European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available on-line at: www.oapub.org/edu

doi: 10.5281/zenodo.1328243 Volume 4 | Issue 12 | 2018

EXAMINING PRESERVICE MATHEMATICS TEACHERS' PERCEPTIONS AND CONCEPT IMAGES OF SEQUENCE AND SUBSEQUENCE CONCEPTS

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Abstract:
This study examined first-grade preservice mathematics teachers' perceptions and concept images of sequence and subsequence concepts. The qualitative research design was utilized in accordance with this purpose. The study sample was composed of 99 preservice mathematics teachers attending the Faculty of Education at a state university during the fall term of 2015/2016 academic year. A form of 6 open-ended questions prepared about the concepts of sequence and subsequence was used as the data collection instrument. Answers of the preservice teachers to the open-ended questions were first scanned as images and then transferred into digital medium. Next, the data transferred into digital medium were subjected to a content analysis with MAXQDA 12 qualitative data analysis software. "Data coding" which is a data analysis method was utilized. It was concluded in the study that the preservice teachers had wrong perceptions and concept images of both sequences and subsequences scientifically.

Keywords: concept of sequence, concept of subsequence, preservice mathematics teachers

1. Introduction

According to most teachers, today, success in mathematics means being able to use formulas, rules and methods instantly and properly and do the calculation in a right way (Soylu & Aydın, 2006). Thus, many students are not aware of which mathematical concepts underlie the operations they use when solving a mathematical problem or question and what mathematics actually means. In other words, students do several operations without knowing the features of concepts on which they study on and without explaining the reasons exactly in mathematical subjects. Indeed, students think of mathematics learning as doing operations by using meaningless formulas and symbols and try to learn mathematics by rote learning (Oaks, 1990; Soylu & Aydın,

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2006). That is why a student solving a given mathematical problem properly does not always mean that he/she can understand or explain the mathematical concept related to that problem exactly (İşleyen & Işık, 2005). Bingölbali & Özmantar (2010) state that trying to teach mathematical subjects through slideshows, direct instruction, formula memorizing or with teacher-centred approaches hinders students' conceptual development. To eliminate this hindrance to perform effective mathematics teaching requires to teach conceptual and operational information in a balanced way (Zembat, Özmantar, Bingölbali, Şandır & Delice, 2013; Birgin & Gürbüz 2009; Baki, 2008; Soylu & Aydın, 2006). Hence, definitions and features of concepts are something to be handled with importance and care in mathematics instruction (Soylu & Aydın, 2006).

Tall & Vinner (1981) state that how students comprehend and think of a concept from the aspect of learning can be analyzed using the concept definition and concept image model. Concept definition can be regarded as a formal definition recognized by the scientific community. For instance, formal definitions for mathematics are general definitions agreed by mathematicians and the mathematical community. Concept image is an informal definition of a concept, involving all mental images that occur in individual's mind and is shaped with individual's impressions and experiences of that concept and the features he/she know about it. Images about the same concept are individual, and several images concerning the concept in mind, perceptions of the concept's features involve several words and/or phrases that evoke the concept (Bingolbali & Monaghan, 2008). Therefore, concept image is an informal definition and may also include conceptual misconceptions as it occurs in minds consciously or unconsciously through individual's experiences. Moreover, concept image does not have to be suitable for or consistent with the concept; it can involve students' conflicted opinions of which they are unaware on the concept (Rösken & Rolka, 2007).

In the literature, there are several studies aiming to identify conceptual misconceptions of primary and secondary education students as well as preservice mathematics teachers studying in faculties of education (Moralı, Köroğlu & Çelik, 2004; Yenilmez & Avcu, 2009; Kaplan, İşleyen & Öztürk, 2011; Özkaya & İşleyen, 2012; Baki & Aydın Güç, 2014) and on conceptual and operational learning (Baki & Kartal, 2002; Soylu & Aydın, 2006; Birgin & Gürbüz, 2009) in regard to certain basic mathematical concepts. In an overlook at the results of these studies in the literature, it is reported that students have several conceptual misconceptions, their conceptual and operational knowledge are not balanced, they have trouble with defining the concepts and cannot tell the relationships between them. On the other hand, no study was observed on the concepts of sequence and subsequence although there are few studies on the definitions of mathematical concepts (Aydın & Köğce, 2008; Dane & Başkurt, 2012; Köğce, 2015). It is reported in the studies on difficulty indices regarding mathematical subjects in the literature that the unit of sequences and series takes the first place in the difficulty index (Durmuş, 2004; Tatar, Okur & Tuna, 2008) and students find it hard to comprehend the subjects in the unit of sequences and series (Akbayır, 2004; Alcock & Simpson, 2004; Alcock & Simpson, 2005; Akgün & Duru, 2007). Furthermore, Çiltaş & Işık (2012) observed in their study aiming to identify the mental models of preservice teachers in the subjects of sequences and series that the preservice teachers failed to do a drawing of an exemplary model which states the concepts of sequence and series properly and tried to explain these concepts and their features through examples.

Due to the abstract, complex and hierarchic (Nesbit, 1996) structure of mathematical concepts, learning a new mathematical concept or information highly depends on learning the previous concepts meaningfully and establish proper relationships between them. Therefore, concepts of sequence and subsequence are some of the basic and important concepts in Analysis I course. Indeed, concepts of sequence and subsequence taught in Analysis I are very close to the concepts of function, convergence, limit, continuity, derivative and integral and among the most basic concepts due to their strong relationship with them. Hence, learning concepts such as convergence, limit, continuity, derivative and integral in a proper way significantly depends on learning the definitions and features of concepts of sequence and subsequence properly in the first place. Despite the importance of concepts of sequence and subsequence in learning other concepts in the Analysis course properly, the fact that sequences and series are at the top in the difficulty indices and preservice teachers cannot provide correct models of these concepts is to be addressed. Hence, the reasons such as concepts of sequence and subsequence are among the basic and important concepts in Analysis I course and there are no studies aiming to explore the definitions and relationships of these concepts necessitated such a study.

Accordingly, this study aimed to examine first-grade preservice mathematics teachers' perceptions and concept images of sequence and subsequence concepts. To that end, the main research question of this study was decided to be "*How do first-grade preservice elementary mathematics teachers explain the concepts of sequence and subsequence and what perceptions and concept images do they have of these concepts?*"

2. Method

In this section, information related to research design, sample, instruments and data analysis were given.

2.1. Research Design

Aiming to identify the first-grade preservice elementary mathematics teachers' perceptions and concept images of the sequence and subsequence concepts, this is a descriptive research and utilized the qualitative research design. Qualitative research ensures that data are read over and over to be divided into codes and categories based on their similarities and differences and research results are presented (Merriam, 1998; Çepni, 2012; Karasar, 2016; Yıldırım & Şimşek, 2016).

2.2. Sample

The sample of the research was composed of 99 first-grade preservice teachers attending the Department of Elementary Mathematics Education in the Faculty of Education at a state university during the fall term of 2015/2016 academic year. This sample was chosen as it had been observed that preservice teachers taking the General Mathematics course in the Department of Elementary Mathematics Education in the previous academic year had certain shortcomings when explaining the mathematical concepts subjected to the research. Hence, first-grade preservice mathematics teachers were purposively chosen for the sample in this study.

2.3. Instrument

A form of 6 open-ended question was created to determine on what level the preservice teachers could explain the concepts of sequence and subsequence and what kind of concept images they had of these concepts (see Table 1). The open-ended questions allowed preservice teachers to state the reasons for their answers and reflected the way they thought of these concepts (Gronlund & Linn, 1990). This is why open-ended questions were utilized as data collection instrument.

Before preparing the question form, the related literature was reviewed in detail. First, 10 open-ended questions (2 questions about definitions of the concepts 8 operational questions about the usage of concepts) were prepared by the research on the concepts of sequence and subsequence in accordance with the research purpose. Next, two field education experts and one field experts were asked to examine these questions. As per the opinions of the field experts, 4 of the questions were excluded from the form because operational questions about the concepts were similar and serve the same purpose. The finalized question form is composed of two parts. The first part involves 2 open-ended conceptual questions and the second part involves 4 open-ended operational questions about these concepts. While the preservice teachers were asked to explain the two concepts with reasons in the first part, they were asked to answer the operational questions about these concepts with reasons in the second part. Questions used in the data collection instrument are presented in Table 1.

Table 1: Question used in the data collection instrument

Chapter I - Questions about Definitions of the Concepts
1. What is a sequence? Define. What are the conditions that a statement is a sequence in your opinion?
2. What is a subsequence? Define. What are the conditions that a statement is a subsequence in your opinion?
Chapter II - Operational Questions about the Concepts
1. From which set should the statement $f(n) = \frac{2n-3}{n-3}$ be defined to which set so it states a sequence?
2. Explain whether inverse of the function defined by $f: N^{+} \to R$, $f(n) = 3n + 3$ would be a sequence with reasons.
3. Write a subsequence of the sequence $(a_{n}) = \left(\frac{n+1}{2n}\right)$ and explain why it is the subsequence of this sequence with reasons.
4. Explain whether the sequence $(b_n) = \left(\frac{3n+1}{2n+1}\right)$ is a subsequence of the sequence $(a_n) = \left(\frac{2n+1}{n+1}\right)$ with reasons.

The data collection instrument revised in accordance with the expert opinions were applied to 50 first-grade preservice mathematics teacher in the fall term of 2014/2015 academic year in a pilot study. The data collection instrument was finalized, and it was determined that it took one class hour (45 minutes) to complete it in the pilot study.

2.4. Data Collection and Analysis

The data of the study were collected by applying the data collection instrument to the participant preservice teachers for one class hour (45 minutes).

A comprehensive literature review was performed for the formal definitions of the sequence and subsequence concepts and criteria were developed to be used in the evaluation of the answers given to the questions in the data collection instrument. The criteria created using the definitions of the sequence and subsequence concepts in the literature are given in Table 2.

Table 2: Criteria used in the data analysis

Concepts	Definitions made in the literature
1. Sequence	• A function defined by $f: N \to R$ is called real number sequence (Doruk & Kaplan, 2013). • Each f function defined from the set N^+ to the set R is called a real number sequence or "sequence" in short (Aydın & Asma, 2003). • A function defined from the natural numbers set N to R is called a real number sequence. For sequences, the representation$f: N \to R,\ n \to f(n) = a_n$ is used (Dernek, 2009). • The function of which domain is positive integers is called sequence. By this definition, the function$f: N \to R,\ y = f(n)$ is a sequence (Kadıoğlu & Kamali, 2013). • Every function of which domain is positive integers is called an infinite sequence or "sequence" in short. If the range of the function is real numbers set R, the sequence is called a "real convergent sequence" (Balcı, 2012). • With $A \neq \emptyset$ being a null set, every function defined by $f: N^+ \to A$ is called a sequence (Altun, 2015).
2. Subsequence	• Let (a_n) and (k_n) be two sequences. If the sequence (k_n) is a monotone increasing sequence, in other words, it is $k_{n+1} > k_n$ and $\forall\ n \in N^+$, in the sequence (a_n), the sequence (a_{k_n}) obtained by writing the general term of (k_n) in place of n is called a subsequence of the sequence (a_n) (Aydın & Asma, 2003). • Given (a_n), with (k_n) being an increasing sequence of natural numbers, the sequence (a_{k_n}) is called a subsequence of (a_n) (Dernek, 2009). • With (k_n) being an increasing sequence of positive integers, the sequence (a_{k_n}) is called a subsequence of (a_n) (Balcı, 2012).

After the question form had been applied, form of each preservice teacher was assigned a number. For example, "PT1" represents the preservice teacher 1. Next, the answers given by the preservice teachers to the open-ended questions were scanned as images and transferred into the digital medium, and MAXQDA 12 qualitative data analysis software was used for analysing the obtained data.

For analysing the data in a reliable way, the answers given by randomly chosen 10 preservice teachers to the question form was grouped in the categories of correct, incomplete, incorrect and blank independently based on the criterion set by the researcher and a field expert in the first place, and then the answers were descriptively analyzed according to their similarities and differences (Yin, 1994; Merriam, 1988). If the concept was defined correctly as given in Table 2 and explained with the correct reason, it was categorized as "Correct-Correct"; if defined correctly but explained with incomplete reason, it was categorized as "Correct-Incomplete"; if defined correctly but explained with incorrect reason, it was categorized as "Correct-Incorrect", and if defined correctly but with no reason, it was categorized as "Correct-Blank". Similar categorization was performed for incomplete and incorrect definitions and reasons.

The degree of agreement of the coding performed by the researcher and the field expert was calculated with the formulation "Reliability= (Number of agreed categories) (Total number of agreed and disagreed categories)" (Miles & Huberman, 1994). The degrees of reliability regarding the concordance of the analyses conducted separately by the researcher and field expert was calculated to be 0.83 for conceptual and operational data on the definition of sequence and 0.82 for conceptual and operational data on the definition of subsequence. (Miles & Huberman, 1994) state that concordance between the two coders being 0.70 and above is sufficient for reliability. Accordingly, it was decided that the concordance between the coders was reliable. Next, these categorized were reviewed by the researcher and field expert together to clarify the similar categories, and different categories were discussed to achieve consensus (Yin, 1994; Merriam, 1988).

The answers given by the remaining preservice teachers in the question form were analyzed by the researcher alone according to these categories. The categories created once the analysis of all the data was completed were submitted to the review by the same field expert along with the criterion taken as basis in the data analysis and finalized in accordance with the recommendations; they were next presented in Table 3 and Table 4 with percentage-frequency values and citations from the actual answers given by the preservice teachers.

2.5. Validity and reliability

The validity and reliability measures required for the qualitative research method were taken in this study (Yıldırım & Şimşek, 2016). Hence, it was ensured that the participant preservice teachers answered each question in consideration of their current status to achieve internal validity during the implementation of the data collection instruments. For the external validity, the findings were presented in consistency with the research questions in an effort.

To achieve the external reliability, the position of the researcher conducting the data analysis within the research process, conceptual framework used for the data analysis as well as the codes and themes were described, and detailed explanations were made on the data collection and analysis methods. For the internal reliability, the researcher and a field expert participated in the analysis steps and the achieved data were presented in a detailed way and in a descriptive approach.

3. Results

The data obtained from the opinions of the preservice teachers on the mathematical concepts are presented and explained in tables (Table 3-Table 4). The data on how the preservice teachers explained the sequence concept and what kind of perceptions and concept images they had of this concept are shown in Table 3.

Table 3: Definitions concerning the concept of sequence

Concept	**Conceptual Knowledge**										
	Definitions			**Reasons**							
		f	**%**	**Correct**		**Incomplete**		**Incorrect**		**Blank**	
				f	**%**	**f**	**%**	**f**	**%**	**f**	**%**
Concept of Sequence	Correct	4	4.04	2	50	2	50	0	0	0	0
	Incomplete	3	3.03	3	100	0	0	0	0	0	0
	Incorrect	91	91.91	0	0	44	48.35	47	51.65	0	0
	Blank	1	0.01								
	Operational Knowledge										
	Question 1										
	Correct	0	0	0	0	0	0	0	0		0
	Incomplete	2	2.02	0	0	0	0	0	0	2	100
	Incorrect	97	97.97	0	0	0	0	42	43.30	55	56.70
	Blank	0	0								
	Question 2										
	Correct	40	40.40	0	0	6	15	34	85	0	0
	Incomplete	1	0.01	0	0	0	0	0	0	1	100
	Incorrect	56	56.56	4	7.14	1	1.79	49	87.5	2	3.57
	Blank	2	2.02								

Citations about Conceptual Knowledge

Correct-Correct

- The function of which domain is a set of positive integers is called sequence (PT63).
- Functions of which domain is a set of natural numbers are called sequences (PT64).

Correct-Incomplete

- For a statement to be a sequence, its domain $T = \{1, 2, 3, ...\}$ has to be a set of counting numbers and its value set has to be a set of real numbers (PT2, PT7).

Incomplete-Correct

- Numbers ranking according to a certain rule. For it to be a sequence, the denominator should be different from zero, and domain should be counting numbers (PT2, PT13, PT51).

Incorrect-Incomplete

- Terms ordered by a certain rule are called sequences. $Rule: 2n + 1 \rightarrow Term: 3, 5, 7, 9 ...$ (PT1, PT38, PT40, PT46, PT68, PT72, PT74, PT88, PT 89).
- Given$n \in N^+$, numbers denoted as a_n and that come together by a certain rule are called sequences (PT9).
- Group of arithmetically or geometrically increasing-decreasing positive integers by a certain pattern is called sequence (PT10-PT12, PT14-PT16, PT18, PT21, PT32, PT34, PT36, PT37, PT42, PT43, PT61, PT66, PT77, PT90, PT91, PT96, PT97).
- It is the number pattern that is infinite by certain rules and increasing rates (PT25, PT41, PT76, PT78, PT94, PT95).
- In the statement, it is the pattern created by writing down at least 1 in place of n. It should be a function (PT26).
- It is a function defined from a set to another (PT67).
- It is to give positive value in place of n in a function (PT30, PT59).
- For a statement to be a sequence, it should be $a_n \in N^+$ and $n > 0$ (PT31).
- A function of which terms are ordered is called a sequence (PT60).
- A function of which general term is a_n is called a function (PT62, PT92).

Incorrect-Incorrect

- Things following each other successively and connected to the previous and next one. It should be constantly increasing or decreasing (PT3, PT22, PT33, PT50, PT55).
- A set which has elements such as number, shape, etc. in a certain period or rule is called a sequence (PT4, PT6, PT20, PT27, PT52, PT53, PT58).
- Things that continue at equal intervals or in an order depending on a fixed rule and of which terms are integers (PT5).
- Numbers defined by real numbers and of which terms are ordered according to a certain arithmetic (PT17, PT19).
- Group of numbers which have a certain ratio or amount of increase among them is called sequence (Ö23, PT24, PT44, PT65, PT71, PT83, PT85, PT86, PT99).
- It is the number pattern that is infinite by certain rules and increasing rates (PT25, PT39, PT79, PT81, PT93).
- Things that are finite or infinite according to a certain rule (PT8, PT28, PT75, PT87).
- Set of numbers that continue increasing or decreasing by a certain rule (PT29, PT46, PT48, PT49, PT56, PT69, PT70, PT73, PT82, PT84).
- If the result is always a positive integer for the integer values of an unknown in a statement, this statement is called a sequence (PT35).
- It is that all positive integers that are ordered denote a mutual equation. Denominator should be different from zero (PT45).
- Sets of number between which the difference is fixed are called equation (PT57).
- Number relations with rules are called sequences (PT80).
- Set of numbers which is defined by rational numbers and is infinite by a rule is called a sequence (PT98).

Blank: (PT54).

Citations about Operational Knowledge

Correct-Incomplete	Question 1	-
	Question 2	• Its inverse is not a sequence. Because this function defined by natural numbers always proves real numbers. But its inverse function does not always prove natural numbers as it is defined by real numbers. For example, for $n = \frac{5}{2}$, $\frac{\frac{5}{2}-3}{3} = -\frac{1}{6}$ is not a natural number (PT10, PT21). • Inverse of $f^{-1}(n) = \frac{n-3}{3}$ should be defined by $R \to N^+$. As it is $f^{-1}(1) = \frac{-2}{3} \notin N^+$ for n=1, it is not a sequence (PT23, PT66, PT67, PT68).
Correct-Incorrect	Question 1	-

	Question 2	• It is not a sequence. It does not have equivalence in negative numbers in the set of N^+ (PT11). • It does not denote a sequence. Because the statement $f^{-1}(n) = \frac{n-3}{3}$ becomes $f^{-1}(2) = \frac{-1}{3}$ negative for $= 2$. All terms should be positive, so it can denote a sequence (PT13, PT17, PT18, PT19, PT40, PT62, PT84). • As the inverse function $f^{-1}(n) = \frac{n-3}{3}$ is defined by $R \to N^+$, its inverse is not a sequence because result cannot be positive for $n \leq 3$ values (PT22, PT32, PT33, PT39, PT60, PT64). • $f^{-1}(n) = \frac{n-3}{3}$ is not a sequence. Because, given $f^{-1}: R \to N^+$, result is not positive but negative for $n = 1$. It does not prove (PT26, PT35, PT75, PT76, PT79, PT80, PT81, PT87, PT89, PT90, PT94). • Since $f^{-1}(1) = \frac{-2}{3}$, $f^{-1}(2) = \frac{-2}{3}$, $f^{-1}(3) = 0$, $f^{-1}(4) = \frac{1}{3}$, the statement $f^{-1}(n) = \frac{n-3}{3}$ does not denote a sequence. Because there is no certain rate between and some of the results are negative (PT36). • Since the numerator of $f^{-1}(n) = \frac{n-3}{3}$, is zero for $n = 3$, there is unidentifiability in question. So, it is not a sequence (PT41, PT82, PT83). • Since $f^{-1}(3) = 0 \notin N^+$ for $n = 3$, it is not a sequence (PT53, PT70). • Inverse of $\mathrm{f}^{-1}(\mathrm{n}) = \frac{\mathrm{n}-3}{3}$ should be defined by $\mathrm{R} \to \mathrm{N}^+$. But this is a conflict because it is defined by $\mathrm{f}^{-1}: \mathrm{R} \to \mathrm{R}$. So, it is not a sequence (PT61). • $\mathrm{f}^{-1}: \mathrm{R} \to \mathrm{N}^+$ is not a sequence. Because it should be defined by $\mathrm{f}^{-1}: \mathrm{R} \to \mathrm{R}$ so it can be a sequence (PT85). • Inverse of $\mathrm{f}^{-1}(\mathrm{n}) = \frac{\mathrm{n}-3}{3}$ does not follow the definition of sequence. Because it is $\mathrm{f}^{-1}(1) \notin \mathrm{Q}$ for $n = 1$ (PT98).
Incomplete-Blank	**Question 1**	• Should be $f: N \to Z$ (PT41). • Should be $f: N \to R$ (PT97).
	Question 2	• $f^{-1}(n) = \frac{n-3}{3}$ is not a sequence (PT47).
Incorrect-Correct	**Question 1**	-
	Question 2	• $f^{-1}(n) = \frac{n-3}{3}$. It is real number when you write down natural number in place of n (PT1, PT6). • $f^{-1}(n) = \frac{n-3}{3}$. When writing down natural number in place of$n$, it is a sequence because the terms $\frac{-2}{3}, \frac{-1}{3}, 0, \frac{1}{3}$, have a certain order (PT49, PT78).
Incorrect-Incomplete	**Question 1**	-

	Question 2	• Since $f^{-1}(n)=\frac{n-3}{3}$ for $n=1$, $\frac{-2}{3}$ for $n=2$ and $\frac{-1}{3}$ for $n=3$ are 0, it is a sequence (PT4).
Incorrect-Incorrect	**Question 1**	• For the statement $f(n)=\frac{2n-3}{n-3}$ to denote a sequence, it should be $R-\{3\}\rightarrow R$ as it will be $n-3\neq 0, n\neq 3$ (PT1, PT9, PT10, PT11, PT13, PT22, PT24, PT26, PT29, PT31, PT33, PT38, PT61, PT62, PT72, PT85, PT86, PT90, PT91, PT94, PT95). • As the terms of the sequence cannot be negative, it should be $2n-3>0\; n-3>0$. That is, it should be defined by all real numbers within the range of $3<n<+\infty$ (PT3, PT6, PT7, PT18, PT19, PT20, PT21). • Since it should be $n\neq 3$ and $\frac{2n-3}{n-3}>0$, it should be $n\neq 3$ and $n>\frac{3}{2}$ (PT4, PT5, PT55). • It should be defined by $\{2,4,6\}\rightarrow\{-1,5,3\}$ (PT8). • As it should be $n\neq 3$, it should be defined by R from the set of $n\in Z-\{3\}$ (PT12, PT40, PT42). • For the statement $f(n)=\frac{2n-3}{n-3}$ to denote a sequence, it should be $n-3>0$ and $2n-3\geq 0$. If the necessary operations are done, it should be defined by the real numbers that prove $n>3$ (PT14, PT27, PT28). • It should be $f(n)=\frac{2n-3}{n-3}>0$ to denote a sequence. If we consider the sign, it is $(f(n))=(-\infty,\frac{3}{2})\cup(3,+\infty)$ for $n-3>0$ and $2n-3\geq 0$ (PT15, PT53). • $f(1)=\frac{1}{2}$, $f(2)=-1$. As its image cannot be negative, it does not denote a sequence. Its image set should consist of positive integers, so it can denote a sequence (PT36). • If $n-3=0$ and if $n=3$ and $2n-3=0$, its domain is $\left(\frac{3}{2},3\right)$ because $n=\frac{3}{2}$ (PT80).

	Question 2	• It should be one-to-one and surjective. It does not denote a sequence as it is not one-to-one and surjective (PT3, PT38). • As the statement $f^{-1}(n) = \frac{n-3}{3}$ is a rational number for each n value, it is a sequence (PT5, PT43). • Since $f(1) = 6, f^{-1}(6) = 1$ for $n = 1$; $f(2) = 9, f^{-1}(9) = 2$ for $n = 2$, inverse of this function is also a sequence. Because it follows the domain and image set rule (PT7, PT56). • As the statement $f^{-1}(n) = \frac{n-3}{3}$ is not a value that makes denominator zero, that is, makes it undefined, it is a sequence (PT8, PT12, PT14, PT24, PT30, PT37, PT42, PT45, PT48, PT50, PT59, PT63, PT74, PT86). • As the statement $f^{-1}(n) = \frac{n-3}{3}$ is defined by $R \to R$, it denotes a sequence (PT9, PT15, PT16, PT29). • As the inverse of $f^{-1}(n) = \frac{n-3}{3}$ covers the values of $f: R \to N^{+}$, it is a sequence (PT20, PT25, PT57, PT65, PT72, PT91). • For the rule $f^{-1}(n) = \frac{n-3}{3}$ to be positive, it should be $n - 3 > 0$. If $n - 3 > 0$, it is a sequence as the terms $(f^{-1}(n)) = (\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \ldots)$ for $n > 3$ are positive (PT27, PT28). • As the inverse of $f^{-1}(n) = \frac{n-3}{3}$ is defined for $n > 0$, it is a sequence (PT31). • As the inverse of $f^{-1}(n) = \frac{n-3}{3}$ is 1 for $n = 6$, 2 for $n = 9$, and 3... for $n = 12$, it is an increasing sequence (PT34, PT55). • Since there is a certain rate among the terms as $f^{-1}(n) = \frac{n-3}{3}$, $n = 1 \to \frac{-2}{3}$, $n = 2 \to \frac{-1}{3}$, it is a sequence (PT44, PT95, PT99). • Since the denominator of $f^{-1}(n) = \frac{n-3}{3}$ is a real number, it is a sequence (PT46). • It is a sequence. Because it follows the rule $N^{+} \to R$ (PT52, PT58, PT69, PT71, PT93). • Since $f^{-1}: R \to N^{+}$ is a function, it is a sequence (PT77, PT88, PT92). • In the inverse of $f^{-1}(n) = \frac{n-3}{3}$, if n is chosen to be an integer to make the statement $n - 3$ 3 and multiples of 3, $f^{-1}: Z \to Z$ becomes a sequence (PT96). • As the statement $f^{-1}(n) = \frac{n-3}{3}$ is proven for each $n \in N$, it is a sequence (PT97).
Incorrect-Blank	Question 1	• It should be defined by $R \to R$ (PT16, PT99). • It should be defined by the set of $Z^{+} \to (3, +\infty)$ (PT17). • Should be $f: N^{-} \to Q$ (PT23). • Should be $f: R - \{3\} \to R$ (PT25, PT30, PT37, PT44, PT45, PT46, PT58, PT59, PT63, PT64, PT66, PT67, PT68, PT70, PT71, PT73-PT79, PT81-PT84, PT92, PT93). • Should be $N^{+} - \{3\} \to R$ (PT32, PT43,PT47, PT51-PT53, PT56). • $\{4, 5, 6, \ldots \infty\} \cup \{0\} \to R$ (PT34). • Should be $\{0, \infty\} \to \{1, \infty\}$ (PT35). • Should be $f: R \to \{R - 3\}$ (PT39, PT48, PT60, PT65, PT69, PT87, PT88, PT98). • Should be $f: R - \{2, 3\} \to R$ (PT49). • Should be $f: N^{+} - \{2, 3\} \to +\infty$ (PT50, PT57). • It should be defined by $f: N \to Z$ (PT89). • It should be defined by the set of integers (PT96).

Question 2	• It is a sequence (PT53, PT73).
Blank:	(PT2, PT51).

Considering the distribution of the data on how the preservice teachers defined the concept of sequence and what perceptions and concept images they had of the concept in Table 3, 4.04% of them defined it in the category of correct answer, 3.03% in the category of incomplete answer, and 91.91% in the category of incorrect answer. Half of the participants who defined the concept of sequence in the category of correct answer provided correct reasons while the other half provided incomplete reasons. All of the participants who defined the concept in the category of incomplete answer provided valid reasons for the definition of sequence. As for the participants who defined the concept in the category of incorrect answer, 48.35% of them provided incomplete reasons and 51.65% of them provided incorrect reasons. Regarding the distribution of the answers given to the first and second operational questions about the concept of sequence in the second part of the data collection instrument, 2.02% of the preservice teachers answered the first question without providing a reason in the category of incomplete answer while 97.97% (43.30% provided incorrect reasons and 56.70% provided no reason at all) answered it in the category of incorrect answer. 40.40% of the preservice teachers (with 15% and 85% providing incomplete and incorrect reasons, respectively) answered the second operational question in the category of correct answer whereas 56.56% (with 7.14% providing correct, 1.79% providing incomplete and 87.5% providing incorrect reasons) answered the question in the category of incorrect answer. As for the citations from the answers given by the preservice teachers both to conceptual and operational questions about the concept of sequence in Table 3, it can be argued that the preservice teachers had scientifically wrong perceptions and concept images of the concept.

The data on how the preservice teachers explained the subsequence concept and what kind of perceptions and concept images they had of this concept are shown in Table 4.

Table 4: Definitions concerning the concept of subsequence

Concept	**Conceptual Knowledge**										
Concept of Subsequence	**Definitions**			**Reasons**							
		f	**%**	**Correct**		**Incomplete**		**Incorrect**		**Blank**	
				f	**%**	**f**	**%**	**f**	**%**	**f**	**%**
	Correct	9	9.09	0	0	1	11.11	8	88.89	0	0
	Incomplete	12	12.12	12	100		0	0	0	0	0
	Incorrect	68	68.68	14	20.59	11	16.18	43	63.23	0	0
	Blank	11	11.11								
	Operational Knowledge										
	Question 3										

Correct	8	8.08	0	0	5	62.5	1	12.5	2	25
Incomplete	1	0.01	0	0	0	0		0	1	100
Incorrect	72	72.72	0	0	0	0	59	81.94	13	18.06
Blank	18	18.18								
Question 4										
Correct	52	52.52	0	0	28	53.85	21	40.38	3	5.77
Incomplete	0	0	0	0		0	0	0	0	0
Incorrect	31	31.31	0	0	0	0	26	83.87	5	16.13
Blank	15	15.15								

Citations about Conceptual Knowledge

Correct-Incomplete

- If all terms of a sequence are also the terms of another sequence, such sequence is called subsequence. For instance, all integers are also elements of the set of real numbers, the sequence of integers is the subsequence of the sequence of real numbers (PT85).

Correct-Incorrect

- If all terms of a sequence are also the elements of another sequence, such sequence is called subsequence. For example; a subsequence of the sequence $\{3, 5, 7, 9\}$ is $\{5, 7, \}$ (PT27, PT28, PT36, PT37, PT77, PT80, PT82, PT83).

Incomplete-Correct

- A sequence which is subset of a sequence is called a subsequence (PT29, PT49, PT55, PT88, PT89, PT94).
- A sequence which involves some elements of a sequence is called a subsequence (PT31, PT70, PT96).
- It is that some elements of a sequence are created with another rule (PT74).
- With (a_n) being a sequence, the sequence $\left(a_{k_n}\right)$ obtained by writing down k_n in place of n is called subsequence (PT78, PT87).

Incorrect-Correct

- Sequences meeting the conditions of a general sequence are called subsequences (PT10, PT19, PT35, PT38, PT53, PT84).
- If we get the same results when attributing values to n, it is a subsequence (PT13, PT20).
- It is that a general sequence's elements with n are written as small units. A general sequence covers the subsequence (PT14).
- A sequence formed by the terms of a sequence within a certain range is called a subsequence (PT17, PT48, PT71, PT72).
- The sequence which has at least one shared element with a sequence is called subsequence (PT9).

Incorrect-Incomplete

- Some elements of a sequence coming together is called a subsequence (PT1, PT2, PT43, PT47, PT58, PT92, PT93, PT97).
- For a statement to be a subsequence, it should involve the elements of the sequence by which it was formed (PT7, PT60).
- If a new sequence can be formed with numbers in a sequence by a separate rule, this new sequence is called the subsequence of that sequence (PT42).

Incorrect-Incorrect

- It should be the derivatives of the sequence (PT3, PT33).
- It is the set that help us achieve the term n. of a sequence (PT4).
- It is a concept involved by a general sequence in it. It must follow the general term (PT5, PT98).
- It is something like a subset (PT6, PT62, PT66, PT95).
- For a sequence to be a subsequence, it should be a constant sequence (PT8).
- It is to create a new sequence by reducing the general rule of a sequence (PT11, PT18, PT40, PT41, PT61, PT91).

- It is a sequence formed by elements of a sequence increasing at a certain rate (PT12, PT98).
- If a result that defines the sequence is achieved when writing down the terms given in a subsequence in place of the uncertainties given in a normal sequence (PT15).
- A series formed by the elements coherent with certain attributes of a sequence that is composed of numbers together is called a subsequence (PT16).
- The new sequence achieved by attributing certain values to n is called a subsequence (PT21).
- If the values attributed to n do not make the denominator of the sequence zero, this value is called subsequence (PT22, PT34).
- Sequence which helps define a sequence is called a subsequence (PT23).
- In the sequence (a_n), the sequence (a_{n-1}) obtained by lessening the variable by 1 is called subsequence (PT24).
- Each of a sequence's terms is called the subsequence of that sequence (PT25, PT44, PT50, PT75).
- If the domain of a function covers the domain of another function, it is a subsequence (PT26).
- It is the set formed by some elements of a function (PT30, PT57).
- Sequences of which domain covers its value set and its value set covers its domain are called subsequences (PT32).
- It is the ordering of numbers from a_1 to a_n by means of formulas (PT46).
- Sequences of which domains share the same domain are called subsequences (PT63, PT67).
- Each subset of a function is called a subsequence (PT64).
- The new sequence obtained by increasing its variable by 1 is called subsequence. For example; a subsequence of the sequence (a_n) is (a_{n+1}) (PT65).
- The sequence of which domain is smaller than the domain of the general sequence is called subsequence (PT68, PT69).
- If the values that render the denominators undefined for the sequences $(a_n)$and (b_n) are different, these sequences are called each other's subsequence (PT73).
- A sequence formed by multiplying the numerator and denominator of a sequence by the same number is called a subsequence (PT81).
- It is the case that a sequence follows another sequence monotonously (PT90).

Blank: (PT39, PT45, PT51, PT52, PT54, PT56, PT59, PT76, PT79, PT86, PT99).

Citations about Operational Knowledge

Correct-Incomplete	Question 3	• The sequence $(a_{n+1}) = (\frac{n+2}{2n+2})$ is a subsequence of the sequence $(a_n) = (\frac{n+1}{2n})$. Because all terms of the sequence (a_{n+1}) are elements of the sequence (a_n) (PT31, PT74, PT85, PT87, PT93).
	Question 4	• Given $a_1 = \frac{4}{3}, b_1 = \frac{3}{2}$ for $n = 1$; $a_1 = \frac{7}{5}, b_1 = \frac{5}{3}$ for $n = 2$, as we get different results when we attribute the same value to n, that is, because one does not cover any element of the other, the sequence (b_n) is not a subsequence of the sequence (a_n) (PT4, PT7, PT13, PT16, PT17, PT25, PT29, PT40, PT42, PT43, PT45, PT48, PT50, PT58, PT69, PT70, PT71, PT74, PT75, PT80, PT84, PT85, PT88, PT89, PT92, PT94, PT96, PT97).
Correct-Incorrect	Question 3	• We can form the subsequence by increasing its variable by one. If one writes down n + 1 in place of n , the sequence $(a_{n+1}) = (\frac{n+2}{2n+2})$ becomes its subsequence (PT65).

	Question 4	• The sequence $(b_n) = (\frac{3n+1}{2n+1})$ is not a subsequence of the sequence $(a_n) = (\frac{2n+1}{n+1})$. Because $a_n = b_n$ for $n = 0$ but it is not a subsequence as it cannot be $a_0\ veya\ b_0$ (PT1). • If the amount of increase of sequences' terms is the same, it becomes subsequence. Each term of the sequence b_n increases by $\frac{1}{15}$, each term of the sequence a_n increases by $\frac{1}{10}$. Since the amounts of increase are different, the sequence b_n is not a subsequence (PT10). • It is not. Because rules are not coherent with each other, their lessened states. For example, it is $a_1 \neq b_1$ for $= 1$. This indicates that the lessened rule is not coherent with the first rule (PT11, PT20, PT87). • Since $b_1 = \frac{4}{3}$, $b_2 = \frac{7}{5}$, $a_1 = \frac{3}{2}$, $a_2 = \frac{5}{3}$, the two of them do not increase at the same rate in the sequence. So, it is not a subsequence (PT12, PT36). • It is not its subsequence. Because they cannot be written down by transforming to each other (PT14). • It is not a subsequence. Because, in the sequence a_n, the result is not integer when we attribute values to n (PT21). • If the variable of the sequence (a_n) is written down by lessening by one, that is, $n-1$ is written down in place of n, it is $(b_n) \neq (a_{n-1})$. As it does not equalize, the sequence (b_n) is not a subsequence of the sequence (a_n) (PT24). • It is not its subsequence. Because the range of the sequence (a_n) does not cover the range of the sequence (b_n). Since $b_1 = \frac{4}{3}$, $a_1 = \frac{3}{2}$ for $n = 1$, it is in the range of (a_n): $(\frac{3}{2}, \infty)$ and the range of (b_n): $(\frac{4}{3}, \infty)$ (PT26). • If $b_k = a_k$, as the equation $\frac{3k+1}{2k+1} = \frac{2k+1}{k+1} \rightarrow 3k^2 + 3k + k + 1 = 4k^2 + 2k + 2k + 1$, $3 = 4$ is not correct, the sequence b_n is not a subsequence of the sequence (a_n) (PT27, PT28, PT38). • Because it is not defined by the same numbers, it is not its subsequence (PT35). • To be a subsequence, it should be $a_{b_n} = a_1$. If $a_1 = \frac{3}{2}$, it is $b_n = 1$. Here, if $\frac{3n+1}{2n+1} = 1$, it is $n = 0$. As it cannot be b_0 in the sequences, it is not its subsequence (PT46). • Because their general terms and approach to infinity are difference, it is not its subsequence (PT53). • $b_n: R - \left\{-\frac{1}{2}\right\} \rightarrow R$ and $a_n: R - \{-1\} \rightarrow R$. As their domains are different, it is not its subsequence (PT66, PT67). • It is not its subsequence. Because, once we simplify or widen the sequence b_n, we cannot get the sequence a_n (PT81, PT98).
Correct-Blank	Question 3	• The sequence $(a_n)$is a subsequence of itself (PT69, PT70).
	Question 4	• It is not its subsequence (PT31, PT49, PT79).

Incorrect-Incorrect

- As there is no undefined value for each value of n, since $n=1\to 1$, $n=2\to\frac{3}{4}$, $=3\to\frac{4}{6}$, $n=4\to\frac{5}{8}$, $A=\left\{1,\frac{3}{4},\frac{4}{6},\frac{5}{8}\right\}$ is a subsequence (PT4, PT7, PT15, PT36, PT37, PT40, PT42, PT43, PT48, PT49, PT53, PT60, PT62, PT71, PT90, PT92, PT94, PT97).
- $a_1=\frac{1+1}{2.1}=1$ is a subsequence term. Because the result is a rational number for n=1 (PT5).
- Since $a_n=\frac{n+1}{2n}=\frac{1}{2}+\frac{1}{2n}$, the sequence $\frac{1}{2n}$ is a subsequence of the sequence a_n. Because there is the constant term of $\frac{1}{2}$ (PT8).
- The sequence $\left(\frac{2n+1}{3n}\right)$ is a subsequence of the sequence a_n as the coefficient at the beginning of "n" are different (PT10).
- By lessening the rule of(a_n), we can have a subsequence $(b_n)=\frac{n+1}{2}$ (PT11, PT19).
- Subsequence of $1\to 1$, $n=2\to\frac{3}{4}$, $n=3\to\frac{4}{6}$ is $\left\{1,\frac{3}{4},\frac{4}{6}\ldots\right\}$. Because it increases at a certain rate (PT12).
- $\left(\frac{n+2}{3n}\right)$ is a subsequence of the sequence (a_n). Because the result is the same for $n=1$ (PT13).
- Since the terms decrease as $a_1=1$, $a_2=\frac{3}{4}$, $a_3=\frac{2}{3}$, $a_4=\frac{5}{8}\ldots$, it is a subsequence (PT16, PT34, PT78).
- $a_1=1$, $a_2=\frac{3}{4}$ are subsequences of the sequence. Because it does not zero the denominator and has a certain value (PT20, PT55, PT66).
- For $n=1$, $a_1=1$ is a subsequence. Because the result is not integer when attributing other values to n (PT21).
- For $n=1$, $a_1=\frac{2}{2}=1$. Since this attributed value does not zero the denominator, a_1 is a subsequence of the given sequence (PT22).
- We can write down its subsequence by lessening the variable of the sequence by 1. If one writes down $n-1$ in place of , they get the subsequence $(a_{n-1})=(\frac{n}{2n-2})$ (PT24).
- For $n=6$, the term $a_6=\frac{7}{12}$ is a subsequence of the sequence (a_n). Because any term of a sequence is its subsequence (PT25, PT64).
- Range of the sequence (a_n) is $(1,\infty)$. As the range $(1,2)$ is within this range, it is a subsequence (PT26).
- For $n=1$, since the term $a_1=\frac{2}{2}=1$ is an element of the sequence (a_n), a_1 is a subsequence (PT27, PT28).
- Since $(a_n)=\left(\frac{n+1}{2n}\right)=\frac{1}{2}+\frac{1}{2n}$, the sequence $(\frac{1}{2n})$ is a subsequence of the sequence (a_n). Because they have the same domain (PT29, PT61, PT63, PT77, PT95).
- For $\mathrm{n}=1$ $\mathrm{a}_1=\frac{2}{2}=1$ and $\mathrm{a}_1\subset\mathrm{a}_\mathrm{n}$, it is a subsequence (PT 30, PT 50).
- Since a sequence of b_n cannot be written down meeting the condition of $\mathrm{a}_\mathrm{n}=\mathrm{b}_\mathrm{n}$, the given sequence has no subsequence (PT38).
- Because it is defined by $\mathrm{a}_\mathrm{n}\colon \mathrm{R}-\{0\}\to\mathrm{R}$, we have to write down a sequence which has the same domain and value set. As the sequence $\mathrm{b}_\mathrm{n}=\frac{3\mathrm{n}-1}{2\mathrm{n}}$ is defined by $\mathrm{R}-\{0\}\to\mathrm{R}$, it is a subsequence of the sequence a_n (PT67, PT68).
- A subsequence of the sequence $(\mathrm{a}_\mathrm{n})=(\frac{\mathrm{n}+1}{2\mathrm{n}})$ is $\mathrm{b}_\mathrm{n}=\frac{3\mathrm{n}-1}{\mathrm{n}+1}$. Because the value which makes the denominator of the sequence (a_n) 0 and the value which makes the denominator of the sequence (b_n) 0 are different (PT73).
- The sequence (b_n)=(2n) is a subsequence of the sequence (a_n). Because all elements of (b_n) is elements of (a_n) (PT75, PT80, PT89).
- If we expand the numerator and dominator of the sequence (a_n) by 2, we get the subsequence $\mathrm{b}_\mathrm{n}=\frac{2\mathrm{n}+2}{4\mathrm{n}}$ (PT81, PT88, PT98).
- Because it is undefined for $\mathrm{n}=0$, it has no subsequence (PT84).

Question 4

- If $b_k = a_k, \frac{3k+1}{2k+1} = \frac{2k+1}{k+1} \rightarrow 3k^2 + 3k + k + 1 = 4k^2 + 2k + 2k + 1$, it is $3 = 4$. Since k does not result in a rational number, it is not a rational number (PT5).
- Once the given two statements are equalized, it is a subsequence as there is equation (PT8).
- If we write down the equation statement b_n in place in the statement a_n, it is $\frac{2\left(\frac{3n+1}{2n+1}\right)+1}{\frac{3n+1}{2n+1}+1} = \frac{8n+3}{5n+2}$. As there is a sequence in this statement, the sequence (b_n) is a subsequence of the sequence (a_n) (PT15, PT55).
- It is its subsequence. Because when we place it as a_1, a_2, we get a smaller sequence than the previous one (PT19).
- As the increasing rate is the same between the terms, the sequence (b_n) is a subsequence of the sequence (a_n) (PT22, PT57).
- As the equation $\frac{3n+1}{2n+1} = \frac{2n+1}{n+1}$ for $n = 0$ is proven, it is its subsequence for $n = 0$ (PT30, PT37).
- It is its subsequence. Because the range of the sequence (a_n) covers the range of the sequence b_n (PT32, PT34).
- It is its subsequence as its terms follow each other like $\mathrm{a}_1 = \frac{3}{2}$, $\mathrm{b}_1 = \frac{4}{3}$ for $\mathrm{n} = 1$ (PT41, PT56, PT60, PT64).
- As it is not its subsequence, it should be $a_n > b_n$. If $\frac{2n+1}{n+1} > \frac{3n+1}{2n+1}$, it is $4n^2 + 8n + 1 > 3n^2 + 4n + 1$, $n^2 + 4n > 0$. As this inequation is proven for each positive number, the sequence b_n is a subsequence of the sequence a_n (PT61).
- It is its subsequence. Because the sequence b_n starts before the sequence a_n (AÖ62).
- It is a subsequence because they have the same domain (PT63).
- It is its subsequence. Because it is defined by the given statement (PT65).
- As the sequence (b_n) is achieved by adding n to the numerator and denominator of the sequence (a_n) , it is its subsequence (PT68).
- As one's numerator is equal to other's denominator, it is a subsequence (PT72).
- As the values which make the denominators of the sequences (b_n) and(a_n) zero are different from each other, the sequence (b_n) is a subsequence of the sequence (a_n) (PT73).
- It is its subsequence. Because both sequences are defined by $R^+ \rightarrow R^+$ (PT77).
- It is its subsequence. Because its terms are ordered and monotonous (PT90).
- Since, in the sequences $b_n = \frac{1}{2n+1} + 1$, $a_n = 1 + \frac{n}{n+1}$, constants and numerators are equal, and the denominators are bigger than the denominator of the sequence b_n, it is a subsequence of the sequence a_n (PT91).
- Since $b_n = \frac{1}{2n+1} + 1$, $a_n = 1 + \frac{n}{n+1}$, as the sequence a_n covers the sequence b_n it is its subsequence (PT93).

Incorrect-Blank	**Question 3**	• Subsequence of the sequence $(a_n) = (\frac{n+1}{2n})$ is $a_1 = 1, a_2 = \frac{3}{4} a_3 = \frac{4}{6} = \frac{2}{3}$ (PT1, PT2,PT45, PT46). • The given sequence has no subsequence (PT9). • The sequence $(\frac{3n+1}{2n})$ is a subsequence of the given sequence (PT17). • The sequence $2n + 2$ is a subsequence of the given sequence (PT35). • It is the sequence $(\frac{2n}{n+1})$ (PT41, PT58, PT59, PT72). • It is the sequence $(\frac{n-2}{n+1})$ (PT82, PT83).
	Question 4	• It is its subsequence (PT9, PT18, PT78, PT82, PT83, PT95).
Blank	**Question 3**	(PT3, PT6, PT18, PT23, PT32, PT33, PT39, PT44, PT47, PT51, PT52, PT54, PT56, PT57, PT76, PT79, PT86, PT99).
	Question 4	(PT2, PT3, PT6, PT23, PT33, PT39, PT44, PT47, PT51, PT52, PT54, PT59, PT76, PT86, PT99)

As for the distribution of the data on how the preservice teachers explained the concept of subsequence and what perceptions and concept images they had of the concept according to Table 4, 9.09% of the preservice teachers (with 11.11% providing incomplete and 88.89% incorrect reasons) defined the concept in the category of correct answer, 12.12% of them (with 100% providing correct reasons) defined it in the category of incomplete answer and 68.68% of them (with 20.59% providing correct, 16.18% incomplete and 63.23% incorrect reasons) defined it in the category of incorrect answer. In addition, 11.11% of the preservice teachers did not answer the question about the definition of the subsequence concept. Regarding the distribution of the answers given to the third and fourth operational questions about the concept of subsequence in the second part of the data collection instrument, 8.08% of the preservice teachers (with 62.5% providing incomplete, 12.5% incorrect and 25% no reasons) answered the third question in the category of correct answer whereas 72.72% (81.84% provided incorrect reasons and 18.06% provided no reason at all) answered it in the category of incorrect answer. Moreover, 18.18% of the preservice teachers did not make any attempt to solve this question. As for the fourth operational question, 52.52% of the preservice teachers (with 53.85% providing incomplete, 40.38% incorrect and 5.77% no reasons) answered it in the category of correct answer while 31.31% (83.87% provided incorrect reasons and 16.13% provided no reason at all) answered it in the category of incorrect answer. As for the citations from the answers given by the preservice teachers both to conceptual and operational questions about the concept of subsequence in Table 4, it can be argued that the preservice teachers had scientifically wrong perceptions and concept images of the concept.

4. Discussion

In this section, the findings were associated with the literature and discussed, and recommendations were made about the achieved results.

According to the findings about the concept of sequence, very few preservice teachers defined the concept correctly. Half of the preservice teachers who defined the concept correctly did it in coherence with the formal definition while the other half narrowed down the scope of the concept and defined it with overspecialization by stating, "…the domain should be the set of counting numbers $T = \{1, 2, 3, \ldots\}$, the value set should be the set of real numbers". Overspecialization refers to thinking of a concept, rule or principle in a narrower and more restricted mentality (Bingölbali & Özmantar, 2010). Furthermore, almost all of the preservice teachers defined the concept of sequence incorrectly, very few of them defined it incompletely.

Almost all of the preservice teachers who defined the concept incorrectly provided the following definitions which were incomplete about its attributes and indicated that they had misconceptions about the concept: "Terms ordered by a certain rule are called sequences."; "Given $Rule: 2n + 1 \rightarrow Term: 3, 5, 7, 9 \ldots;\ n \in N^+$, numbers denoted as a_n and that come together by a certain rule are called sequences"; "Group of arithmetically or geometrically increasing-decreasing positive integers by a certain pattern is called sequence."; "It is the number pattern that is infinite by certain rules and increasing rates"; In the statement, it is the pattern created by giving at least 1 to n."; "It should be a function."; "It is a function defined from a set to another."; "It is to give positive value in place of n in a function."; "For a statement to be a sequence, it should be $a_n \in N^+$ and $n > 0$."; "Function of which general term is a_n is called a function." It can be also understood from the following definitions that the other half of the preservice teachers who defined the concept incorrectly had perceptions and concept images not even close to the formal attributes of the subsequence: "Things following each other successively and connected to the previous and next one."; "It should be constantly increasing or decreasing."; "A set which has elements such as number, shape, etc. in a certain period or rule is called a sequence."; "Things that continue at equal intervals or in an order depending on a fixed rule and of which terms are integers."; "Numbers defined by real numbers and of which terms are ordered according to a certain arithmetic."; "Group of numbers which have a certain ratio or amount of increase among them is called sequence."; "It is the number pattern that is infinite by certain rules and increasing rates."; "Things that are finite or infinite according to a certain rule."; "Set of numbers that continue increasing or decreasing by a certain rule."; "If the result is always a positive integer for the integer values of an unknown in a statement, this statement is called a sequence."; "It is that all positive integers that are ordered denote a mutual equation, denominator should be different from zero."; "Sets of number between which the difference is fixed"; "Number relations with rules"; "Set of numbers which is defined by rational numbers and is infinite by a rule."

Regarding the findings on the operational questions (question 1 and question 2) which required using the concept of sequence, similarly, almost all of the preservice teachers answered the first question, and more than half of the preservice teachers answered the second question incorrectly by providing incorrect reasons or no reasons at all. The result obtained concerning the answers given to these two questions which required using the definition and attributes of sequence indicates that the preservice teachers had scientifically wrong perceptions and concept images of the sequence concept.

According to the findings about the concept of subsequence, few preservice teachers defined the concept correctly. On the other hand, almost all of the preservice teachers who defined the concept correctly reduced the concept of subsequence to the concept of subset, and therefore, provided incorrect reasons with the following statements: "If all terms of a sequence are also the elements of another sequence, such sequence is called subsequence."; "For example; a subsequence of the sequence $\{3, 5, 7, 9\}$ is $\{5, 7, \}$." It can be inferred from this result that the preservice teachers mistook the concepts of subsequence and subset and did not know under which circumstances a sequence would be a subsequence of another sequence.

Furthermore, about four fifth of the preservice teachers defined the concept of subsequence incorrectly or did not define it at all. About one fifth of the preservice teachers who defined the concept incorrectly provided correct or incomplete reasons, and more than half of them tried to explain it by providing the following definitions which are perceptions and concept images not coherent with the formal attributes of subsequence: "Derivatives of sequence"; "A set that help us achieve the term n. of a sequence easily"; "It is a concept involved by a general sequence in it."; "It must follow the general term."; "It is something like a subset."; "For a sequence to be a subsequence, it should be a constant sequence."; "It is to create a new sequence by reducing the general rule of a sequence."; "It is a sequence formed by elements of a sequence increasing at a certain rate."; "If a result that defines the sequence is achieved when writing down the terms given in a subsequence in place of the uncertainties given in a normal sequence."; "A series formed by the elements coherent with certain attributes of a sequence that is composed of numbers together is called a subsequence."; "The new sequence achieved by attributing certain values to $n$is called a subsequence."; "If the values attributed to $n$do not make the denominator of the sequence zero, this value is called subsequence."; "Sequence which helps define a sequence is called a subsequence."; "In the sequence(a_n), the sequence (a_{n-1}) obtained by lessening the variable by 1 is called subsequence."; "Each of a sequence's terms is called the subsequence of that sequence."; "If the domain of a function covers the domain of another function, it is a subsequence."; "It is the set formed by some elements of a function."; "Sequences of which domain covers its value set and its value set covers its domain are called subsequences."; It is the ordering of numbers from a_1 to a_n by means of formulas."; "Sequences of which domains share the same domain are called subsequences."; "Each subset of a function is called a subsequence."; "The new sequence obtained by increasing its variable by 1 is called subsequence."; For example, a subsequence of the sequence (a_n) is (a_{n+1})."; "The sequence of which domain is smaller than the domain of the general sequence is called subsequence."; "If the values that render the denominators undefined for the sequences $(a_n)$and $(b_n)$are different, these sequences are called each other's subsequence."; A sequence formed by multiplying the numerator and denominator of a sequence by the same number is called a subsequence."; "It is the case that a sequence follows another sequence monotonously."

Regarding the findings on the operational questions (question 3 and question 4) which required using the concept of subsequence, similarly, about three fourth of the preservice teachers answered the third question and about one third of the preservice teachers answered the fourth question incorrectly by providing incorrect reasons or no reasons at all. On the other hand, about half of the preservice teachers answered the fourth operational question about the concept of subsequence correctly although about all of them provided incomplete or incorrect reasons. The result obtained concerning the answers given to these two questions which required using the definition and attributes of subsequence indicates that the preservice teachers had scientifically wrong perceptions and concept images of the subsequence concept.

Possible reasons for these results achieved concerning the concepts of sequence and subsequence might be the epistemological, psychological and pedagogical obstacles in the instruction of these concepts (Soylu, Akgün, Dündar & İşleyen, 2011). Moreover, when teaching a concept, concentrating only on technical information and definitions and ignoring the relationship between that concept and others is another aspect of the instructional obstacles. In other words, presenting the students this information through rote learning and without having them contemplate on it might have caused them to develop such misconceptions and concept images. It is reported in the studies on difficulty indices regarding mathematical subjects in the literature that the unit of sequences and series takes the first place in the difficulty index (Durmuş, 2004; Tatar, Okur & Tuna, 2008) and students find it hard to comprehend the subjects in the unit of sequences and series (Akbayır, 2004; Alcock & Simpson, 2004; Alcock & Simpson, 2005; Akgün & Duru, 2007). Furthermore, Çiltaş & Işık (2012) observed in their study aiming to identify the mental models of preservice teachers in the subjects of sequences and series that the preservice teachers failed to do a drawing of an exemplary model which states the concepts of sequence and series properly and tried to explain these concepts and their features through examples. It can be argued that the results achieved on the concepts of sequence and subsequence in this study are in parallel with the results in the literature. Because it was concluded in this study, that the preservice teachers had wrong perceptions and concept images of both sequences and subsequences scientifically.

Due to the fact that the preservice teachers were incompetent at providing the definitions of the concepts of sequence and subsequence and could not provide a formal definition and considering that they did important part of learning about these concepts during high school, the approach of rote learning should be avoided when teaching these concepts and materials that will positively affect the retentive and interrelated learning of concepts such as concept maps and concept networks should be used. It was also observed that the preservice teachers answered the questions requiring the use of sequence and subsequence because they did not know about both concepts. In this sense, it is necessary to know that mathematics is not just about formulas and equations. When teaching the subjects, they should be presented in a holistic way, definitions and concepts should not be ignored, and importance should be attached to the establishment of interrelations between the concepts rather than adopting a formula-based study approach.

Acknowledgements or Notes

The author would like to express his gratitude to the referees for their valuable comments and suggestions. The author declares that there is no conflict of interests regarding the publication of this paper. A part of this study was presented in 12th National Science and Mathematics Education Congress (UFBMEK), Trabzon, Turkey, 2016.

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