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AN ANALYSIS OF HOW PRESERVICE MATH TEACHERS CONSTRUCT THE CONCEPT OF LIMIT IN THEIR MINDS

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Abstract:
Recently, people prefer learning information operationally rather than conceptually. In this context, this study was carried out to uncover how preservice math teachers construct in their minds the conceptual definition of limit within the scope of the Calculus Course. The participants of this study consisted of 62 (30 female, 32 male) sophomore students studying in the Elementary Mathematics Teacher Education Department at Uludağ University Faculty of Education in the 2016–2017 academic year. Midterm and final exam questions requiring the use of prior knowledge were used to help collect data. Interviews were conducted with three participants who were chosen for their success. In these interviews, five questions were asked by the researchers to uncover the mathematical thinking levels and abstraction processes of the participants. The methods of semi-structured interviews and observations were used to collect data. The data were video-recorded and transcribed. The transcripts were analyzed and interpreted according to the cognitive actions of the RBC- model and the steps in Sfard's theory of mathematics learning. Based on the analysis, the participants were found to be more successful in operational information than in conceptual information. Although the preservice teachers were able to accomplish operational learning, it can be said that they could not fully accomplish conceptual learning because they could not identify algebraic representations and could not use reasoning on these representations. Interviews with the participants revealed that they memorized the characteristics of the concept of limit to be successful in the exams. However, conceptual learning did not take place. Understanding how participants learn is believed to benefit the educators who teach the concept of limit.

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1. Introduction

The concept of limit is among the most fundamental concepts of calculus since it is directly linked to many important concepts such as derivatives, integrals, continuity and the approximation theory (Cornu, 1991; Çıldır, 2012). Although limit has the characteristics of a fundamental concept, only a small portion of individuals can achieve adequate understanding of the concept of limit in everyday mathematics education (Tall & Vinner, 1981; Sierpinska, 1987). Limit is a word with Latin origins, and some of the other meanings of it are as follows: Limiting, boundary, maximum extent, limitation, and the number in which images below the function are stacked in numbers that are close to this number, except for a specified number.

The mathematical definition of the concept of limit was expressed by Augustin-Louis Cauchy (1780–1857) as follows: "*When the values successively assigned to the same variable indefinitely approach a fixed value, so as to end by differing from it as little as desired, this fixed value is called the limit of all the others*" (URL-1). Another definition was made by Karl Weierstrass (1815–1897): "*$c \in I$, if for each $\varepsilon>0$ there is at least one $\delta> 0$ real number for the real numbers x providing the inequality of $0< |x-c|< \delta$ such that $|f(x) - L| < \varepsilon$, then L is called the limit of function f at point c and is indicated by $lim_{x\to c} f(x)=L$" (URL-2).*"

The fact that there are infinite processes in the concept of limit creates an obstacle for individuals to easily understand the concept of limit. By teaching the concept of limit within the context of the Calculus Course, it is aimed, in general, to have students gain skills such as the ability to calculate the limit value of various functions at a certain point, understand the relationship between the graph and the limit of a function, use the theorems of the concept of limit, and be able to determine whether the limit exists through the formal definition. Undoubtedly, students' ability to attain these gains depends on their ability to have a sufficient conceptual understanding of the formal definition of the concept of limit (Baki & Çekmez, 2012).

Studies in the literature show that only a small number of individuals understand the formal definition of the limit concept sufficiently through instruction (Ervynck, 1981; Quesada, et al., 2008). The definition of limit is so full and difficult to be understood, which has led students to develop the idea of highlighting an informal definition in the teaching of the concept of limit, and pushing the formal definition to the background (Fernandez, 2004; Gass, 1992). On the other hand, a number of researchers think differently and make the formal definition of the concept of limit as the fundamental point of transition to abstract thinking, making inferences about formal mathematical expressions and using formal proof techniques (Ervynck, 1981; Swinyard & Lockwood, 2007). Novak (1993) has stated that the goal of education is to lead students to meaningful learning. He has defined meaningful learning as the necessity of establishing links between newly introduced concepts and prior knowledge. Concept learning is defined differently for each approach. It is explained by a link between the stimulant and the reaction established by the individual in the behavioural approach, while it is expressed as remembering information and restructuring it through flexible perceptions in the cognitive approach.

Denbel (2014) has stated that university students see limit as something unattainable, an approach, a boundary or a dynamic process. Moreover, Denbel has expressed that students make many misconceptions; the limit of a function is the image of the infinite status of limit and a misunderstanding of the uncertainty in the derivative definition of limit.

As the concept of limit plays a role in teaching the concepts of continuity, derivatives and integrals, there are many studies in the literature on the concept of limit. A great majority of the studies intend to identify the difficulties that students experience while making sense of the concept of limit in their minds, the contribution of different instructional methods in understanding the concept of limit and the misconceptions about this concept (Akbulut & Işık, 2005; Altun, 2009; Kabaca, 2006). By taking advantage of their own teaching experience, Queseda et al. (2008) have listed the difficulties that students face when learning the concept of limit as follows: the use of quantifiers in the definition and students' seeing the role of quantifiers in proving the existence of limit for the first time, and the fact that students' previous mathematical experiences do not give them any chance to understand the relationship between algebraic expressions of inequalities within the definition of limit. The difficulties also include the trouble in expressing inequalities in a limit problem, having difficulty in finding the value ranges of variables by using inequalities within the limit definition, and having difficulty in making algebraic changes in inequalities to determine the relationship between the ε and δ variables. In his research on the difficulties preservice teachers experience when making the formal definition of the concept of limit, Barak (2007) has found that preservice teachers are not able fully comprehend the definitions of Ɛ and δ. The preservice teachers say that this statement is only the definition of limit. They are unable to explain exactly what the Ɛ and δ symbols mean, and these symbols do not articulate anything clear in their minds. Problems that involve limitlessness are not a simple subject for students (Baştürk & Dönmez, 2011; Tangül, Barak & Özdaş, 2015). Besides that, it has been stated that preservice teachers cannot fully comprehend the inequality within the definition of limit and cannot find the equality between the values of Ɛ and δ. A number of researchers have researched how students learn about limits and have carried out certain instructional studies to eliminate information-related restraints and misconceptions (Akbulut & Işık, 2005; Biber & Argün, 2015; Bukova-Güzel, 2007; Çıldır, 2012; Dönmez & Baştürk, 2010; Kula & Bukova-Güzel, 2015; Przenioslo, 2004; Roh, 2007; Tangül et al., 2015).

2. Literature Review

2.1 RBC + C model

Since the abstraction process is not something directly observable (Dreyfus, 2007), it has become necessary to identify observable actions that can provide information about the abstraction process. This model has been defined by Hershkowitz, Schwarz and Dreyfus (2001) as the RBC (Recognizing, Building-with, Constructing) Theory of Abstraction in order to analyze mathematical abstraction processes. The Nested RBC Model of Abstraction (Recognizing, Building-with, Constructing) Model emphasizes the need for abstraction and stimulation (encouragement) in the process of constructing (forming) the concept in the mind. Hershkowitz, et al., (2001) has defined the epistemic actions of the abstraction process as recognizing, building-with, and constructing. In RBC, epistemic actions have been observed to be non-sequentially sorted when a person constructs a concept in his mind (Dreyfus, 2007). It is emphasized that the actions have a structure that can be intertwined with one another and can accommodate each other (Özmantar, 2004).

Recognizing means an individual's ascribing a meaning to mathematical structures in the learning environment by using formal or informal information that exists in his repertoire (Hershkowitz, et al., 2001). This includes the recognition of structures that individuals are familiar with about the mathematical structure being studied (Bikner-Ahsbahs, 2004; Hassan & Mitchelmore, 2006); in other words, the use of the structures when necessary (Dreyfus, 2007).

The act of building-with is observed when an individual is faced with the need to use mathematical structures, which he has recognized, in describing or associating with something, in defending a proposal, in making assumptions, or in problem-solving in order to produce new information (Hershkowitz, et al., (2001); Dreyfus, 2007). During the process of building-with, individuals, who need familiar structures for the purpose of generating new information, use their existing structural knowledge to create a solution that is feasible for the problem. This process, in which the act of building-with is intertwined with the epistemic action of recognizing, requires that what is known be associated with the new content (Bikner-Ahsbahs, 2004; Hershkowitz, et al., 2001). An individual's building-with action is a critical point in the abstraction process, and when not observed, it can give him a hint to activate him (Dreyfus, 2007).

Creating is the process in which the individual restructures the recognized structures by subjecting them to a partial change, and thus, he constructs new structures/meanings based on that action (Bikner-Ahsbahs, 2004). This is because the individual is unable to create a new structure without accomplishing other cognitive actions using his knowledge and experience. Creation occurs as a result of the realization of the two other cognitive actions (Dreyfus, 2007). The creation of a structure will also take place when the individual intensely contemplates on the mathematical subject alone (Dreyfus, Hershkowitz & Schwarz, 2001).

The problem may enable a student to perform the act of creating knowledge while another student performs the act of recognizing. In other words, the occurrence of these actions is not clear and precise. This depends on the student's past experiences, personal skills, and whether stimulants trigger the student's knowledge (Dreyfus et al., 2001). However, Dreyfus (2007) has indicated that the new structures created using abstraction are fragile, which makes it difficult to maintain the new structure. Consolidation can happen if a person associates structures with each other, if the structures are used in creating a new structure, and if these structures are deeply considered. The act of consolidation can arise while students study the mathematical subjects that they know well and also when they use a situation or a concept — which they have just abstracted — for an advanced abstraction (Dreyfus & Tsamir, 2004).

Hershkowitz et al. (2001) in a study on ninth graders has revealed that abstraction takes place during problem-solving. Özmantar and Monaghan (2007) have stated the factors that influence the process of abstraction in their study on the absolute value function. Yeşildere and Türnüklü (2008) have examined the effects of different mathematical powers on the abstraction process.

RBC + C model has been examined using various mathematical concepts in different studies: The greatest integer function (Altun & Yılmaz, 2008), characteristics of algebraic and arithmetic operations (Dreyfus et al., 2001), ratios and proportions (Hassan & Mitchelmore, 2006) and probability (Dreyfus, Hadas, Hershkowitz & Schwarz, 2006; Hershkowitz, 2004; Schwarz, Dreyfus, Hadas & Hershkowitz, 2004). Moreover, it has been researched in the subjects of functions (Hershkowitz et al., 2001), absolute value (Özmantar, 2004; 2005), linear equations (Sezgin-Memnun & Altun, 2012), infinity (Tsamir & Dreyfus, 2002), and triangles (Yeşildere & Türnüklü, 2008).

2.2 Sfard's Theoretical Model for The Learning of Mathematical Concepts

The common characteristic of the explanations to date about what abstraction is that abstraction has been discussed by researchers in the context of a process. Many researchers have attempted to identify the steps of this process. For example, Sfard (1991) argued that abstract concepts would be perceived as operational and structural, and he defined abstraction in a theoretical structure. He stated that abstraction consisted of the steps of internalization, condensation and reification. In the theory based on the work of Piaget, it is emphasized that the students' understanding of mathematical objects is bidirectional (operational–structural), the operational structure precedes the conceptual structure, and the transition from the operational comprehension to structural comprehension takes place in a three-stage process (Sfard, 1991, p.18).

A mathematical concept is a complex network of ideas developed from mathematical definitions and mental structures (Sfard, 1991; 1992). According to Sfard (1991), when a new concept is learned, a natural starting point is established through a definition. Certain mathematical definitions consider concepts as objects which are the components of a larger existing system. This is considered a structural conceptualization. On the other hand, concepts can also be defined in terms of actions that lead to operations, algorithms or an operational understanding. A structural understanding requires the ability to visualize a mathematical concept as a "real thing" that exists as part of an abstract mathematical structure. In contrast, an operational understanding requires more than some actions that need to occur or a potential that requires a procedure. Although these two approaches appear to be different, they are actually complementary to each other. They can be regarded as two sides of a coin. Both are crucial to creating a profound understanding of mathematics.

The steps in Sfard's theory of mathematics learning can be listed as understanding operations between the same objects (interiorization), understanding the transformation of operations to other objects (condensation), and obtaining a new object (reification).

When the basic principles of the two approaches, which were highlighted to make sense of the idea of abstraction in this study, are examined, the presence of certain similar points can be seen. One of the similarities is that the two opinions accept abstraction as a process. However, the opinions are separated from each other in the later steps. Noss (2002, p.5) has described this difference as follows: The idea of conceptualization or abstraction as a piece of information is in a separate area from the action, tools, language, or the signal system outside it. The idea of abstraction in this sense is mathematically important since it creates a system with its own concepts and rules that are used to transfer these concepts (Piaget, 2000). This characteristic of formal mathematical abstraction is central to its own benefit. It is questioned whether situational abstraction and mathematical abstraction can be completely separated from the conditions (context) surrounding their own creation processes.

From a sociocultural standpoint, in the cognitive perspective, it is not an exception to find common grounds between certain examples. However, the depth of mathematical understanding becomes important when the similarities are interpreted. Although Noss (2002) has discussed the suspicion of addressing abstraction through the cognitive perspective in the context of situational abstraction, this suspicion is also likely to be found in other researchers with a sociocultural perspective. Another point in which the two opinions are divided is that the role of the context is perceived differently in the fulfillment of abstraction.

After an examination of the existing perspectives for explaining abstractions, the theories that addressed abstraction through socio-cultural perspectives were considered to be more suitable for the present study.

Several examples of learning were found that fitted the nested RBC model, but none was found that fitted the empirical abstraction model; nevertheless, we believe there is a place for a kind of empirical abstraction in higher mathematics (White & Mitchelmore, 2010). In this research study, the RBC- model and Sfard's theoretical model for the learning of mathematical concepts will be used to examine the way students construct their knowledge of limits through observable cognitive actions, and it will be researched how the learning process advances as the interventions predicted by the theoretical structure are implemented. The main premise is to question the change in the conceptual state of the preservice teachers who can make operational interventions in the concept of limit.

2.3 The Goal of the Paper

The need for this study stems from the following fact as expressed by one of the researchers: According to the experience we have gained from the Calculus I, II, III and IV courses we have thought in most of 27 years, we know that it is difficult to understand functions without an understanding of variables, it is difficult to understand limits without an understanding of functions, and it is difficult to understand differentials and integrals without an understanding of limits. Although almost half of the undergraduate students calculated limits correctly, it was seen in their exam papers that they were unable to be fully consistent and they experienced confusion when limits were asked conceptually.

It was aimed in this study to uncover the processes of comprehension developed by preservice math teachers for the conceptual definition of limit. It is thought that understanding the change that was experienced in this process will contribute to the teaching of the concept of limit.

To this end, the research problem of the study was determined to be: "As a result of the teaching of the concept of limit in the context of the Calculus Course, how do preservice teachers understand the formal definition of the concept of limit?" Answers to the following sub-problems were sought:

- What is the conceptual and operational understanding of preservice math teachers in limit questions?
- What is the understanding developed by preservice teachers in terms of learning limits conceptually?
- How do preservice math teachers construct limits in their minds?

3. Methods

In this study, a case study approach was used, as one of the qualitative research types. It was decided to use this method because the study accommodated both descriptive and explanatory characteristics (Yin, 2003), which are the two characteristics of a good case study. The sample of this study carried out to uncover the understanding developed by preservice teachers to learn limits conceptually during the Calculus Course consisted of 62 (30 females and 32 males) elementary school preservice math teachers. Participants were sophomores studying in the secondary mathematics education program, and it was found out that all of the participants regularly participated in the Calculus Course.

3.1 Data Collection Instrument

More than one data collection instruments were utilized during the research process. A test with 5 questions as a pre-test at the midterms and with 7 questions as a post-test at the final examinations was used, and all questions were open-ended (Appendix 1). The test was prepared by taking into account the studies on the definition of the concept of limit in the literature and expert opinions. The test was considered and administered as the midterm and final exams of the Calculus Course. The purpose for administering the exams was to see the actual and top success performances of the participants. The participants answer to the questions in the test were categorized in terms of accuracy. The numbers of participants in the categories were presented in Table 7. In addition to that, excerpts were taken from the participants' answers to the questions in the test, and examples were given for the mistakes and misunderstandings that emerged. As a result, an attempt was made to understand the extent to which the participants constructed limits conceptually in their minds. In light of the sub-problems of the study, 5 questions and objectives related to the concept of limit were selected from the questions of the test. The analysis of the data was carried out through these objectives and questions.

Table 1: Objectives and Questions Selected from the Midterm and Final Exams

Selected Main Objectives	Test Type	Question Number
Operational Expression of Limits	Midterm	2
Proof of limits using the $\varepsilon - \delta$ technique	Midterm	4
Using operational knowledge to reach the concept of limit	Midterm	5
Expressing the limit at a point a of a specific function at a range (the use of a theorem)	Final	3
Formal definition of the limit of a function	Final	7

On the basis of the midterm and final questions, the common point of the objectives of the selected questions was how the formal definition of limit developed during the learning process and how the participants' operational and conceptual knowledge had changed in the process of doing mathematics. Moreover, depending on the problem situation of the study, the participants in the lower 27% slice and upper 27% slice were determined and were labeled as the lower and upper groups based on the general score of the participants' midterm and final exam averages. A total of three participants were interviewed, including one participant from the upper and lower groups each and a participant from the average level. An attempt was made through the interviews to explain the participants' ideas about the concept of limit. The interview questions were developed by the researchers of the study (Appendix 2). The interview was conducted by preparing a story situation and five questions to determine the status of the participants according to the RBC- model (Appendix 2). A story was directed to the participants. And then, the 5 interview questions were coded as Koray, Ali and Saliha. The researcher was coded as A.

From among these answers, more excerpts were presented from the transcript of the interview with Koray.

3.2 Data Analysis

The data collected in the study were analyzed in two stages. In the first stage, the objectives selected for the midterm and final exams (see Table 1) were discussed using two different theories. In the second stage, interviews were conducted with the preservice teachers who were selected according to the results of these analyses. The questions chosen by the researchers from the midterm and final papers, considering the conceptual and operational learning of limits, were interpreted in order to compare the steps of the two theories. First, a preliminary analysis framework was created based on the steps of these two theories. Next, the participants' solutions were examined according to this framework. In addition to that, the analysis of the interview questions prepared in order to further analyze the preservice teachers' definitions of the concept of limit and their status according to the steps of the two theories was again tried to be explained based on the characteristics of the RBC- model. The audio recordings obtained in the interviews were first transcribed and then analyzed separately by the two researchers. For the analysis steps, the transcripts of the preservice teachers' answers to the questions were taken into consideration. The transcripts were subject to a thematic analysis. Thematic analysis is a form of recognizing the pattern in the data, in which the emerging themes become categories of analysis (Fereday & Muir-Cochrane, 2006). The analysis and review take a closer look at the selected data and give the ability to construct codes and categorizations based on the characteristics of the data to uncover themes that are relevant to a theory. Predefined codes can be used, especially if they complement other research methods used in the study. For the thematic analysis, the researchers defined the themes for the RBC- model and Sfard's theory of mathematics learning, and carried out this thematic analysis according to the answers the individuals had given in the transcripts. The thematic analysis template prepared by the researchers for the interview questions is shown below.

A. Recognize: The recognizing step is divided into two:

- R1: Analogy (Simile),
- R2: Customization (Seeing that two things are identical).

B. Building With: What is expected of the individual here is to get something processed or solved. It is divided into two:

- B1: Using the previously created mathematical structure (understanding the mathematical situation, explaining the mathematical situation),
- B2: Combining similar kinds of information and using them for a solution (Problem-solving, contemplating about the process).

C. Construction: This step, also called forming a structure, can be divided into three.

- C1-Requiring a new structure,
- C2-Creating a new structure,
- C3-The person's consolidating the new structure so as to facilitate the act of recognition.

Table 2 below shows the preliminary analysis indicators of the interview questions according to the RBC- model.

Table 2: RBC Preliminary Analysis Data

Selected Questions	**R**	**B**		**C**		
		B1	**B2**	**C1**	**C2**	**C3**
1 (Midterm question 2)	✓	✓				
2 (Midterm question 4)	✓	✓				
3 (Midterm question 5)	✓	✓	✓			
4 (Final exam question 3)	✓	✓	✓			✓
5 (Final exam question 7)	✓		✓		✓	

Another preliminary analysis was conducted on the midterm and final exam questions that were related to limits. The reason for the selection of the five questions listed below, for which the preliminary analyses are shown, is the fact that they cover the operational and conceptual learning of limits. These five questions were selected and classified as they were suited to the steps of the RBC- model and Sfard's theory of mathematics learning. In this classification, which question belonged to which step was determined by ensuring consensus of the researchers. Table 3 below shows the skills expected from the preservice teachers when solving the questions and the steps in the theory to which these skills belong.

Table 3: Comparison of the questions in terms of Sfard and RBC steps

Task 1 — Midterm 2		
Investigate the limit: $\lim_{x \to 1/2} [\![x]\!]$.		
Sfard's	**The Skill Expected in the Solution**	**RBC**
(Interiorization)	Recalling the greatest integer function	**(Recognize)**
	Expressing the existence of right limit and left limit	**(Building with)**
Task 2 — Midterm 4		
Prove that $\lim_{x \to -3}(2x+5) = -1$ using the $\varepsilon - \delta$ technique.		
Sfard's	**The Skill Expected in the Solution**	**RBC**
(Interiorization)	The operational part of limits	(Recognize)
(**Condensation**)	*Expressing* $\varepsilon - \delta$	(Building with)
	Being able to prove	**(Construction)**
Task 3 — Midterm 5		
Given that the limit $\lim_{x \to c} f(x)$ exists and the limit $\lim_{x \to c} g(x)$ does not exist, find the f(x) and g(x) functions that provide the conditions where the limit $\lim_{x \to c}(f(x) + g(x))$ does not exist.		
Sfard's	**The Skill Expected in the Solution**	**RBC**
(Interiorization)	Investigation of the limits of three functions	(Building with)
(**Condensation**)	Being able to write the functions $f(x)$ *and* $g(x)$	**(Construction)**
Task 4 — Final 3		
Given the function $f(x)$ =$b - \lvert x-a \rvert \le f(x) \le b + \lvert x-a \rvert$, investigate the limit of f(x) at point a by explaining it.		
Sfard's	**The Skill Expected in the Solution**	**RBC**
(Interiorization)	Finding the right limit and left limit at a point a.	(Recognize)
	Finding the limit of a function with the help of the Sandwich theorem	(**B**uilding with**)**
Task 5 — Final 7		
Describe the concepts of a function and the limit of the function.		
Sfard's	**The Skill Expected in the Solution**	**RBC**
In this question, a description is obtained according to the two theories, taking into consideration the functions and features that participants identify.		

3.3 Reliability and Validity

Triangulation and interrater reliability methods were utilized to ensure the reliability in this study (Lincoln & Guba, 1985). In order to ensure triangulation, the suitability of the categories to the steps of the theory was determined by using the preservice math teachers' written texts and the transcripts of their voice recordings. While the data from the written texts were used predominantly to define categories, the voice recordings were used to validate the categories. For example, based on the written texts, it was determined whether they conformed to the three steps of the RBC- model. The classifications were finalized after they were confirmed through the voice recordings. For the interrater reliability, the predictions, observations and explanations of the preservice math teachers were assessed as raw data by two independent researchers. The researchers assigned them to independent thematic categories in connection with the steps of the theory. It was seen that the consensus of the researchers was over 80% after the coding. The researchers had repeatedly discussed inconsistencies in these categories until consensuses had been achieved.

4. Results and Discussion

Table 4 shows the preservice teachers' achievement statuses in the midterm and final exams of the Calculus Course. Based on the data obtained from the preservice teachers, the frequency and percentage distributions of the answers to the questions about limits are given in Table 4.

Table 4: Midterm and Final Exam Achievement Status

	N	$\bar{x}$
Midterm Exam	62	57
Final Exam	62	64

The preservice teachers were more successful results in the midterm exam than in the final exam. The maximum score in both exams was 100 points. The final exam grades of the preservice teachers were approximately 12% higher than their midterm exam scores.

Four of the five questions asked in the midterm exam were directly related to the concept of limit. The ability of the teacher candidates to use limits in mathematical operations, their ability to conceptually explain limits and their ability to formally define the concept of limit have demonstrated a wide range of outcomes. An example of this situation can be seen in Table 5 below.

Table 5: Classification of the Midterm Exam Questions According to the Limits Objectives

Formal Definition of Limits	**f**	**%**
Question 4	12	18
Conceptual Explanation of Limits	**f**	**%**
Question 5	13	21
Ability to use the concept of limit in operations	**f**	**%**
Question 2	40	64
Question 3	20	32

* Only the first question out of the five questions in the midterm exam was not related to the knowledge on limits directly.

Only 18% of the preservice teachers were able to reach the formal definition of limit in the midterm exam by using the ε-δ technique, which is the formal definition of the concept of limit. The preservice teachers who responded incorrectly expressed the concept in their own sentences instead of the above definition. They usually saw a limit as a delimiter. They showed examples of delimiters from daily life. The rest of the class left the question blank. Considering the solutions, it was seen that the capability to use limits in mathematical operations had reached approximately 30% success. The use of limits in operations rather than the formal definition of limit demonstrated that it gave better results for the preservice teachers.

Of the seven questions directed to the preservice teachers in the final exam, only three were about the concept of limits. The skills of the preservice teachers about the formal definition of the concept of limit in the final exam demonstrated different results compared to the ones in the midterm exam. This can be seen in Table 6 below.

Table 6: Classification of Final Questions according to the Limits Objectives

Proof using the $\varepsilon - \delta$ technique (Formal Definition)	**f**	**%**
Question 7	8	13
Ability to use the concept of limit in operations	**f**	**%**
Question 2	42	68
Question 3	32	51

Only 13% of the preservice teachers were able to reach the formal definition of limit in the final exam by using the ε-δ technique. Moreover, there were also those who used different methods in identifying limits. Some of them were inclined towards the graphical representation of limits but could not answer the question correctly. The majority of the preservice teachers were seen to develop the use of limits at the operational level, which was used in the midterm exam.

The questions that were chosen from among the questions directed to examine the process of change in the conceptual and operational understandings of the preservice teachers about the concept of limit in the mid-term and final exams during the instruction and were believed to give more ideas about the construction of limits in the mind were expressed in the data collection instruments. Questions that could not be solved without using conceptual knowledge were labeled as conceptual. Questions that could be solved using limit calculations and a series of arithmetic mathematics were labeled as operational. Questions that require conceptual knowledge following operational knowledge are included in Table 7.

Table 7: Success Status in Operational and Conceptual Types of Question

		Correct		**Incorrect**		**Unanswered**	
Question Type	**Questions**	**f**	**%**	**f**	**%**	**f**	**%**
Operational	1	40	64	8	13	14	23
Operational and Conceptual	2	12	18	16	27	34	55
	4	32	51	12	20	18	29
Conceptual	3	13	21	21	34	28	45
	5	8	13	16	25	38	62

As seen in Table 7, the average number of correct answers given by the preservice teachers in response to questions involving conceptual information was 11, and the mean percentage was 17. The response rate of questions 1 and 4 from among these five questions was higher than that of the others. The percentage of correct answers given to question 5 was less than the percentage of correct answers given to the other questions. In the question group where the operational information was included, the participants' average number of correct answers was 40 and the average percentage was 64. The preservice teachers' average number of correct answers to the questions including operational and conceptual information was less than that of the questions requiring only operational knowledge. What attracts attention here is that 62% of the preservice teachers did not answer question 5. Question 5 was a question about the formal definition of limits. The success of the preservice teachers decreased from the operational information to the conceptual information.

Based on the interviews with the preservice teachers, their statements were as follows:

Koray: *"Yes, unfortunately, I memorized, I did not understand its logic."*
Ali: "*Since we've learned it for the exam, it does not go beyond memorization, and it is forgotten very quickly."*

An overall analysis of the data shows that participants were more successful in operational information rather than in conceptual information.
In the interviews with the preservice teachers, in order to understand how they constructed the concept of limits in their minds, the way they constructed the concepts of limits cognitively was questioned using the RBC- model.

In order to remind the preservice teachers about limits and their knowledge of the concept of limits prior to the interviews, they were told an old story, the Zenon paradox. The reality of this was questioned over the story. What was intended here was to create icebreakers for the preservice teachers to get used to the interview environment.

Koray was first asked to write a function with a limit. However, Koray focused on domains first.

K 21: "*Uh-huh. I understood, but will it be defined at all points, or in real points.*"
K 23: "*Uh-huh. For example, suppose that the domain is all the real numbers defined in all the real numbers in the $y=x^2$ parabola and the value set is $(0, \infty)$. Because it is the square of a number, it will be a positive number. If I approach every point of this, for example, depending on x, if I approach 1, -1, and other points from the right or from the left (these are sample things), I'll have the same value if I approach from both sides. This is the first example that comes to my mind. There's continuity here, but there may not be in the end."*
K 25: *"Of course, for a limit to exist, its right and left limits from distance Δ must be equal."*

Koray appears to have recalled domains, as understood from what he expressed here. Following that, he recalled the domain and value sets for the function. After that, Koray, who could then establish the concept of function, was able to remember the following features related to limits: Continuity is the equation for the right and left limits. The "Recognize" step of the RBC model took place.

K 39: "*In order to have continuity, it should be equal to the image of limit at that point. Mathematically, it should be* $lim_{x\to a} f(x)$*= f(a).*"

Koray was able to provide explanations at the B1-Uncerstanding level by describing limits mathematically.

K 40: "*If it satisfies these two conditions, it has a limit at each point for the domain and image set.*"

His being able to explain the reason — for the information that he expressed mathematically above — shows that he was at the B1-Understanding level.

In this question, Ali, thought that it would be right to approach the limit concept from the right- and left-hand side. However, in contrast to Koray, Ali could not reach a solution due to lack of prior knowledge.

Ali 38: "*Let me see the zero, then, professor. If I write -1 in the function, it is becoming 1. If I write 1, it becomes zero.*"
…
A 44: "*So, do you think there's a limit at this point?*"
Ali 45: "*No, there is not.*"
A 46: "*Why?*"
Ali 47: "*It's right and left limits are not equal*."

Ali begun to approach the point from the right and left by starting from a very distant range (The point he determined is zero. He checked the limit beginning with 1 from the right side, and beginning with -1 from the left side).

As a second question, the preservice teachers were asked to write a function that had no limit. Here, the teacher candidates tried to respond by reversing their solutions they carried out in the first question.

A question was directed to Koray to calculate the limit of the greatest integer function. In this question, the possibility of existence of limit in different types of functions was questioned by mathematical operations.

K 52: "*You know, I would first draw a graph of this. The greatest integer functions. The small one, that is, I took .5. That is why it was equal to the greatest integer.*"

Koray was able to remember the properties of the greatest integer function.

In this question, it was observed that the prior knowledge of Saliha, another teacher candidate, was weak, and even some of her mathematical constructions in her mind had been formed corruptly.

K 53: *"I don't take the greatest integer that is the closest to it, that is, smaller than it, for example, -1. I will wander around 0 until 1; then, I will wander around 1. When I come to two later on."*

...

He was able to explain the solution of the greatest integer function by specifying its reasoning. This is an indication that Koray was at the B1-Explanation level.

K 59: *"If I take the delta range, regardless of whether I approach from the left or from the right, because it equals to zero, there is a limit at this point, and it is zero."*

The above explanations and solutions of Koray are an indication that he was at the B2-Problem-solving level.

As the fourth question, the limit of a piecewise function was asked. In this question, their ability to use the properties of the limit in mathematical operations was questioned. Whether the right and left limits were equal was questioned particularly in the critical points determined in the piecewise function.

K 69: *"I also remember from last year. There are critical points here for their limits to be known. That is, except for these critical points, their limits from the right and left are not equal. So, I shall take the critical points."*

Koray stated that he recalled his prior knowledge. Especially his being able to remember that the right- and left-hand limits at the critical points of a piecewise function are equal indicates that he was at the "Recognize" level according to the RBC model.

K 79: *"...the critical points. Their limits should be equal at these points. I mean, they should be equal when I approach from the left and right."*

The above explanations of Koray are indicative of the fact that he was at the (Building-With) B1-Explanation level according to the RBC model.

K 83: *"Now, as I approach from the left, I look at the function, and it is "2." I don't know that this time as I'm approaching from the right. So, if I substitute "-1" for x, it's "–a+b." This is the first equation. It is obviously "-2" when we approach from the right. The other is "3a+b" when approached from the left. There are two unknown equations. I can solve them by stacking them. From there, I find the values "a=-1" and "b=1.""*

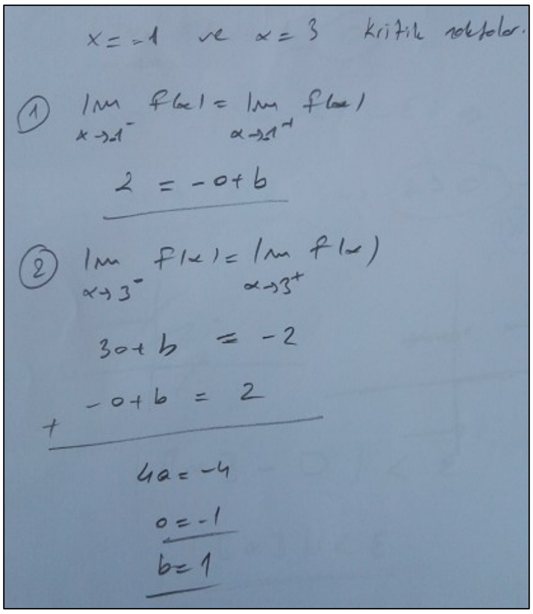

Figure 1: Koray's Limit Approach Example

Koray's above considerations and the actions he carried out afterwards are indicative of the fact that he was at the (Building-With) B2-Contemplating about the Process level according to the RBC model.

> A 95: "*What should be a and b for a limit of the point — that is highlighted in the piecewise function question — to exist in all real numbers?*"
> Ali 96: "*I have to look at the function from the right- and left-hand side. I have to look at -1 from the right- and left-hand side. I need to equate it to 2. I also have to take a look at the left and right of 3. Shouldn't I equate it to -2? From there I find a and b the same way.*"
> ...
> Ali 109: "*If we approach 3 from the right, it is -2 for the values greater than 3.*
> *If we approach from the left, it will be ax+b.*"
> Ali 110: "*For this to have a limit, what should ax+b be equal to? It should be equal to -2, then. Shouldn't I substitute 3 for x?*"
> …
> "*Now, shouldn't we substitute -1 (for x) professor? We need to equate it to 2.*"
> …
> "*You see, professor, so that the left and right are the same.*"
> ...
> "*The results become a=1 and b=1.*"

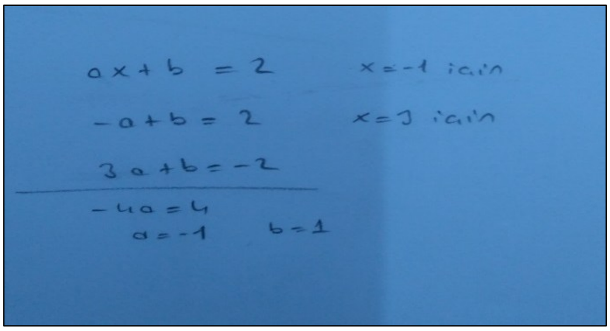

Figure 2: Ali's Limit Approach Example

Ali was lacking prior knowledge (the equality of right and left limits) when doing operations with limits. However, to perform "building-with," he could use the opportunities offered by the researcher to do operations (Building-With) at the B1-Understanding level according to the RBC model.

In the fifth question, the researcher made the following statements:
When we define the limit (or rather in the literature), a definition has been developed using the δ-Δ technique. Now, considering these two questions, can you tell us about its relationship with δ-Δ? There are two different questions. One was a piecewise function, and the other was the greatest integer function. You said we should approach from the right- and left-hand side. You said it should be continuous. Actually, these are two key concepts that lead to the definition of limits. And considering these, they defined limits. On the basis of these two questions, what are the relationships between them using the δ-Δ technique?

> K 98: "*I understand, what I remember about the δ-Δ technique: Let* $lim_{x \to c} f(x)$*=L. When we approach this from the right and left side as much as Δ, we can get such a δ > 0 number.*"

These explanations of Koray are indicative of the fact that he was at the Recognize level according to the RBC model.

> K 101: "*I think, for example, how can a piecewise function be made? We will question whether there is a limit by using the δ-Δ technique. Here, we have reached the values of δ and Δ. I need to find a relationship between these two values; that is, I need to liken this to that.*"

The above explanations of Koray are indicative of the fact that he was at the (Building-With) B1-Understanding level according to the RBC model.

> K 103: "*Here, if f(x)-L‹δ is provided, the number Δ should be searched. While searching for this, there is a limit using the δ-Δ technique, if a number like Δ is found even though* $|x-a| < \Delta$, $\delta > 0$."

The fact that he was able to explain the existence of limit by using the δ-Δ technique is an indication that Koray was at the level of B1-Explanation.

K 109: *"Let me provide a transition between Δ and δ. When -1, 0 and 1 becomes negative, it needs to change a little. It will not come down when we write -1. It must come up because it is an absolute value. The right side is already positive. So, let's look at the left side. The numbers are going to go to -1. But, since it is an absolute value, -1 will turn into +1. Then when I come to -1, I'll get a value of +1. It is going to be like that. On the positive side, I have to write with an x on the y axis so I can connect it to the others. If I can't establish a connection with the one inside, I can't establish a connection between the Δ and δ. I couldn't know exactly how to connect them."*

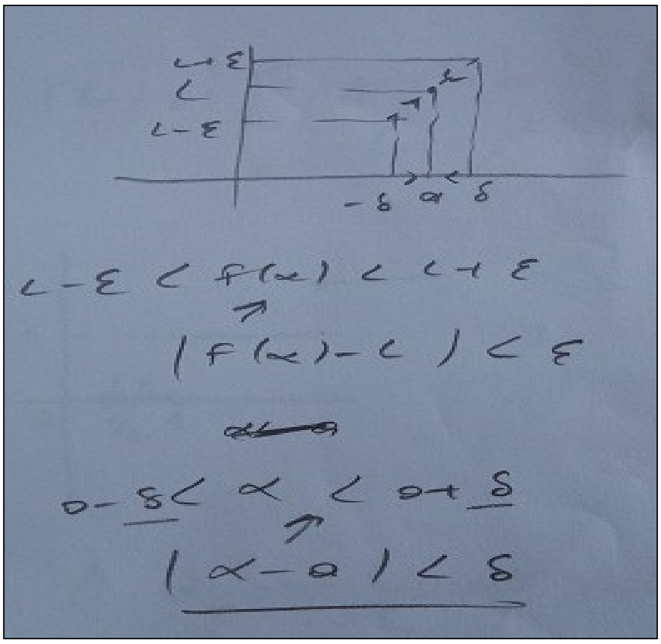

Figure 3: An example from Koray's Definition of Formal Limit

Koray's operations that were mentioned above, as well as his ideas, showed that he was at the (Building-With) B2-Contemplating about the Process level according to the RBC model. But, they also showed that he could not reach C.

K 116: "*Professor, I remember it like this: I need to establish a relation between the two functions and I must write δ in the form of Δ. For example, it may turn out to be twice of that. That's a bit of a memorization, but I couldn't establish a full relation with the function. This means I've memorized it."*

From the above statements of Koray, it is understood that C did not occur while he was making transitions between Recognize and Building-With.

K 119: "*This is because we are approaching a point, you know. That is, let's say we have -Δ and +Δ in the graph. When we write L+δ and L-δ, and my point is point a, now if I approach it from the right- and left-hand side, the values shuttle between L+δ and L-δ.* "

It was seen that he was able to do the graphical representation at the C level according to the RBC model, but was unable to explain algebraically."

K 122: "*I had difficulty in the first question. I couldn't establish a relation between the two, but I need to establish a relation. If δ is greater than zero, how do I prove that Δ is greater than zero? Of course, if δ is greater than zero, I will prove that Δ is greater than zero. Δ will be connected to δ, but it should not be negative. That's how I remember it. I mean, I had difficulties in the first question and I couldn't see it.*"

The above dialogue proves that the concept of limit could be shown graphically at the C level according to the RBC model, but it could not be expressed algebraically.

A133: "*So, we memorized it?*"
K 134: "*Yes, unfortunately, I memorized, I did not understand its logic.*"
A 135: "*How did it happen, then?*"
K 136: "*Now, I have grasped the logic, and it's been nice. Thanks.*
The above-mentioned dialogue between the researcher and Koray is an indication of memorization in the learning of mathematical knowledge. That is, it is an indication that operational learning occurs but that conceptual learning does not fully take place."
Ali 163: "*I couldn't do it. I can't think, professor.*"
…
Ali 164: "*We've learned the lesson a bit for something. Just for the exam.*"
…
Ali 165: "*Since we've learned it for the exam, it does not go beyond memorization, and it is forgotten very quickly.*"

Based on what Ali said, learning was at the memorization level, meaning that learning did not take place because the participants constructed erroneous structures in their minds.

5. Conclusion

In the first section of this study, the success of preservice teachers in limit questions during the midterm and final exams were evaluated. Then, it was seen how these questions could be interpreted according to Sfard's theory of mathematics learning and the RBC- model. In this respect, the levels of the limit questions were understood as well as what kind of a construction process the participants were in according to these theories. Finally, how the preservice teachers constructed the concept of limit was revealed through the interviews with the group consisting of the participants with good, moderate and low achievement characteristics, who were selected based on the exam grades.

In this study, the Recognize step played an important role for the preservice teachers — who were able to construct the concept of limit in their minds — to fully create the domain and value sets of a function and to correctly construct the equations of continuity and limits from the right- and left-hand sides. It can be said that the inability of the other preservice teachers to construct the concept of limit was due to their inability to accurately create the domain and value set of the function. At this stage, the importance of the "Recognize" step in constructing the concept of limit is seen. A critical consideration in the process of constructing the concept of limit is to have a good understanding of functions, and domain and value sets.

In the Building-With step, the preservice teachers who could construct the concept of limit in their minds could explain their operations about limits and were aware of what they were doing. Deficiencies in the Recognize step were effective in the fact that the preservice teachers were not able to fully construct the concept of limit in their minds. Due to the lack of prior knowledge in the process of doing operations with limits, somehow, they were able to conclude memorization-based operations with the help of their feelings or with the support of the researchers. It can be said that during the period of Building-With, the Recognize action was quite effective but not of critical importance, since somehow, the operations were accomplished. Although Recognize was more critical than Building-With, Building-With was needed to transfer knowledge to the Construct step.

The preservice math teachers' understanding of the concept of limit in the context of Calculus Course was questioned and explained in this study which revealed that the preservice teachers had misunderstandings about the formal definition of the concept of limit and had difficulties in understanding the definition. It is seen that a great majority of the preservice teachers included in the study were not able to fully express the formal definition of the concept of limit. What emerged from the in-depth questioning of the ideas of the individuals who were able to express the definition was that the preservice teachers memorized the definitions. And, the reason why they memorized was to be successful in the exams.

Although the majority of the participants were successful in the questions involving the operational part of limits, they were not able to achieve the same success in the questions involving the conceptual part of them. This result of the study is similar to those found in studies in the literature (Barak, 2007; Queseda et al., 2008; Baki & Çekmez, 2012; Sezgin Memnun, Aydın, Özbilen & Erdoğan, 2017). The success of the participants in the operational questions was higher compared to their success in the conceptual questions. This result parallels the study of Bekdemir, Okur and Gelen (2010). This may be stemming from the fact that conceptual learning and operational learning are not balanced in mathematics, that there is more conceptual learning than operational learning, and therefore, students cannot apply concepts or definitions learned in the mathematics course (Soylu & Aydın, 2006).

The RBC- model was utilized to investigate the preservice teachers' process of constructing the knowledge of limits in their minds. In this process, RBC was helpful in identifying the components of the limit knowledge in the preservice teachers' minds. It was discovered that the interviewed preservice teachers had developed an understanding of limits and had no difficulty in using this concept in mathematical operations. It was also observed that they could not make a formal definition of limit but a certain conceptual schema was formed in their minds. This shows that they could not fully construct limits in their minds. The preservice teachers memorized the formal definition without fully understanding it, which was inferred from the following: They were successful in terms of determining the quantifiers and variables within the definition of limits; they were able to find a delta variable for a given epsilon value; they were able to carry out operations until a certain point; and for the remaining steps of operations, they said that "they are things" that are similar to what I have done so far." In summary, the preservice teachers were unable to comment on the variables within the definition (its formal definition), although they could show different representations about the definition of the limit (graphical representation of limits).

The result obtained here also demonstrated that the preservice teachers who could not transfer their operational knowledge to conceptual knowledge in the process of creating the knowledge of limits in their minds could not reach the construct step, either. There was a substantial correspondence between "Sfard's definition of the transition process from operation to understanding" and "RBC's transition process from the Building-With step to the Construct step."

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