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AN EXAMINATION OF MEANINGS AND ERROR TYPES ASSOCIATED WITH PRE-SERVICE ELEMENTARY TEACHERS' POSED PROBLEMS FOR THE MULTIPLICATION AND DIVISION OF FRACTIONS

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Abstract:
The current study aims to examine meanings and error types associated with pre-service elementary teachers' semi-structured problems for the multiplication and division of fractions. A total of 83 junior pre-service elementary teachers were recruited in the spring semester of the 2016-2017 academic year. A researcher-developed Problem Posing Test consisting of eight items was used to collect the data of this study. The findings indicated that the pre-service elementary teachers were not proficient in posing appropriate problems for the multiplication and division of fractions. Furthermore, while the most frequent error type found was a failure in expressing the multiplication operation in the question root for the multiplication of fractions, it was assigning natural number meaning to fractions for the division of fractions.

Keywords: error types, multiplication of fractions, division of fractions, problem posing

1. Introduction

Although problem posing refers to different kinds of activities, including posing a new problem and reformulation of a given problem (Brown & Walter, 1993; Silver, 1994), it is clear that it is critical for mathematics. Furthermore, the National Council of Teachers of Mathematics (2000) explains that problem posing is also important for students to continue to survive in their daily lives as problems in the real world may not be clearly given. Rather, they need to determine these problems and reformulate them if necessary. When problem posing is considered in an educational context, it helps teachers create a more student-centered environment in which students learn content conceptually (English, 2003). Specifically, in mathematics, researchers accept the

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importance of problem posing activities to improve students' knowledge and hence their learning (Barlow & Cates, 2006; Silver, 1994). Silver and Cai (2005) state that problem posing allows students to participate in "*authentic mathematical activity*" (p. 129). As a result of this activity and posing their own problems, students feel "*some ownership of the mathematics*" (Barlow & Cates, 2006, p. 69). Similarly, Cunningham (2004) mentions that students who pose problems are more open to discuss their problems with their classmates to improve the quality of their problems. Lavy and Bershadsky (2003) also emphasize that these students would be more creative and active compared to students who do not engage in problem prosing activities. In addition to the benefits of problem posing activities for students, they are helpful for teachers as well, as problem posing activities are like a mirror reflecting students' mathematical thinking. In spite of its benefits for students and teachers, students are not given enough opportunities to pose problems in a classroom environment (Silver, 1994). To create such a learning environment, first of all, teachers need to know to pose appropriate problems. Therefore, it becomes important to examine whether or not pre-service elementary teachers are able to pose appropriate problems.

1.1 Problem Posing

Because of the important contributions of problem posing activities, researchers defined problem posing in different ways. Silver (1994) explains that problem posing is "*both the generation of new problems and the re-formulation, of given problems*" (p. 19). There are also other researchers who accept problem posing as a formulation of a given problem or posing a related problem from a given problem (De Lange, 2003). Problem posing can also be accepted as a process rather than a product (Abu-Elwan, 2002). In this process, students use mathematical terminology and establish connections among the steps of the problem posing process (Rudnitsky, Etheredge, Freeman, & Gilbert, 1995).

Similar to different definitions of problem posing, researchers developed different kinds of problem posing frameworks (Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman, 2005; Silver, 1994; Stoyanova & Ellerton 1996). The first of these frameworks developed by Silver (1994) accepts that problem posing can occur before, in addition to during or after solving a problem. Christou and his colleagues (2005) proposed another framework which distinguishes problem posing situations considering students' thinking process. This model consists of four processes: "*editing quantitative information, selecting quantitative information, comprehending and organizing quantitative information, and translating quantitative information from one form to another*" (Christou et al., 2005, p. 151).

Another framework which was used in this study as well was developed by Stoyanova and Ellerton. Stoyanova and Ellerton (1996) explain that problem posing situations can be three types, namely, free, semi-structured, or structured. In the first of these situations, free problem posing situation, students pose problems without any restriction. For example, considering one of the topics of this study, asking students to pose a problem related to the multiplication of fractions can be an example of a free problem posing situation. On the other hand, if students are given a restriction such as a

diagram, symbolic form, picture, or table and are asked to pose a problem using the restriction, then it would be an example of a semi-structured problem posing situation. Finally, in structured problem posing situations, students pose a problem by reformulating or manipulating a given problem to themselves. As can be concluded from the above paragraphs, there are different frameworks to analyze problem posing situations and each of these frameworks can be used for different purposes. However, Stoyanova and Ellerton's framework (1996) is a better fit, as the focus of this study is on pre-service elementary teachers' semi-structured problems for the multiplication and division of fractions.

1.2 Multiplication and Division of Fractions

Conceptual knowledge of fractions *"…is crucial in order for students to bridge the gap from arithmetic to algebra*" (Darley, 2005, p. 115). Similarly, the National Mathematics Advisory Panel (2008) also explains this connection between fractions and algebra by stating "*The most important foundational skill not presently developed appears to be proficiency with fractions*" (p. 18). Despite its importance, the topic of fractions is difficult for students, even for teachers. One of these difficulties results from learning fractions and operations with fractions as disconnected (Mewborn, 2001).

Another difficulty results from attributing natural number meaning to fractions (Ni & Zhou, 2005). Considering one of the topics of this study, multiplication, students think that multiplication always makes the result bigger (Greer, 1994). Students' thinking is correct as students first learn multiplication at an elementary school level and with natural numbers. For example, if students are given a problem statement such as "If 1kg of tomatoes costs ₺3, how much does it cost for 4kgs tomatoes?", they would possibly multiply these two quantities given in the problem and conclude that the result is bigger than both of the quantities. However, with the introduction of fractions, students can be given such a problem: "If 1kg of tomatoes costs ₺3, how much does it cost for 1/2kgs tomatoes?". Although students would use multiplication to find the result, the result would be smaller. In the same way, the division of fractions may not yield a smaller value, contrary to the division with natural numbers (Vamvakoussi & Vosniadou, 2010). To help students discover that some of the properties of multiplication/division of natural numbers are not valid for the multiplication/division of fractions, teachers need to explain the different meanings of these operations. While the multiplication of fractions has repeated addition and part of part meanings, the division of fractions has two different meanings: measurement and equal-sharing (Cathcart, Pothier, Vance, & Bezuk, 2003).

Problems focusing on the measurement meaning give the number of objects in a group and ask the number of groups; on the other hand, problems focusing on the equal-sharing meaning give the number of groups and ask the number of objects in a group (Bulgar, 2003). Although most teachers prefer to pose and solve focusing on the measurement meaning, effective teachers need to identify which one is the most suitable and explain the reasoning behind his/her choice (Graeber, Tirosh, & Glover, 1986; Tirosh, 2000). If teachers do not understand these different meanings, they cannot

explain why the reciprocal of the second fraction is a must when solving a fraction division problem and they instead prefer to solve it procedurally (Kamii & Dominick, 1998; Li & Huang, 2008). Therefore, one of the purposes of this study is to examine pre-service elementary teachers' posed problems in terms of the different meanings of the multiplication and division of fractions.

If teachers do not conceptually know the multiplication and division of fractions, they may not understand why their students have problems in these operations and may not help their students make sense of these operations. One of the ways to help students is to pose appropriate problems for the given operations (Redmond & Utley, 2007). In order to gain insight on whether or not pre-service elementary teachers would be able to help their future students, pre-service elementary teachers' semi-structured problems for the multiplication and division of fractions were examined. Specifically, the research questions were as follows:

1) What are the meanings, including part of part and repeated addition, associated with pre-service elementary teachers' posed problems for the multiplication of fractions?
2) What are the different types of errors associated with pre-service elementary teachers' posed problems for the multiplication of fractions?
3) What are the meanings, including measurement and equal-sharing, associated with pre-service elementary teachers' posed problems for the division of fractions?
4) What are the different types of errors associated with pre-service elementary teachers' posed problems for the division of fractions?

2. Method

2.1 Participants

The participants of this study were 83 junior pre-service elementary teachers selected by a convenience sampling technique among the pre-service elementary teachers enrolled in the second mathematics methods course. All the pre-service elementary teachers took the first mathematics methods course. These courses are designed to help pre-service elementary teachers improve their mathematical knowledge in teaching and be better prepared to teach mathematics for grades 1 through 4. The study was conducted without reflecting any names of the pre-service elementary teachers. Hence, a pseudonym such as PT1, PT2, PT3, … PT83 was given to each pre-service elementary teachers.

2.2 Data Collection

A researcher-developed test made up of eight items related to the multiplication and division of fractions given in symbolic forms was used to examine the pre-service elementary teachers' semi-structured problems. Each item in the test consisted of two fractions in combination with a proper and mixed fraction. For the purposes of this

study, the pre-service elementary teachers' problems for the second and sixth items were examined and these items were given in Table 1.

Table 1: Second and Sixth Items in the Problem Posing Test (PPT)

Item Number	Symbolic Form	Characteristic of Item
2.	Write a story problem for $\frac{2}{3} \times \frac{1}{2} =?$	Multiplication of proper fractions where the result is a proper fraction
6.	Write a story problem for $\frac{5}{6} \div \frac{2}{3} =?$	Division of a proper fraction by a proper fraction where the result is a mixed fraction

This test was conducted just after providing the pre-service elementary teachers information on what basic ideas are necessary to teach fractions, what helps students learn fractions conceptually, and what possible misconceptions or errors are related to fractions. For each of the above items, a blank piece of paper was passed out to the pre-service elementary teachers to pose their problems on it. The pre-service elementary teachers had approximately 20 minutes to pose problems for the given items above.

2.3 Data Analysis

Before determining the meanings focused on the pre-service elementary teachers' problems, the pre-service elementary teachers' answers were categorized into a problem, not-a-problem, or unable to pose a problem to get insight into whether or not the pre-service elementary teachers are able to pose problems. Then, on the basis of the study purposes, the meanings emphasized in their posed problems were categorized as part of part or repeated addition for the multiplication of fractions. Similarly, the meanings associated with the pre-service elementary teachers' posed problems for the division of fractions were categorized as measurement or equal-sharing. Finally, given that another purpose of this study was to identify the errors that pre-service elementary teachers made in their problems, these problems were analyzed based on the common errors provided in the related literature (Luo, 2009; Luo, Lo, & Leu, 2011; Lo & Luo, 2012; Kilic, 2013; McAllister & Beaver, 2012; Rizvi, 2004). Specifically, the errors made in the problems for $\frac{2}{3} \times \frac{1}{2} =?$ were classified as confusing units (ME1), attributing natural number meaning to the result of the operation (ME2), failure in expressing the operation in the question root (ME3), logical error (ME4), or using numerical quantifying (ME5). Finally, the pre-service elementary teachers' errors made in the problems for $\frac{5}{6} \div \frac{2}{3} =?$ were classified as confusing units (DE1), assigning natural number meanings to fractions (DE2), posing a problem using ratio-proportion (DE3), failure in establishing part-whole relationship (DE4), dividing to the denominator of the divisor (DE5), using the multiplication operation instead of the division operation (DE6), or posing problem through inverting and multiplying the divisor fraction (DE7). To ensure reliability, the pre-service elementary teachers' answers were categorized independently by the author of the paper and a researcher having studies related to

fractions. Since the inter-rater reliability was calculated as 93% which was over 70%, the data analysis was reliable (Miles & Huberman, 1994).

3. Results

The results of this study were presented in two main sections: multiplication of fractions and division of fractions. Under these sections, the meanings and errors associated with the pre-service elementary teachers' posed problems were provided.

3.1 Multiplication of Fractions

3.1.1. Meanings Associated with Problems Posed for the Multiplication of Fractions

The meanings employed for the second symbolic form given in the problem posing test were analyzed regardless of whether the problems were correct or not and the distribution of these meanings was given in Table 2.

Table 2: The distribution of meanings associated with the second item of the PPT

	Problem			Not a Problem	Unable to Pose
	Part of Part	Repeated Addition	Others		
Write a story problem for $\frac{2}{3} \times \frac{1}{2}$ =?	34	3	7	14	25

As seen in Table 2, two types of meanings, part of part and repeated addition, that can be used in problems involving multiplication of fractions were used to pose a story problem for the given symbolic form, with the pre-service elementary teachers predominantly employing the part of part meaning. One appropriate posing of these problems was as follows:

> "$\frac{2}{3}$ *of a bottle is filled with milk. I will use* $\frac{1}{2}$ *of the milk to make a dessert. How much of the whole milk will be used to make the dessert?"*

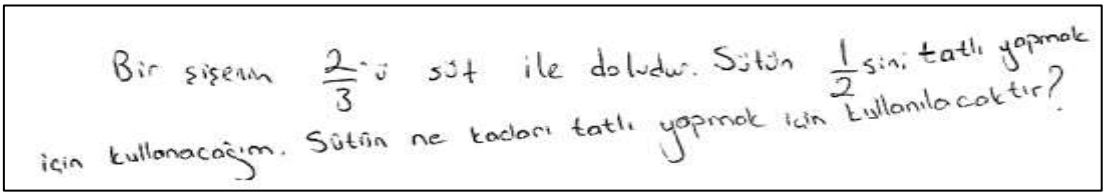

Figure 1: Appropriate Posing: Part of Part

Three pre-service elementary teachers tried to pose story problems based on the repeated addition meaning, which refers to a certain number of groups of the same size; however, it is impossible to pose an appropriate problem focusing on the repeated addition meaning for $\frac{2}{3} \times \frac{1}{2}$ =?. Normally, students can easily understand that a×b is equal to adding b to itself a times; however, it is difficult to apply this understanding to the multiplication of fractions. The reason for this difficulty is that repeated addition of

equally sized groups is not applicable beyond natural numbers. For example, adding $\frac{1}{2}$ exactly $\frac{2}{3}$ times is hard to conceptualize. One of the wrong problems posed by the pre-service elementary teachers for $\frac{2}{3} \times \frac{1}{2}$=? was as follows:

> *"Elif ate $\frac{1}{2}$ of a bread for breakfast. Elif ate $\frac{2}{3}$ times as much bread for dinner as she did for breakfast. How much bread did Elif eat for dinner?"*

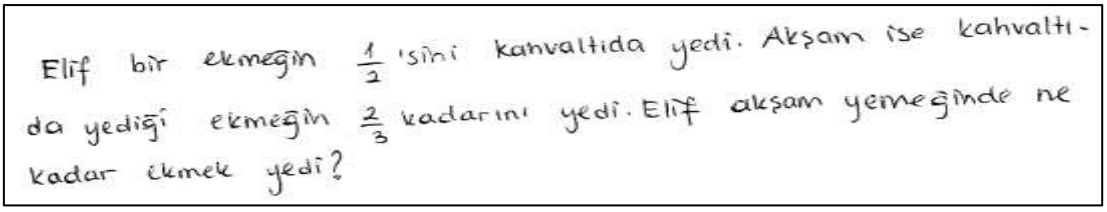

Figure 2: Inappropriate Posing: Repeated Addition

As can be seen in Table 2, while 14 of the pre-service elementary teachers' statements were not accepted as a problem, as they were similar to "What is $\frac{1}{2}$ times $\frac{2}{3}$?", 25 of them could not pose a problem for the given symbolic form.

3.1.2. Error Types Associated with Problems Posed for the Multiplication of Fractions

To identify what kind of errors were made in the pre-service elementary teachers' posed problems for the second item in the PPT, the problems with error(s) were identified and their frequencies were given below:

Table 3: The distribution of answers associated with the second item of the PPT

	Errorless	With Error	Not a Problem	Unable to Pose
Write a story problem for $\frac{2}{3} \times \frac{1}{2}$=?	34	10	14	25

Considerable amount of the pre-service elementary teachers successfully posed a story problem requiring multiplication of two fractions and all of them focused on the part of part meaning of the multiplication operation. As mentioned above, three pre-service elementary teachers made an error resulting from focusing on the repeated addition meaning of the multiplication operation in their problems. The remaining pre-service elementary teachers who made an error either failed in expressing the multiplication operation in the question root or attributed natural number meaning to the result of the multiplication. The distribution of these error types was given in Table 4.

Table 4: The distribution of error types associated with the second item of the PPT

	ME1	ME2	ME3	ME4	ME5
Write a story problem for $\frac{2}{3} \times \frac{1}{2}$=?	0	1	6	0	3

The pre-service elementary teachers who failed in expressing the multiplication operation in the question root instead used the addition, subtraction, or division operation in their problems. One of these problems, in which PT_1 used the subtraction operation rather than the multiplication operation, was as follows:

"While Ayse ate $\frac{1}{2}$ of a cake, Elif ate the $\frac{2}{3}$ of the cake that Ayse had eaten. How much more did Ayse eat than Elif?"

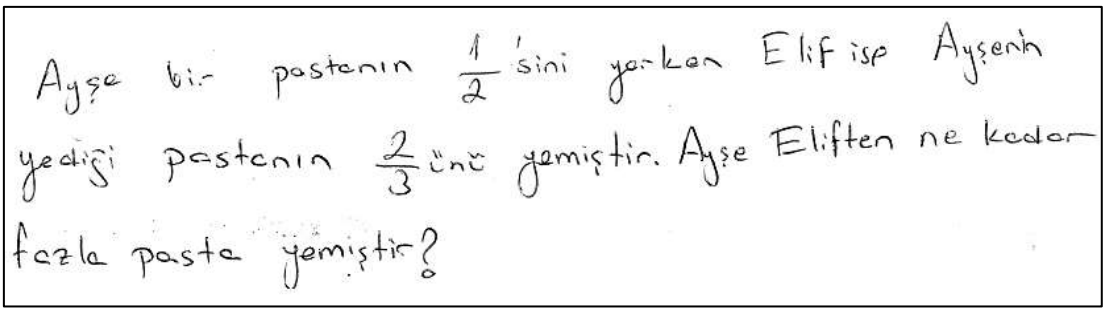

Figure 3: A problem with ME3

Two other pre-service elementary teachers posed problems requiring the addition operation, and one of these problems was given below:

"Ali walked $\frac{1}{2}$ of the way from his home to school. After he sat down to take a rest, he walked $\frac{2}{3}$ of the way that he had walked. How much of way did Ali walk altogether?"

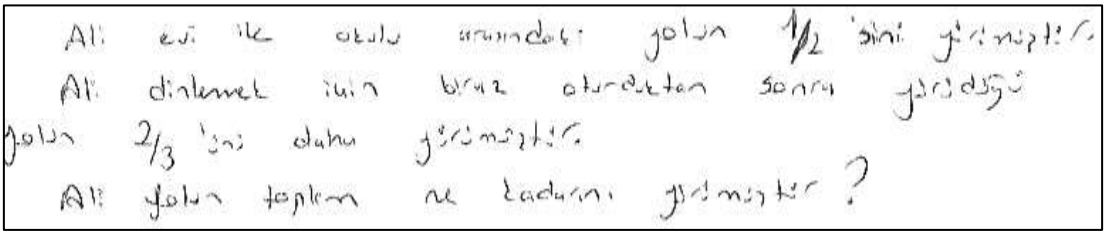

Figure 4: Another problem with ME3

3.2 Division of Fractions

3.2.1. Meanings Associated with Problems Posed for the Division of Fractions

Analyses of the problems posed by the pre-service elementary teachers showed that nearly half of the pre-service elementary teachers (39) could not even pose a problem for the given symbolic form, as can be seen in Table 5.

Table 5: The distribution of meanings associated with the sixth item of the PPT

	Problem			Unable to Pose	Not a Problem
	Measurement	Equal-Sharing	Others		
Write a story problem for $\frac{5}{6} \div \frac{2}{3}$ =?	30	1	1	39	12

Furthermore, most of the pre-service elementary teachers focused on the measurement meaning of the division operation rather than the equal-sharing meaning. One example of these problems was the following: "I am planning to serve $\frac{5}{6}$ of a cake and a serving size will be $\frac{2}{3}$ of the cake. How many servings can I make from $\frac{5}{6}$ of the cake?"

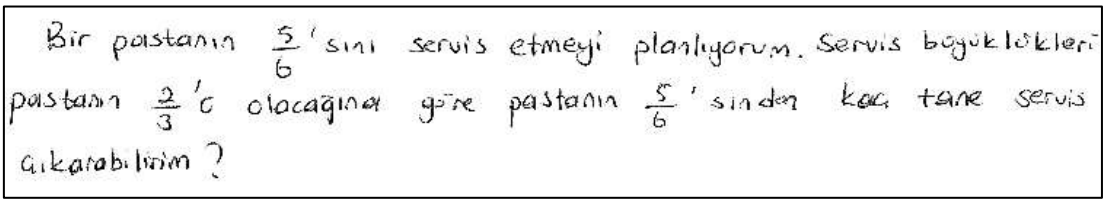

Figure 5: Appropriate Posing: Measurement

There was only one problem that employed the equal-sharing meaning of the division operation, and it was written by PT72: "Ali can read $\frac{2}{3}$ of the book in $\frac{5}{6}$ of an hour. How many hours does it take Ali to read the book?

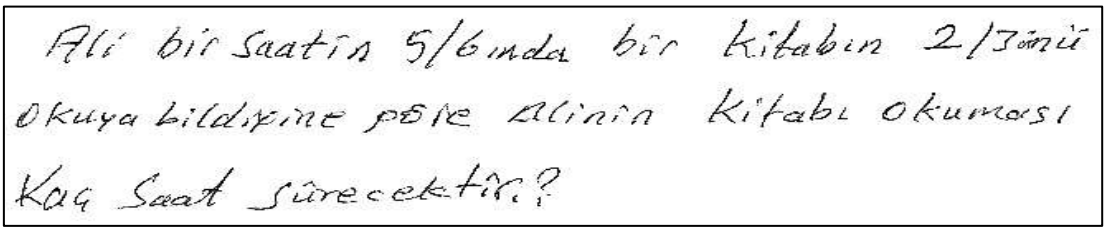

Figure 6: Appropriate Posing: Equal Sharing

3.2.2. Error Types Associated with Problems Posed for the Division of Fractions

Before identifying the error types made by the pre-service elementary teachers, problems with error(s) were identified and the frequencies of them were given below in Table 6:

Table 6: The distribution of answers associated with the sixth item of the PPT

	Errorless	With Error	Not a Problem	Unable to Pose
Write a story problem for $\frac{5}{6} \div \frac{2}{3}$=?	9	23	12	39

As can be understood from the table, of the pre-service elementary teachers who posed a problem for the sixth item in the PPT, only nine of them posed a problem without an error. One of these problems focused on the equal-sharing meaning of the division operation, and eight of them focused on the measurement meaning. That is, while 30 pre-service elementary teachers employed the measurement meaning in their problems, only 8 of them correctly posed a problem for the given symbolic form.

When the problems with errors were examined, it was seen that the pre-service elementary teachers made only two types of errors, DE2 and DE6. Furthermore, nearly

all of the errors were of the type DE2, which refers to assigning natural number meaning to fractions. The distribution of the error types was given in Table 7.

Table 7: The distribution of error types associated with the sixth item of the PPT

	DE1	DE2	DE3	DE4	DE5	DE6	DE7
Write a story problem for $\frac{5}{6} \div \frac{2}{3}$=?	0	22	0	0	0	1	0

A typical problem with a DE2 error posed by the pre-service elementary teachers was as follows: "I would like to serve $\frac{5}{6}$ of a cake to my guests. If I have to serve $\frac{2}{3}$ a cake to each guest, how many persons can I serve from $\frac{5}{6}$ of the cake?".

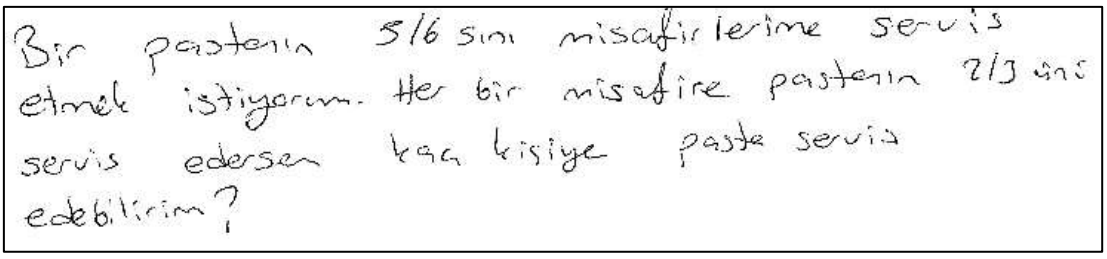

Figure 7: A problem with DE2

Although the result of the mathematical expression given in Table 6 was a mixed fraction, PT_5 attributed the natural number meaning to the result by using the statement "how many persons." The only problem with the DE6 error was posed by PT_{82} and it was given below:

> "$\frac{5}{6}$ *of a bottle is filled with milk. Aslı drank* $\frac{2}{3}$ *of the milk during her breakfast. How much of the milk is drunk by Aslı?".*

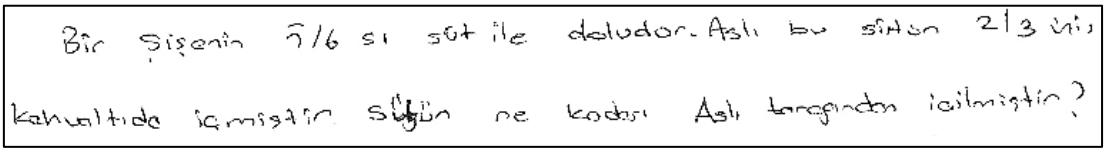

Figure 8: A problem with DE6

4. Conclusion, Discussion, and Recommendations

This study explored the pre-service elementary teachers' problems for the multiplication and division of fractions considering the meanings of these operations. The errors associated with the pre-service elementary teachers' problems were also examined. For the multiplication of fractions, the analysis showed 37 out of 83 pre-service elementary teachers used the part of part and repeated addition meaning of the multiplication operation when posing their problems. In addition, 25 of the pre-service elementary teachers could not pose a problem, and 14 of them wrote statements similar to "How much is the multiplication of $\frac{2}{3}$ and $\frac{1}{2}$?", which were not accepted as a problem.

The pre-service elementary teachers who were able to pose a problem mostly focused on the part of part meaning of the multiplication of fractions in their problems. All of these problems were correct for the given symbolic form. This result supports the findings of Luo (2009), who stated that pre-service elementary teachers were more successful in posing problems focusing on the part of part meaning than the repeated addition meaning of the multiplication of fractions. Similarly, the pre-service elementary teachers who tried to focus on the repeated addition meaning of the multiplication of fractions could not pose problems correctly.

For the division of fractions, almost half of the pre-service elementary teachers left the division item blank. Thirty of the posed problems focused on the measurement meaning of the division operation and only one of them focused on the equal-sharing meaning. The reason for preferring to employ the measurement meaning in the problems may result from giving too much emphasis on the part-whole meaning of the fractions in textbooks (Lamon, 2006). However, those pre-service elementary teachers who attempted to pose problems focusing on the measurement meaning were largely unsuccessful.

The errors made by these pre-service teachers for the multiplication of fractions were in three types, which were attributing natural number meaning to the result of the operation (ME2), failure in expressing the operation in the question root (ME3), and using numerical quantifying (ME5). Actually, the error type, failure in expressing the operation in the question root (ME3), is not specific to the multiplication of fractions, as previous studies related to the addition, subtraction, and division of fractions found similar results (Isik & Kar, 2012; Redmond & Utley, 2007; Rizvi, 2004; Toluk-Ucar, 2009). For the division of fractions, the most common error, comprising 22 out of 23 errors, was DE2, which resulted from attributing natural number meaning to the result of the operation. These findings are similar to the previous studies, as researchers also mention that in/pre-service teachers still have difficulties in problem posing activities (Chapman, 2012; Leung & Carbone, 2013). To overcome their difficulties, teacher educators can include this kind of activities in their courses (Kilic, 2013; Rizvi, 2004; Toluk-Ucar, 2009). Apart from raising the pre-service elementary teachers' awareness of the importance of problem posing, the findings of this study contribute to the literature by providing an insight into how future instruction needs to be employed to help pre-service elementary teachers pose appropriate problems. Furthermore, the reasons for the specific errors made by pre-service elementary teachers and the strategies to overcome these errors can be also examined in future studies.

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