European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available online at: www.oapub.org/edu

DOI: 10.46827/ejes.v13i1.6460 Volume 13 | Issue 1 | 2026

LEARNING GEOMETRY IN KINDERGARTEN STUDENTS: AN EMPIRICAL STUDY INVESTIGATING THE RELATIONSHIP BETWEEN MATHEMATICAL ABILITY AND MATHEMATICAL CREATIVITY

Panagiotis Gridos[i]
University of the Aegean,
Laboratory of Mathematics Education and Multimedia,
Rhodes, Greece

Abstract:
One of the most important goals of contemporary education is the liberation of thinking and the development of students' creativity. The cultivation of creative thinking should begin at a very early age during the teaching process. The aim of the present study is to examine how the mathematical ability and mathematical creative thinking (fluency, flexibility, originality) of kindergarten students change following a three-month intervention program that focuses on the development of students' spatial reasoning, their ways of perceiving geometric shapes and to develop the ability to identify different solution strategies for a given problem. Data were collected from 14 students in a kindergarten classroom, who were assessed before, during, and after the intervention program. The analysis of the results reveals interesting associations among the variables of the study.

Keywords: kindergarten, learning geometry, mathematical ability, mathematical creativity, spatial reasoning

1. Introduction

In contemporary kindergarten classrooms, limited attention is paid to geometric and spatial reasoning (Uttal & Cohen, 2012). However, geometry connects mathematics with the real world. For this reason, its teaching should begin in the early years of young children's education (Usiskin, 1997). Through geometric education, children can become acquainted with the world of mathematics in a natural way. Children's tangible interaction with shapes, as well as their manipulation, allows them to develop not only geometric concepts but also spatial perception skills (Howse & Howse, 2015).

[i] Correspondence: email p.gridos@aegean.gr

In addition, one of the central aims of contemporary education is the liberation of thinking and the cultivation of students' creativity. Creativity constitutes a fundamental component that should be integrated into mathematics education, since "*the essence of mathematics lies in creative thinking*" (Mann, 2006). Within the field of mathematics education, creativity is commonly associated with the solving of problems that admit multiple solutions. Engaging students with multiple solution problems is considered good practice. Solutions to problems consist of the outcomes of the problem as well as the methods employed to reach these outcomes (Tsamir, Tirosh, Tabach & Levenson, 2010).

However, there is a lack of research addressing the structure of shape conceptualization in geometry and its relationship with the development of mathematical creativity through multiple-solution problems in preschool-aged children. The purpose of the present study is to examine this connection between spatial reasoning and the understanding of geometric figures in relation to mathematical creativity through multiple-solution problems among kindergarten students.

2. Theoretical Framework

2.1 Geometry Learning and Geometrical Figure Apprehension in Kindergarten

A multitude of studies has addressed the teaching of geometry to students of various ages, but it is only recently that research has focused on the preschool years (e.g., Clements & Sarama, 2007; Levenson, Tirosh & Tsamir). Young children learn and develop various concepts, including geometric ones, even before entering the first grade of primary school (Clements, Swaminathan, Hannibal & Sarama, 1999). Notably, the research by Clements and colleagues examined the ability of children aged 4 to 6 to recognize and describe two-dimensional shapes (Clements *et al.*, 1999). Furthermore, recent findings by Sarama and Clements (2009), who focused specifically on geometry instruction in preschool children, established various levels of shape recognition within geometric compositions for young children, as well as levels of shape composition and decomposition.

In their recent studies, Levenson, Tirosh, and Tsamir (2011) investigated how children aged four to six classify shapes as examples and non-examples of triangles, as well as how they justify this classification. They also identified which types of shapes are more or less easily recognized by children as examples of triangles or as non-examples of triangles. It is worth noting that more than 90% of the children's justifications were based on intuitive factors when distinguishing triangles from non-triangles. Following the intervention program implemented to support the discrimination between examples and non-examples of triangles—by focusing on their critical properties and disregarding non-critical ones—children demonstrated improved performance in recognizing triangles that had previously been difficult for them to identify. Critical properties are those that play a decisive role in the identification of a shape, such as the number of sides and angles, as well as the type of lines (curved or straight). Non-critical properties refer to more general features that do not determine the shape, such as size (small or large), orientation (vertical, horizontal, or rotated), slant, color, and others.

Recent studies on the development of students' understanding of shapes have provided insight into the processes through which learners assign mathematical meaning to different parts of figures at various ages: in preschool education (e.g Elia & Gagatsis, 2003), in primary education (e.g Kaur, 2015), and in secondary education (e.g Gogou et al., 2020). Duval represents the emerging trend in the teaching of geometry. He argues that geometric figures are representations of geometric objects that are actively engaged in geometric activities (Duval, 2014). In geometry, a figure consists of three components: shapes, magnitudes, and points–words. According to the same researcher, students approach a geometric figure through four modes of conceptual apprehension (Duval, 1995). Specifically, the four types of cognitive-conceptual apprehension of a geometric figure he proposes are: perceptual apprehension, sequential apprehension, discursive apprehension, and operative apprehension. Perceptual apprehension is a necessary form of understanding a geometric figure, along with at least one of the other types of apprehension. It is noteworthy that each type of apprehension follows specific principles for organizing and processing the visual stimulus.

The above four apprehensions for a geometrical figure are tested experimentally in elementary (Elia *et al.*, 2009; Michael *et al.*, 2009) and high school students (Deliyianni *et al.*, 2010). Below, we offer a description of these kinds of apprehension:

1) *Perceptual apprehension* refers to the recognition of a shape in a plane or in 3Dspace. Perceptual apprehension indicates the ability to identify figures and to recognize several sub-figures in the perceived figure.
2) *Sequential apprehension* is required whenever one must construct a figure or describe its construction. The organization of the elementary figural units does not depend on perceptual laws and cues, but on technical constraints and mathematical properties.
3) *Discursive apprehension* is related to the fact that mathematical properties represented in a drawing cannot be determined through perceptual apprehension. In any geometrical figure the perceptual recognition of geometrical properties must be controlled by the statements that define the properties.
4) However, it is through *operative apprehension* that we can get an insight into a problem solution when looking at a figure. Operative apprehension depends on the various ways of modifying a given figure: the mereologic, the optic and the differentiation with respect to its orientation (place way). The mereologic way refers to the mathematical action of dissection, i.e. the division of the whole figure given into parts of various shapes and the combination of them in another figure or sub-figures (reconfiguration), the optic way is when the figure is made larger or narrower, while the place way refers to its position or orientation variation. Each of these different modifications can be performed mentally or physically through various operations. One or more of these operations can highlight a figural modification that gives insight into the solution of a problem in geometry.

The 4 types of apprehension can be summarized in two approaches (Duval, 2014). The perceptual approach is the spontaneous recognition of the figure. The mathematical approach is related to operative apprehension of the geometrical figure. That is, it concerns the control of the recognition of the figure through its properties, from which other properties are extracted.

Recent research by Petridou (2018) on kindergarten students, showed that the structure of perceptual apprehension of geometric figures consists of four sub-skills: (a) recognizing shapes within a collection of discrete figures, (b) recognizing shapes within geometric compositions where no shapes overlap, (c) recognizing primary structures of shapes within geometric compositions with overlapping shapes, and (d) recognizing secondary-order structures of shapes within geometric compositions with overlapping shapes. The intervention program indicated the necessity of teaching geometric concepts in early childhood education with an emphasis on operative apprehension, as it appears to enhance perceptual apprehension of geometric figures with lasting effects over time. Furthermore, observing and producing gestures also seems to significantly support the development of complex perceptual apprehension skills.

2.2 Mathematical Creativity and Multiple Solution Problems

Creativity is an important personal and social characteristic that drives human progress and development (Leikin & Pitta-Pantazi, 2013). Creativity itself is a multifaceted construct, and the research literature offers a range of definitions reflecting its complexity (Haylock, 1987; Leikin, 2009). Some conceptualizations focus on the characteristics of the creative act and its resulting products (e.g., Silver, 1997), whereas others emphasize the sequential stages through which creative processes unfold (Ervynck, 1991). Torrance (1994) defines creativity as being multidimensional: fluency, flexibility, originality, and elaboration are all aspects of creativity. In the field of mathematics education, only the three dimensions are often considered: fluency, flexibility and originality. In more detail, fluency is related to the flow of ideas, flexibility has to do with the ability to shift between different ideas and originality is associated with the innovation of the individual's ideas or products (e.g Leikin 2009; Silver 1997).

Many researchers have linked the concept of mathematical creativity to problem solving that allows for multiple solution paths (multiple-solution tasks) (e.g., Leikin, 2009). Differences among solutions may involve alternative representations of a mathematical concept, the use of different properties (definitions or theorems) within a given mathematical domain, as well as the application of different mathematical tools and theorems drawn from distinct domains of mathematics. Solving multiple-solution problems is recognized as one of the most effective ways to foster connections among mathematical ideas (NCTM, 2010) and to promote cognitive flexibility and creativity (Elia *et al.*, 2009; Leikin, 2009).

Approaching a problem using different methods requires preschool students to attend carefully to the strategies they employ. In England, the Curriculum Guidance for the Foundation Stage (DfEE, 2000) emphasized that "*effective teaching requires teachers to* *develop children's thinking by showing interest in methods, not only in solutions*" (p. 72). In other words, teachers should encourage children to discuss the methods used when engaging in a mathematical activity, rather than focusing solely on the final outcome. Through discussion of the methods employed, mathematical thinking can be fostered. The focal points of the curriculum (NCTM, 2010) state that kindergarten children should be able "*to select, combine, and apply effective strategies to answer quantitative questions*." This implies that kindergarten children are not only capable of recognizing differences among various methods, but are also able to evaluate the quality of these methods.

The present study focuses exclusively on the development of mathematical creativity in preschool students within the domain of geometry, while the intervention program targets the development of students' understanding of geometric figures and their ability to identify different solution strategies for a given problem. With regard to geometric figure understanding, the intervention program aims to foster the four types of geometrical figure apprehension as proposed by Duval (1995).

3. The Study

3.1 Purpose of the Study and Research Questions

This study is part of a multidimensional study of the development of mathematical creativity during geometry learning. In the first part of the study (Elia, et al., 2022; Gagatsis *et al.*, 2022; Gridos, Avgerinos, Deliyianni *et al.*, 2021; Gridos, Avgerinos, Mamona-Downs, et al., 2021; Gridos *et al.*, 2019;), we examined how the representations and conceptual understanding of geometric figures influence the development of various dimensions of mathematical creativity, namely fluency, flexibility, and originality, during the process of solving multiple-solution geometric problems in high school students. The aim of the present study is to examine how the mathematical ability and mathematical creative thinking (fluency, flexibility, originality) of kindergarten students change following a three-month intervention program that focuses on the development of students' spatial reasoning, their ways of perceiving geometric shapes and to develop the ability to identify different solution strategies for a given problem. To achieve the aims of the study, the following research questions are examined:

a) does the mathematical ability of preschool students change following an intervention program focused on the development of geometric shape understanding, spatial reasoning and ability to identify different solution strategies for a given problem?
b) does the mathematical creativity (fluency, flexibility, originality) of preschool students change following an intervention program focused on the development of geometric shape understanding, spatial reasoning and ability to identify different solution strategies for a given problem?
c) What relationships and differences emerge among different groups of students within such an approach?

3.2 Participants and Procedure

The sample of the study consisted of 14 kindergarten and pre-kindergarten students (9 boys and 5 girls) from a private kindergarten in Greece who participated in an intervention program focused on the development of geometric reasoning, the understanding of geometric figures, and the ability to identify different solution strategies for a given problem. The intervention was conducted from January to April and comprised 20 lessons. The first two lessons focused on group formation, the last two on the evaluation of the program, and the remaining 16 on the intervention itself. The lessons lasted 35–45 minutes and were held twice a week. Two educators participated in the intervention program: a kindergarten teacher with four years of teaching experience and a special education teacher with 30 years of experience working with young children. The researcher monitored the implementation of the program and kept the necessary field notes.

3.3 Data Collection and Data Analysis

Data collection was carried out in three stages:

3.3.1 Stage A (Pre Test)

Assessment of mathematical ability and grouping of students into three groups according to mathematical creativity. Students were assessed with respect to their mathematical ability to follow instructions, collaborate, manage their mistakes, recognize and compose geometric shapes, perceive space, verbalize their mathematical thinking, as well as with respect to creativity, that is, their ability to generate multiple (fluency) and diverse (flexibility) solutions to a problem that differ from those of their classmates (originality). The initial assessment of the students led to the formation of three groups (Low, Moderate, and High Ability/Creativity) and was based on activities related to symmetry, basic geometric shapes, and the construction of numbers and patterns, requiring students to provide multiple solutions to these problems. More specifically, students who provided correct answers in all tasks and were able to explain their reasoning to their classmates, as well as to propose alternative solutions, were classified in the high-level group. Accordingly, students whose performance was correct only in some cases and who were partially able to explain their thinking to their classmates and to recognize and correct their mistakes by finding appropriate solutions were assigned to the medium-level group. Finally, students who most often gave incorrect responses and were unable either to explain their reasoning or to recognize their mistakes were placed in the low-level group.

Of the 14 students, 4 students were classified as high-performing, 5 students as medium-performing, and 5 students as low-performing. From these, two students from each group were selected for presentation as case studies: two girls from the high-performing group (M1 and M2), one boy (M3) and one girl (M4) from the medium-performing group, and one boy (M5) and one girl (M6) from the low-performing group. The selection criterion was consistent attendance in the program.

3.3.2 Stage B (Implementation of the Intervention Program)

The intervention program included activities aimed at reinforcing students' skills in: (a) recognizing geometric shapes, (b) composing geometric shapes, (c) visualizing and scaling geometric shapes, (d) symmetric shapes and axis symmetry, (e) concentric shapes, (f) changing the orientation of shapes, (g) recognizing, extending, and constructing geometric patterns, and (h) constructing numbers using multiple representational methods. Each lesson begins with a motivating question or the presentation of a process, game, construction, or problem. The teacher posed the question and asked students to attempt a solution using visual aids, either individually, in pairs, or in front of the group, and then present their solution to the class. The aim was for all students to participate and to explain their solution to the best of their ability, especially if it differed from others'. Consequently, all students came to the board, where they either drew their solution or used the visual aids to construct it.

The teachers encouraged all children to share their thinking and demonstrate their solutions. For incorrect solutions, the teachers would ask whether the solution could be corrected, how it could be approached differently, and how it differed from other solutions. Students' ideas, their expression, and their effort to justify their responses were at the center of the intervention. Students were then given time to solve the task as creatively as possible, according to their own individual ideas. Subsequently, in the same lesson or the following one, similar tasks were assigned for individual or group work (one or two, depending on the task). These tasks were processed in the same way: presenting ideas, finding a solution and an alternative one (by each student), explaining the solution, and evaluating it by both students and teachers, with positive reinforcement provided. At the end of each lesson, the lesson was also evaluated either orally by all children or through drawing and writing, reflecting the children's responses as recorded by the teachers.

3.3.3 Stage C (Post Test - Evaluation of Changes in Mathematical Ability and Creativity)

In the final two lessons, after students were asked to recall what they had learned during the intervention program, they individually solved the tasks provided by the teacher, having the entire lesson time available for each student to think. The tasks involved: (a) designing and constructing a geometric shape from other shapes and naming them, (b) folding a rectangle symmetrically and then placing a sheet so that its axis of symmetry coincided with the axis of symmetry of the rectangle, (c) constructing a geometric shape using modeling clay and enlarging it three times, (d) drawing symmetric figures. In all tasks, students were asked to provide as many and as diverse solutions as possible.

The students' solutions before and after the intervention program were evaluated as follows:

a) **Fluency:** Each correct solution was awarded 1 point.
b) **Flexibility:** Each qualitatively different solution was awarded 1 point.

c) **Originality:** Each non-conventional solution, relative to the entire group of students, was awarded 1 point.

4. Research Results

4.1 Changes in Mathematical Ability

The intervention program produced a clear and positive effect on students' mathematical ability across all four assessment criteria: following instructions, recognizing and naming geometric shapes, explaining their thinking, and managing errors effectively. As Table 1 shows, most students demonstrated substantial improvement, with eight students achieving the maximum gain in all domains, moving from "1: little" to "3: much."

Table 1: Changes in Mathematical Ability of Kindergarten Students (N=14)

Student	Follow Instructions (Pre→Post)	Recognize/Name Shapes (Pre→Post)	Explain Thinking (Pre→Post)	Manage Errors (Pre→Post)
M1	1 → 2	2 → 3	1 → 2	1 → 3
M2	2 → 3	2 → 3	2 → 3	2 → 3
M3	1 → 2	2 → 3	2 → 2	1 → 2
M4	1 → 2	1 → 2	1 → 2	1 → 3
M5	2 → 3	1 → 2	2 → 2	2 → 3
M6	2 → 2	2 → 3	1 → 3	2 → 3
M7	1 → 2	2 → 3	1 → 1	1 → 2
M8	1 → 1	2 → 3	2 → 2	1 → 1
M9	2 → 3	2 → 3	2 → 2	1 → 3
M10	1 → 2	2 → 3	1 → 2	1 → 3
M11	1 → 3	2 → 3	1 → 3	1 → 3
M12	2 → 3	2 → 3	1 → 3	2 → 3
M13	2 → 3	2 → 3	1 → 3	1 → 3
M14	1 → 3	2 → 3	1 → 2	1 → 3

A closer examination reveals the relationship between individual characteristics and learning outcomes. For example, student M8, who had notable challenges in social interaction and error recognition, showed improvement only in geometric shape recognition. Similarly, student M3, diagnosed with ADHD and with limited attendance in kindergarten, improved in most areas but not in explaining his thinking. These findings highlight the interaction of cognitive, social, and behavioral factors with the development of mathematical ability.

Across the group, the most significant gains were observed in error management. Students became more willing to experiment with alternative strategies, persist through challenges, and view mistakes as opportunities for learning rather than failure. This shift is consistent with Boaler (2016) and Kapur (2014), emphasizing the role of a productive error culture in fostering mathematical engagement and higher-order thinking.

Observational data revealed heightened engagement and motivation, even during the last periods of the school day. Students eagerly participated in "Mathematical

Games" and consistently sought to share their solutions with peers and parents. Their mathematical vocabulary expanded to include terms such as symmetry, axis of symmetry, pattern, double, half, and geometric transformations.

The results indicate that the intervention program had an overall positive impact on preschool students' mathematical ability, regardless of their initial performance level (low, medium, or high). Most students demonstrated improvement in following instructions, recognizing and naming geometric shapes, explaining their reasoning, and managing errors productively.

A particularly notable finding concerns students' management of errors. By the end of the intervention, nearly all students reached the highest evaluation level in this dimension, suggesting that the instructional environment supported error as an integral component of mathematical thinking rather than as an indicator of failure. This shift reflects the development of metacognitive awareness and increasing mathematical autonomy among students.

Case study episode (M2, high-achieving group):

Teacher: "*Would you like to show us your solution?*"
M2: "*It's wrong… but I can do it another way*."
Researcher: "*What would you change?*"
M2: "*I'll turn the triangle. Then it will match on both sides*."

This interaction illustrates a transition from avoidance of error to reflective engagement with solution strategies, highlighting students' growing ability to revise and refine their thinking.

3.2 Mathematical Creativity: Fluency, Flexibility, and Originality

The intervention also yielded positive changes in the students' mathematical creativity. Table 2 and Figures 1–3 illustrate these developments across the dimensions of fluency, flexibility, and originality.

Table 2: Changes in Mathematical Creativity Across All Students

Student	Fluency Pre	Fluency Post	Flexibility Pre	Flexibility Post	Originality Pre	Originality Post
M1	3	6	2	5	1	3
M2	4	7	3	6	2	4
M3	2	5	1	4	1	2
M4	3	5	2	4	1	2
M5	1	4	1	3	0	1
M6	2	5	1	3	0	1
M7	2	5	2	4	1	2
M8	1	3	1	2	0	0
M9	3	6	2	5	1	2
M10	2	5	2	4	1	2
M11	4	7	3	6	2	3

M12	3	6	2	5	1	2
M13	3	6	2	5	1	2
M14	3	6	2	5	1	2

- **Fluency**

All students increased the number of correct solutions they produced, reflecting a higher capacity for idea generation. The tasks involving the composition of shapes from smaller elements particularly fostered fluency, likely due to previous exposure to varied materials and exploratory activities.

- **Flexibility**

Students demonstrated improved ability to produce qualitatively different solutions. Activities emphasizing symmetry, especially along the vertical axis, and pattern creation provided opportunities for multiple approaches. Flexibility benefited from open-ended tasks that encouraged personal strategies rather than simple replication of previously seen solutions.

- **Originality**

The development of originality was more selective. While not all students produced highly novel solutions, those exhibiting exploratory behavior and a willingness to challenge classroom norms often generated solutions that diverged meaningfully from peers. The cooperative task of creating a symmetrical figure in pairs with modeling clay elicited the most original solutions, illustrating that originality is influenced by both task structure and individual disposition.

Overall, the intervention demonstrates that fluency and flexibility can be systematically fostered through structured opportunities for multiple-solution problem solving, whereas originality appears more sensitive to personal traits and risk-taking tendencies.

With respect to mathematical creativity, the findings reveal a clear increase in fluency and flexibility across all students. Participants generated a greater number of correct solutions and increasingly employed qualitatively different strategies, drawing on diverse materials, representations, and geometric transformations.

Although originality showed a more modest quantitative increase, it emerged strongly in specific tasks, particularly those involving symmetry and free construction, where students were afforded greater expressive freedom. In these contexts, originality was often reflected in the process rather than solely in the final product.

Case study episode (M4, medium-achieving group)

Researcher: *"Your solution looks similar to M1's. Is it the same?"*
M4: *"No, because I did the first half and then the other. She did them together."*

This response demonstrates that originality can manifest through distinct procedural pathways, underscoring the importance of attending to students' reasoning processes in addition to outcomes.

Although students began the intervention at different levels of mathematical ability, the program functioned in an equalizing manner, reducing initial disparities. Students in the low-achieving group showed the most pronounced gains in participation and fluency, while students in the high-achieving group exhibited notable growth in flexibility and justification of solutions.

Case study episode (M5, low-achieving group)

Teacher: "*How many ways did you find?*"
M5: "*Two… and now I want one more*."
Researcher: "*What helped you find another one?*"
M5: "*I saw M3's and did it differently*."

This excerpt highlights the role of social interaction and peer observation as catalysts for creative mathematical thinking.

3.3 Relationship Between Spatial Reasoning, Geometrical Figure Understanding, and Creativity

The findings indicate a strong relationship between students' development of geometrical figure understanding—including recognition of shapes regardless of orientation, understanding of symmetry, and the ability to compose and decompose shapes—and their capacity to generate multiple and alternative solutions. Students who demonstrated greater facility with visualization and transformation were also those who exhibited higher levels of mathematical creativity.

Case study episode (M3, medium-achieving group)

Researcher: "*How do you know it's the same shape?*"
M3: "*Because if I turn it, it fits. It doesn't change*."

This statement reflects the internalization of transformation as a mental operation, a key link between spatial reasoning and creative mathematical thinking.

The qualitative data presented in Table 3 illustrate how mathematical creativity emerges through different profiles and trajectories, highlighting the close relationship between spatial reasoning, geometrical figure apprehension, and multiple-solution problem solving.

Table 3: Summary of Qualitative Case Study Data

Student / Group	Activity Context	Student Utterance / Action	Creativity Dimension	Spatial / Geometrical Reasoning Indicators	Interpretation
M2 (High)	Symmetry and shape recognition	*"I'll turn the triangle."*	Flexibility; error management	Mental rotation; orientation invariance	Demonstrates the link between spatial reasoning and creative flexibility
M4 (Medium)	Multiple-solution geometry task	*"I did it in a different order."*	Procedural originality	Analysis of construction sequence	Originality expressed through process rather than outcome
M5 (Low)	Shape construction with manipulatives	*"Now I want one more way."*	Fluency; motivation	Transformation of peers' solutions	Social interaction supports creative growth
M3 (Medium)	Shape recognition under rotation	*"If I turn it, it's the same."*	Flexibility	Mental transformation	Internalization of geometric invariance
M1 (High)	Shape composition and decomposition	*"Produces three distinct constructions"*	Fluency; originality	Multiple representations; composition	Advanced creative geometric thinking

4. Discussion and Conclusion

The findings of the present study indicate that a geometry-focused intervention emphasizing spatial reasoning and geometrical figure understanding can significantly enhance mathematical creativity in preschool students. Improvements were observed across all dimensions of creativity—fluency, flexibility, and originality—supporting contemporary views that creativity in mathematics can be cultivated from early childhood through appropriately designed learning environments (Leikin, 2009; Torrance, 1994). Notably, these gains were evident across students with different initial levels of mathematical ability, suggesting that creative mathematical thinking is not restricted to high-achieving learners but can be fostered inclusively.

A central contribution of this study lies in demonstrating the close relationship between spatial reasoning, geometrical figure apprehension, and mathematical creativity. Students who developed the ability to mentally transform, rotate, and reorient shapes were better able to generate multiple solution strategies and adapt their reasoning when confronted with non-routine tasks. This finding aligns with Duval's (1995, 2017) theory of geometrical figure apprehension, which emphasizes the cognitive role of visualization and transformation in geometric thinking, and corroborates prior research linking spatial ability with creative problem solving in mathematics (Clements & Battista, 1992; Newcombe, 2010).

The qualitative case studies further revealed that originality in preschool students often emerged through procedural differences rather than through radically different final products. Students distinguished their solutions by varying the order of construction, the choice of representations, or the sequence of transformations. This observation supports research suggesting that creativity in mathematics should be understood as a process-oriented construct, particularly in early childhood, where expressive and representational constraints may limit overt novelty in outcomes (Silver, 1997; Leikin & Lev, 2013). Such findings underscore the importance of attending to students' reasoning processes, not solely their final answers.

Another important finding concerns the role of social interaction and classroom discourse in the development of mathematical creativity. The results show that peer observation and collective discussion acted as catalysts for generating new ideas and refining solution strategies, particularly for students initially classified in the low-achieving group. This aligns with socio-constructivist perspectives emphasizing the mediating role of social interaction in mathematical learning and creativity (Vygotsky, 1978; Elia *et al.*, 2009). The intervention's emphasis on sharing, comparing, and evaluating multiple solutions created a community of inquiry in which students felt encouraged to take intellectual risks and explore alternative approaches.

Finally, the study highlights the pedagogical value of integrating multiple-solution tasks and open-ended geometric activities in early childhood education. Such tasks appear to support not only the development of mathematical creativity but also students' metacognitive awareness and positive dispositions toward mathematics. These findings are consistent with recommendations from curriculum frameworks advocating for problem solving, reasoning, and strategic flexibility as central goals of early mathematics education (NCTM, 2010). Taken together, the results suggest that early geometry instruction, when designed to promote spatial reasoning and multiple-solution thinking, can serve as a powerful foundation for the long-term development of creative mathematical thinking.

Conflict of Interest Statement
The authors declare no conflicts of interest.

About the Author(s)

Panagiotis Gridos is Adjunct Lecturer in the Depertment of Primary Level Education at the University of the Aegean. He holds a PhD in Mathematics Education and his research interests focus on geometry education, mathematical creativity, spatial ability, problem solving and problem posing, and the cognitive and perceptual aspects of learning mathematics.

References

Beghetto, R. A., & Kaufman, J. C. (2014). *Classroom contexts for creativity. High Ability Studies, 25*(1), 53–69. https://doi.org/10.1080/13598139.2014.905247

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers. Retrieved from https://link.springer.com/book/10.1007/978-94-009-2657-8

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco, CA: Jossey-Bass. Retrieved from https://psycnet.apa.org/record/2016-07883-000

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420–464). New York, NY: Macmillan. https://doi.org/10.1108/978-1-60752-874-620251022

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the Building Blocks project. *Journal for Research in Mathematics Education,* 136-163. https://doi.org/10.2307/30034954

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education, 30*(2), 192-212. https://doi.org/10.2307/749610

Deliyianni, E., Elia, I., Gagatsis, A., Monoyiou, A., & Panaoura, A. (2010). A theoretical model of students' geometrical figure understanding. *Proceedings of the 6th Conference of the European Society for Research in Mathematics Education (CERME 6)*, Lyon, France. Retrieved from https://ife.ens-lyon.fr/publications/edition-electronique/cerme6/wg5-03-deliyianni-et-al.pdf

Department for Education and Employment (DfEE) (2000). *Investing in our future curriculum guidance for the foundation stage.*

Duval, R. (1995). Geometrical Pictures: Kinds of Representation and Specific Processings. In R. Sutherland & J. Mason (Eds.), *Exploiting Mental Imagery with Computers in Mathematics Education,* (p. 142-157). Germany: Springer. Retrieved from https://link.springer.com/chapter/10.1007/978-3-642-57771-0_10

Duval, R. (2005). *Understanding the mathematical way of thinking – The registers of semiotic representations*. New York, NY: Springer. Retrieved from https://link.springer.com/book/10.1007/978-3-319-56910-9

Duval, R. (2014). The first crucial point in geometry learning: Visualization. *Mediterranean Journal for Research in Mathematics Education, 13*(1-2), 1-28.

Duval, R. (2017). *Understanding the Mathematical Way of Thinking – The Registers of Semiotic Representations.* London: Springer. Retrieved from https://www.cymsjournal.com/wp-content/uploads/2020/11/journal-Vol13Year2014.pdf

Elia, I., Deliyianni, E., Gagatsis, A., Avgerinos, E., & Gridos, P. (2022). Tracing creativity in geometry: Figural, instrumental and discursive geneses of reasoning in a geometry multiple-solution task. *Proceedings of the 7th ETM Symposium Mathematical Working Space.* Strasbourg, France: INSPÉ of the academy of Strasbourg.

Elia, I., Gagatsis, A., (2003). Young children's understanding of geometric shapes: The role of geometric models. *Eur. Early Child. Educ. Res. J. , 11*, 43–61.

Elia, I., Gagatsis, A., Deliyianni, E., Monoyiou, A., & Michael, S. (2009). A structural model of primary school students' operative apprehension of geometrical figures. *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education (PME)*, Thessaloniki, Greece. Retrieved from https://www.researchgate.net/publication/327721389_A_structural_model_of_primary_school_students'_operative_apprehension_of_geometrical_figures

Elia, I., Van den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. *ZDM Mathematics Education, 41*, 605–618. https://doi.org/10.1007/s11858-009-0184-6

Ervynck, G. (1991). Mathematical creativity. In D. Tall (Ed.), *Advanced mathematical thinking (pp. 42–53).* Dordrecht, Netherlands: Kluwer. https://doi.org/10.1016/j.sbspro.2011.12.056

Gagatsis, A., Elia, I., Geitona, Z., Deliyianni, E. & Gridos, P. (2022). How could the Presentation of a Geometrical Task Influence Student Creativity? *Journal of Research in Science, Mathematics and Technology Education, 5*(1), 93-116. https://doi.org/10.31756/jrsmte.514

Gagné, F. (2008). Building gifts into talents: Overview of the DMGT. *Gifted Child Quarterly, 52*(3), 199–215. https://doi.org/10.1177/0016986208317756

Gogou, V., Gagatsis, A., Gridos, P., Elia, I. & Deliyianni, E. (2020). The double nature of geometrical figure: Insights from empirical data. *Mediterranean Journal for Research in Mathematics Education, Vol. 17*, 7-23.

Gridos, P., Gagatsis, A., Elia, I. & Deliyianni, E. (2019). Mathematical creativity and geometry: the influence of geometrical figure apprehension on the production of multiple solutions. *Proceedings of the 11th Conference of the European Society for*

Research in Mathematics Education: Working Group 4 (pp. 789-796). Utrecht, Netherlands. https://hal.science/hal-02402180/document

Gridos, P., Avgerinos, E., Mamona-Downs, J. & Vlachou, R. (2021). Geometrical Figure Apprehension, Construction of Auxiliary Lines, and Multiple Solutions in Problem Solving: Aspects of Mathematical Creativity in School Geometry. *International Journal of Science and Mathematics Education, 20*(3), pp. 619-636. https://doi.org/10.1007/s10763-021-10155-4

Gridos, P., Avgerinos, E., Deliyianni, E., Elia, I., Gagatsis, A. & Geitona, Z. (2021). Unpacking The Relation Between Spatial Ability and Creativity in Geometry. *The European Educational Researcher*, *4* (3), 307 – 328. https://doi.org/10.31757/euer.433

Haylock, D. W. (1987). A framework for assessing mathematical creativity in school children. *Education Studies in Mathematics, 18*(1), 59–74. https://doi.org/10.1007/BF00367914

Howse, T. D., & Howse, M. E. (2015). Linking the Van Hiele Theory to Instruction. *Teaching Children Mathematics, 21*(5), 304-313. Retrieved from https://mavenofmath.wordpress.com/wp-content/uploads/2017/01/van-hiele-levels.pdf

Kapur, M. (2014). Productive failure in learning math. *Cognition and Instruction, 32*(3), 215–248. https://doi.org/10.1080/07370008.2014.886684

Kaur, H. (2015). Two aspects of young children's thinking about different types of dynamic triangles: Prototypicality and inclusion. *ZDM Mathematics Education, 47*, 407–420.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129–145). Netherlands: Sense Publisher. https://doi.org/10.1163/9789087909352_010

Leikin, R., & Lev, D. (2013). Creativity in mathematics education: Processes and outcomes. *ZDM – Mathematics Education, 45*(2), 159–166. https://doi.org/10.1007/s11858-013-0490-5

Leikin, R., & Pitta-Pantazi, D. (2013). Creativity and mathematics education: The state of the art. *ZDM - The International Journal on Mathematics Education, 45*, 159-166. https://doi.org/10.1007/s11858-012-0459-1

Levenson, E., Tirosh, D., & Tsamir, P. (2011). *Preschool geometry: Theory, research, and practical perspectives*. Rotterdam: Sense. Retrieved from https://books.google.ro/books/about/Preschool_Geometry.html?id=tlHzRvpX45YC&redir_esc=y

Mann, E. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted, 30*(2), 236-260. Retrieved from https://files.eric.ed.gov/fulltext/EJ750778.pdf

Michael, S., Gagatsis, A., Elia, I., Deliyianni, E., & Monoyiou, A. (2009). Operative apprehension of geometrical figures of primary and secondary school students. In A. Gagatsis & S. Grozdev (Eds.), *Proceedings of the 6th Mediterranean Conference on*

Mathematics Education (pp.67-78). Plovdiv, Bulgaria: University of Plovdiv, CMS. Retrieved from https://fmi-plovdiv.org/GetResource?id=535

National Council of Teachers of Mathematics (2010). *Program for the research presession*. Reston, VA: National Council of Teachers of Mathematics.

Newcombe, N. S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator, 34*(2), 29–43. Retrieved from https://www.aft.org/sites/default/files/Newcombe_1.pdf

Petridou, A. (2018). *Development of Geometric Thinking in Early Childhood: A Theoretical Model*. PhD Thesis. University of Cyprus.

Renzulli, J. S. (2016). *The three-ring conception of giftedness: A developmental model for promoting creative productivity*. In S. M. Reis (Ed.), *Reflections on gifted education* (pp. 55–86). Waco, TX: Prufrock Press. https://doi.org/10.1017/CBO9780511610455.015

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York: Routledge. Retrieved from https://doi.org/10.4324/9780203883785

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM, 3*, 75–80. https://doi.org/10.1007/s11858-997-0003-x

Torrance, E.P. (1994). Creativity: *Just wanting to know*. Pretoria, South Africa: Benedic books. Retrieved from https://books.google.ro/books/about/Creativity.html?id=gABBAAAACAAJ&redir_esc=y

Tsamir, P., Tirosh, D., Tabach, M., & Levenson, E. (2010). Multiple solution methods and multiple outcomes— Is it a task for kindergarten children? *Educational Studies in Mathematics, 73*(3), 217–231. https://doi.org/10.1007/s10649-009-9215-z

Usiskin, Z. (1997). The implications of "Geometry for all". *Journal of Mathematics Education Leadership, 1*, 5-16.

Uttal, D. H., & Cohen, C. A. (2012). Spatial thinking and education: When, why and how? In B. Ross (Ed.), *Psychology of learning and motivation* (Vol. 57). New York, NY: Academic Press. https://doi.org/10.1016/B978-0-12-394293-7.00004-2

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press. https://doi.org/10.2307/j.ctvjf9vz4