) in the teaching and learning process of mathematics. So, students needed to be allowed to participate actively in the lesson delivery process. Again, teacher educators should refocus their teaching to engage preservice mathematics teachers in problem-posing contexts.

8. Limitations and Recommendations of the Study

The study employed a mixed-methodology approach, combining a case study with qualitative and quantitative methods, which involved a small sample of twenty-five participants. The findings from this study are therefore a possible limitation for generalisability. Further study can be conducted using a mixed method with a quantitative overture that can be extended to a larger population. Secondly, the duration for this study was six weeks. It is therefore suggested that a longitudinal approach can be adopted to study any variability in the behaviour of study participants.

Declarations

Ethics approval and Consent to Participate

The authors declared that the research was conducted ethically and in accordance with all relevant institutional guidelines and regulations of Akenten Appiah-Menka University of Skills Training and Entrepreneurial Development (AAMUSTED) as required. The study was approved by the institutional ethics committee with reference number AAMUSTED/IERC/2024/010. Written informed consents were approved by the participants.

Consent for Publication

Not applicable

Availability of Data and Materials

Data generated or analysed during this study are available from the corresponding author on approved request.

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Author Contributions

The corresponding author was responsible for the concept, design, execution of the intervention and data collection, interpretation, and writing of the manuscript. The co-authors supervised and improved the processes as part of a PhD work.

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Competing Interest Statement

The authors declare no competing interests.

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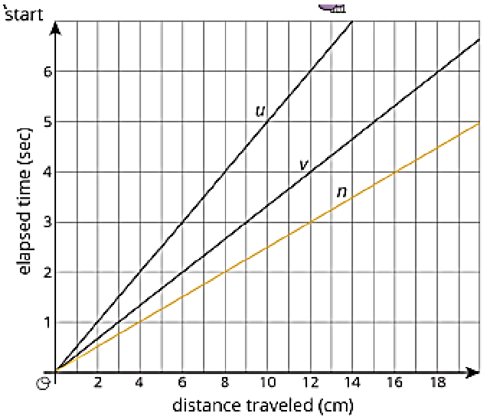
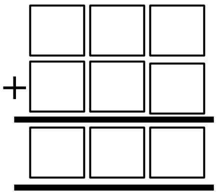
Appendices

Appendix A

Knowledge Level of Posed Problems According to Biggs & Collis (1982)		
Code	**Description**	**Explanation**
PSL	Pre-structural Level (misses the point)	Preservice teachers' responses to tasks at this level of the SOLO taxonomy were often unrelated to the intended outcomes, indicating a lack of understanding. Or no responses were provided.
USL	Unistructural Level (provided one relevant step in the problem)	At this level, responses indicate relevant information, but demand for working memory is minimal. PSTs posed problems, focusing on one aspect (step/procedure), leading to inconsistent responses and limited understanding. The amount of used working memory has increased over pre-structural levels.
MSL	Multi-structural Level (provided two or more relevant steps in the problem.	At the MSL level, PSTs often have fragmented and relevant ideas, leading to inconsistencies in their responses. They can use multiple steps (aspects) in the problems posed, but cannot connect them, resulting in unrelated information. This results in responses that lack a relational connection, making the problems they pose difficult for a comprehensive understanding.
RL	Relational Level (provided more interconnected steps that lead to appropriate solutions)	At the relational level, students demonstrate understanding by connecting multiple ideas (or steps) and integrating task components. Working memory considers individual and interrelationships, leading to a qualitative shift from concrete to abstract learning. This level represents higher-order thinking, where PSTs comprehend all aspects of a problem, their place within the whole, and their associations.
EAL	Extended Abstract Level (provided a problem with steps capable of generalising to other situations)	At this level, PSTs use concise data, interconnectedness between and among steps or procedures, and hypotheses to construct new knowledge. They use prior knowledge and experience to logically conclude beyond what was learned. This level enables progressive thinking and reasoning beyond expected tasks, making it a new form of thinking.

Appendix B

s/n	**Free problem-posing situation**	**Generate problems from constraints/semi-structured situations (Visual representation)**	**Transformed structured situation (Symbolic representation)**
1	**(a) Mathematics content:** Number operations (for JHS students). **Situation:** Ann has 34 marbles, Billy has 27 marbles, and Chris has 23 marbles. **Students' Task:** Based on this information, write at least three mathematical problems at different difficulty levels that you can challenge your classmates to solve. (Silver & Cai, 2005, p. 130).	**(b) Mathematics content:** Linear equation with one unknown (for Junior High School students). **Situation:** A factory is planning to make a billboard. A master worker and his apprentice are employed to do the job. The master-worker alone will complete the job in 4 days, but the apprentice alone will take 6 days. **Students' Task:** Please create at least three mathematical problems based on the situation. You may add conditions to the problems you create. (Singer *et al.*, 2011, p. 155).	**(c) Mathematics content:** Ratios and Percentages (for JHS students) **Situation:** A farmer uses one-third of his land to plant cassava, two-fifths of the remaining land to plant maise, and the rest to plant vegetables. If vegetables cover an area of 10 acres, what is the total area of the farmer's land? **Students' Task:** You are to transform the problem so it can be solved. [WAEC, 2015, BECE Mathematics Q1(b)].
2	**(a) Mathematics content:** Number operations and investigation (for JHS students). **Situation:** Fill in the spaces provided in the boxes with the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9. **Students' Task:** Find as many sums of the digits as possible by putting one in each of the spaces	**(b) Mathematics content:** Linear graph (for Junior High School students). **Situation:** You are provided with the following travel graph of three friends. 	**(c) Mathematics content:** Linear equation with one unknown (for JHS students) **Situation:** When seven is added to three times a number, the answer is the same as when twelve is subtracted from four times the same number. Write down the equation. **Students' Task:** You are to transform the

	provided without repeating any. (Crespo, 2003, p. 254)	**Students' Task:** Based on the situation above, please create at least three mathematical problems to challenge your friends. (Cai & Hwang, 2022, p. 321). 	problem in order it can be solved. [WAEC, 2022, GABECE Mathematics Q3(a)].